

Adult Learning Mathematics – A Research Forum

Celebrating 25 years: A lot done, a lot more yet to do

Editors

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Preface

The seminal idea for this volume was planted in the spring of 2015 during the preparation of a paper on the metamorphosis of the organization Adults Learning Mathematics – a Research Forum over its first twenty years. During that process, the proceedings from the first twenty-one years of the organization were reviewed to evaluate if, and how, the organization had honed its focus over that time. In doing so, I was struck by the breadth and depth of the work that had been presented over the years.

The seed was nourished during the following academic year when, with the help of a research assistant, a database was constructed that catalogued each proceedings article by author, country of origin, and the primary and secondary themes that might typify each. The methodology was qualitative in nature and the themes that emerged fell into six broad categories: classroom methods and materials, affective factors, teachers, student voices, adult theory, and adults as citizens, workers, and parents. While each category plays an important role in advancing the mathematics education of adults, three communicated instructional innovations that revise traditional practices: classroom methods and materials, professional development of teachers, and the citizen as worker.

Fully grown, the seedling became a trilogy of monographs highlighting each of the three categories. The established goal was the celebration of the 25th anniversary of ALM as a mature force within the mathematics education community. And so, at ALM-23, an editorial team was formed to review the articles from each area, choosing a sampler of representative work from close to 700 proceedings articles published during the first 23 years of the organization. The trustees were asked to review the proposal for the trilogy and fund the publication to be released at the 25th annual conference to be held in London, England.

As the selection process progressed, it was decided that one tripartite volume was a more practical product and the editor(s) of each part agreed to select a representative sample of work that would total roughly 150 pages. The candidates were forwarded to the editor-in-chief who then set about locating the principal authors, contacting them to ask permission to reprint their articles. At times this was a daunting task as many people had changed institutions or retired over the elapsed period. A few could not be found or did not respond to repeated contact attempts but, in the end, the majority of the candidate authors were found and responded with an enthusiastic “Yes.”

The volume you now hold is the fruit of that process. It represents two years of planning and work by the editorial team. The actual construction of the manuscript was done with the help of a research assistant from Saint Peter’s University and a professional copy editor who has the great misfortune of being the sister of the editor-in-chief. Their contributions were invaluable and the manuscript would never have come to fruition without their patient labor during the download, conversion, and formatting phases of each chapter.

This volume represents only a sample of the high-quality work that lies published in the annual proceedings. The work of many people did not make the cut during the selection process due to the size restrictions of the volume and the intent to show as much breadth as possible in one tome. It is the hope of the editors that this book will only whet the reader’s appetite to explore the multitude of articles that space excluded. The database that informed the selection committee is available on the ALM website and can be sorted by author or themes if a particular article sparks the interest of the reader.

A note about editing – each chapter was formatted to a uniform size for the volume with font sizes imposed on titles and references. Other than that, the original papers were included as written. Authors had been asked if they wanted to amend their papers and a few did so.

Foreword

Diana Coben

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It is my great pleasure to write the Foreword to this Silver Anniversary Volume celebrating the first quarter century of the international forum bringing together researchers and practitioners in adult mathematics/numeracy teaching and learning in order to promote the learning of mathematics by adults: Adults Learning Mathematics – A Research Forum (ALM).

This collection brings together contributions from ALM researchers and practitioners over 25 years, yet it is only the tip of a much larger iceberg. To date almost 700 papers have been published in successive ALM Conference Proceedings and in peer-reviewed articles in the *Adults Learning Mathematics International Journal (ALMIJ)*. A selection of these papers is included here, chosen by the editors for their intrinsic interest and their contribution to the development of the field. The book is organized into five sections, reflecting some of the wide range of interests and diverse contexts explored over the years: classroom methods; classroom materials; institutional support; workplace and vocational education; and professional development. The chapters offer a smörgåsbord of delights for everyone with an interest in adults learning mathematics.

To put this collection into perspective it is necessary to say something about the organization Adults Learning Mathematics – A Research Forum (which I shall refer to by its acronym: ALM) and, more broadly, the field of research, practice and general interest regarding adults learning mathematics.

As a field of practice adults learning mathematics has existed for as long as adults have been learning and using mathematics, that is to say, for a very long time. Indeed, writing is believed to have originated around 9000 years ago as a method of recording adult activities which necessarily entail the use and learning of mathematics: keeping accounts and running an economy, activities which must logically predate the creation of written records (Schmandt-Besserat, 1992a, 1992b).

In terms of general interest, the twentieth century saw the publication of Lancelot Hogben's classic book *Mathematics for the Million* (Hogben, 1968), first published in 1936 for a general adult readership and since reprinted many times. Its enduring appeal is testimony to the fascination of the subject. Later in the twentieth century, John Allen Paulos' best-seller, *Innumeracy: Mathematical illiteracy and its consequences*, aimed to point out how much "innumeracy pervades both our private and our public lives" (Paulos, 1988, p. 5). In recent years there has been a burgeoning of mathematics-focused websites, many of them aimed at children learning mathematics and their teachers and parents but which may also attract other adults, as well as adult-focused websites such as the *Centre for the Popularisation of Mathematics* (Brown & Yates, 1989-) and the U.S. National Council of Teachers of Mathematics' *Math Forum for Parents and Concerned Citizens* (NCTM, 1994-2018).

As a field of serious research adults learning mathematics is much younger, dating from the twentieth century, with for example in the UK, Moore's (1957) pioneering PhD study, 'A Survey of the Use of Arithmetic in the Daily Life of Adults', followed more than two decades later by Brigid Sewell's (1981) report on adults' use of mathematics in daily life and Laurie Buxton's (1981) account of his work to help an otherwise competent head teacher overcome the crippling effects of mathematics anxiety. The 1960s and 1970s national adult literacy campaigns, for example, in Cuba (Abendroth, 2009) and the UK (Withnall, 1994), triggered a parallel interest in adult numeracy and adults learning mathematics more generally, although 'numeracy' has often been the poor relation of 'literacy' in

these contexts. Meanwhile, in the United States, Sheila Tobias' *Overcoming Math Anxiety* was originally published in 1978 (Tobias, 1993). She pointed out that "millions of adults are blocked from professional and personal opportunities because they fear or perform poorly in mathematics. Most of these adults are capable of learning more mathematics. Theirs... is not a failure of intellect, but a failure of nerve" (Tobias, 1993, p. 9). Meanwhile, Marilyn Frankenstein's *Relearning Mathematics: A Different Third R - Radical Math* aimed to empower adults who have been made to feel a failure in mathematics (Frankenstein, 1989).

Against this background, ALM sprang into being from the worldwide interest generated by my article 'What do we need to know? Issues in numeracy research' (Coben, 1992). I pointed out the differences between the cultures of, on the one hand, academic research and on the other hand adult numeracy and mathematics education, the perceived difficulty of mathematics as subject matter, the lack of relevant research and the lack of contact between teachers and other practitioners and researchers with an interest in adult mathematics education. I proposed that a forum was needed to remedy this situation and carry the field forward. Joan O'Hagan and I organised a one-day meeting at Fircroft College in Birmingham, UK, which was attended by researchers and practitioners from the UK and Europe, with many messages of support from around the world. From that meeting a group was formed to bring the international forum into being, with the first ALM conference (ALM1) held the following year, also at Fircroft College.

Since then annual ALM conferences have been held in various countries, each organised by a local organising team supported by the ALM Trustees and with bursaries wherever possible, with the aim of allowing those without financial support from their institutions or other source to take part. ALM conferences thus have a strong 'local' flavour of the country and region in which they take place. Having the opportunity to meet face-to-face once a year has been an important element in the development of ALM's 'family feeling' and relationships made and fostered through ALM have led to fruitful collaborations, as evidenced by the number of co-authored chapters in this volume. Successive ALM conference proceedings have been published since 1995, together with the creation of the ALM website¹, and, since 2005, the online peer-reviewed journal, *ALMIJ*²; these have made an enormous contribution to the development of the field. Furthermore, the work of ALM members has been invaluable in successive adult-focused working groups and topic study groups at the major international conference on mathematics education, the International Congress of Mathematics Education (ICME), held every Leap Year since 1969 (IMU, 1969-). For a small, voluntary, independent membership organisation³, ALM has consistently punched above its weight.

Meanwhile, interest in adults' mathematical capabilities has been fuelled from the mid-1990s on by successive international surveys of the skills of adults of working age undertaken under the auspices of the Organisation for Economic Cooperation and Development (OECD). The first such survey, the International Adult Literacy Survey (IALS) from 1996 (OECD & Statistics Canada, 2000; Statistics Canada & OECD, 2005) surveyed "quantitative literacy"⁴ amongst other cognate skills. Successor

¹ www.alm-online.net

² <http://www.alm-online.net/alm-publications/alm-journal/>

³ Adults Learning Mathematics - A Research Forum is a company limited by Guarantee, registered in England and Wales No. 3901346, and a Registered Charity No. 1079462.

⁴ Quantitative literacy is defined in IALS as "the knowledge and skills required to apply arithmetic operations, either alone or sequentially, to numbers embedded in printed materials, such as balancing a chequebook,

OECD surveys, the Adult Literacy and Lifeskills (ALL) Survey (OECD & Statistics Canada, 2011; Statistics Canada & OECD, 2005) and the Programme for the International Assessment of Adult Competencies (PIAAC) Survey of Adult Skills (OECD, 2013b, 2016) surveyed “numeracy⁵” alongside “literacy” (reading, not writing) and “problem-solving in technology-rich environments” (Tout & Gal, 2015).

I updated my original ‘What do we need to know?’ article a decade on from the original (Coben, 2003). In it I stressed the need to continue the conversation between researchers and practitioners and bring others into the conversation, including researchers in related fields, policy-makers, providers of adult mathematics education and others with a wider interest in adults learning mathematics as well as adults who were learning - and doing – mathematics. ALM members and friends continued to debate the nature of the field, including the appropriateness or otherwise of the term “field” - other metaphors included the moorland, the planetary system, the building site and the sky with clouds (Coben et al., 2003). These discussions reflected the diverse foci of adults learning mathematics, a feature noted in 2010 by Tine Wedege, who stated that: “the subject area encompasses formal adult mathematics education as well as adults’ informal mathematics learning in the communities of everyday practice” (Wedege, 2010, p. 13). ALM’s focus has always been “lifewide” (Edwards, 2006, p. 25). In 2015, Kathy Safford-Ramus charted the development of ALM through its literature in her 20-year survey of the adaptation of the organization to shifts in clientele, global economics and national policy fluctuations (Safford-Ramus, 2015).

ALM’s scope to date is indicated in the contributions to this book. They cover a host of topics, including adult numeracy, which is strongly represented in this collection, especially in the sections on classroom methods and materials and professional development. Bridging mathematics education, comprising pre-tertiary stand-alone ‘access’ and developmental courses and in-context support within tertiary education programmes, developed in Australasia and elsewhere from the 1970s on, with associated research undertaken by pioneering Bridging Mathematics educators (Taylor & Galligan, 2006). This work is represented here by chapters in the section entitled ‘Institutional Support’. Meanwhile, the renewed focus on literacy (including numeracy)⁶ signalled in the fourth annual *Education for All Global Monitoring Report* (UNESCO, 2006) has not so far resulted in a strong focus on adult numeracy and mathematics learning in what is often called development education practice or research (Rogers & Street, 2012). An honourable exception is the REFLECT programme, which aims to improve the meaningful participation of people in decisions that affect their lives – decisions requiring numeracy as well as literacy, as shown by Kate Newman’s (2003) chapter in this volume. Workplace and vocational education is strongly represented in the collection, with issues of

figuring out a tip, completing an order form or determining the amount of interest on a loan from an advertisement.” (OECD & Statistics Canada, 2000, p. x).

⁵ Numeracy is defined in PIAAC as “the ability to access, use, interpret and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life.” (OECD, 2013a, p. 34).

⁶ In this UNESCO Report, “‘literacy’ refers to a context-bound continuum of reading, writing and numeracy skills, acquired and developed through processes of learning and application, in schools and in other settings appropriate to youth and adults” (UNESCO, 2006, p. 30).

authenticity and ‘transfer’ of mathematical knowledge and understanding between the classroom, everyday life and the workplace explored by contributors to the book.

Finally, I honour with pride and pleasure all those ALM members, friends and colleagues who have contributed to the development of ALM and to the field (or moorland) of adults learning mathematics over the past 25 years. As Richard Edwards notes “The moorland is not an open backcloth upon which we wander, but is made in different ways [...] through our wanderings” (Edwards, 2006, p. 34). Accordingly, I also acknowledge the paths yet to be trodden. For example, the voices of adults learning mathematics are filtered through the educator or researcher reporting on their work: how could adult learners become part of the conversation so that they really are active participants in, rather than passive subjects of, research? Also, ALM’s internationalism remains confined to a relatively small number of countries, due at least in part to global disparities in income, education and access to international travel. Language, also, remains an issue. For a while conference abstracts were at least available in Spanish translation thanks to the efforts of a Spanish-speaking member, Juan Carlos Llorente, but the language of ALM has (so far at least) been English, reflecting the organisation’s conception and birth in England and the fact that English currently functions as the *lingua franca* of much international academic and educational discourse (O’Regan, 2014).

So, in conclusion, I end with the thought that “Research and scholarship is often thought of as a settling of things, but it is equally an unsettling” (Edwards, 2006, p. 33). I hope this volume may be such an unsettling: spurring readers on to create an increasingly inclusive future with and for adults learning and doing mathematics, wherever, whenever, however, and in whatever language they do so.

References

- Abendroth, M. (2009). *Rebel Literacy: Cuba's National Literacy Campaign and critical global citizenship*. Duluth, MN: Litwin Books.
- Brown, R., & Yates, M. (1989-). Centre for the Popularisation of Mathematics. Retrieved from <http://www.popmath.org.uk/centre/index.html>
- Buxton, L. (1981). *Do You Panic About Maths? Coping with maths anxiety*. London: Heinemann Educational.
- Coben, D. (1992). What do we need to know? Issues in numeracy research. *Adults Learning*, 4(1), 15-16.
- Coben, D. (2003). What do we know and what do we need to know? Researching adults learning mathematics – 10 years on. In J. Evans, P. Healy, D. Kaye, V. Seabright, & A. Tomlin (Eds.), *Policies and Practices for Adults Learning Mathematics: Opportunities and risks. Proceedings of the 9th International Conference of Adults Learning Mathematics (ALM9) - A Research Forum* (pp. 24-33). London: ALM and King's College, London.
- Coben, D., Leddy, E., Holder, D., Baker, E., Swain, J., & Tomlin, A. (2003). Teacher research into adult numeracy: Exploring the moorland (and the planetary system, the building site and the sky with clouds). In D. Coben (Ed.), *What Counts as Evidence for What Purposes in Adult Literacy, Numeracy and ESOL? Papers from the first NRDC international conference* (pp. 128-135). Nottingham: University of Nottingham and NRDC.
- Edwards, R. (2006). Beyond the moorland? Contextualising lifelong learning. *Studies in the Education of Adults*, 38(1), 25-36. doi:<https://doi.org/10.1080/02660830.2006.11661522>
- Frankenstein, M. (1989). *Relearning Mathematics: A Different Third R - Radical Maths*. London: Free Association Books.
- Hogben, L. (1968). *Mathematics for the Million: How to master the magic of numbers* (4th ed.). London & New York: W. W. Norton Inc.
- IMU. (1969-). International Congress on Mathematical Education (ICME). Retrieved from <https://www.mathunion.org/icmi/conferences/icme-international-congress-mathematical-education>
- Moore, N. A. (1957). *A Survey of the Use of Arithmetic in the Daily Life of Adults*. (PhD), Birmingham University, Birmingham.
- NCTM. (1994-2018). Math Forum: Parents and Concerned Citizens. Retrieved from <http://mathforum.org/parents.citizens.html>
- Newman, K. (2003). Communication and power. In J. Evans, P. Healy, D. Kaye, V. Seabright, & A. Tomlin (Eds.), *Policies and Practices for Adults Learning Mathematics: Opportunities and risks. Proceedings of the 9th International Conference of Adults Learning Mathematics (ALM9) - A Research Forum* (pp. 181-188). London: ALM and King's College, London.

- O'Regan, J. P. (2014). English as a Lingua Franca: An immanent critique. *Applied Linguistics*, 35(5), 533–552. doi:<https://doi.org/10.1093/applin/amt045>
- OECD. (2013a). *Literacy, Numeracy and Problem Solving in Technology-Rich Environments - Framework for the OECD Survey of Adult Skills. Revised version*. Retrieved from Paris: http://www.oecd.org/document/0/0,3746,en_2649_201185_49886336_1_1_1_1,00.html
- OECD. (2013b). *OECD Skills Outlook 2013. First Results From the Survey of Adult Skills*. Retrieved from Paris: http://www.oecd-ilibrary.org/education/oecd-skills-outlook-2013_9789264204256-en
- OECD. (2016). *Skills Matter. Further results from the Survey of Adult Skills*. Retrieved from Paris: <http://www.oecd.org/skills/skills-matter-9789264258051-en.htm>
- OECD, & Statistics Canada. (2000). *Literacy in the Information Age: Final report of the International Adult Literacy Survey*. Retrieved from Paris and Ottawa: <https://www.oecd.org/edu/skills-beyond-school/41529765.pdf>
- OECD, & Statistics Canada. (2011). *Literacy for Life: Further results from the Adult Literacy and Life Skills Survey. Second International ALL Report*. Retrieved from Paris: <http://www.statcan.gc.ca/pub/89-604-x/89-604-x2011001-eng.pdf>
- Paulos, J. A. (1988). *Innumeracy: Mathematical illiteracy and its consequences*. London: Viking Penguin.
- Rogers, A., & Street, B. (2012). *Adult Literacy and Development: Stories from the field*. Leicester: NIACE.
- Safford-Ramus, K. (2015). ALM as chameleon: Tracing the evolution of the organization through its literature – In A. Hector-Mason, L. Jarlskog, & D. Kaye (Eds.), *Opening our Mathematical Eyes. Proceedings of the 22nd International Conference of Adults Learning Mathematics – A research Forum. Hosted by American Institutes for Research Washington DC, July 12 to 15, 2015* (pp. 129-134). Washington DC: Adults Learning Mathematics (ALM) – A Research Forum.
- Schmandt-Besserat, D. (1992a). *Before Writing: A catalog of Near Eastern tokens* (Vol. 2). Austin, TX: University of Texas Press.
- Schmandt-Besserat, D. (1992b). *Before Writing: From counting to cuneiform* (Vol. 1). Austin, TX: University of Texas Press.
- Sewell, B. (1981). *Use of Mathematics by Adults in Daily Life*. Leicester: Advisory Council for Adult Continuing Education (ACACE).
- Statistics Canada, & OECD. (2005). *Learning a Living: First results of the Adult Literacy and Life Skills Survey*. Retrieved from Paris: <http://www.oecd.org/education/innovation-education/34867438.pdf>
- Taylor, J. A., & Galligan, L. (2006). Research into research on adults in Bridging Mathematics: The past, the present and the future. In M. Horne & B. Marr (Eds.), *Connecting Voices in Adult Mathematics and Numeracy: Practitioners, researchers and learners. Proceedings of the Adults Learning Mathematics (ALM) 12th Annual International Conference jointly with the Australian Council for Adult Literacy (ACAL) and in cooperation with the Australasian Bridging Mathematics Network (BMN) Melbourne, 3rd–7th July, 2005* (pp. 11-). Melbourne: Adults Learning Mathematics - A Research.
- Tobias, S. (1993). *Overcoming Math Anxiety* (Revised and expanded ed.). New York: W. W. Norton and Company.
- Tout, D., & Gal, I. (2015). Perspectives on numeracy: Reflections from international assessments. *ZDM Online*, 47(4), 691-706. doi:10.1007/s11858-015-0672-9
- UNESCO. (2006). *Literacy for Life*. Retrieved from Paris: <http://unesdoc.unesco.org/images/0014/001416/141639e.pdf>
- Wedge, T. (2010). The problem field of Adults Learning Mathematics. In G. Griffiths & D. Kaye (Eds.), *Numeracy Works for Life: Proceedings of the 16th International Conference of Adults Learning Mathematics – A Research Forum* (pp. 13-24). London: Adults Learning Mathematics (ALM) and LLU+, London South Bank University.
- Withnall, A. (1994). Literacy on the agenda: The origins of the adult literacy campaign in the United Kingdom. *Studies in the Education of Adults*, 26(1), 67-85.

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Classroom Methods

Workshop

What Constitutes Effective Practice in Adult Numeracy?

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This workshop focused on a project based at King's College London, 'A study of effective practice in inclusive adult numeracy teaching' which aimed to correlate a range of factors with learners' progress. A total of 472 learners in 47 classes participated in the study across the two academic years 2003/04 and 2004/05. The research took place in a range of settings in different geographical areas of England. The workshop focused on the question: what constitutes effective numeracy practice? We conclude that there is no 'one size fits all' approach to teaching adult numeracy – flexibility is the keynote.

Our project, 'A study of effective practice in inclusive adult numeracy teaching', was one of a suite of five projects initiated by the National Research and Development Centre for Adult Literacy and Numeracy (NRDC) and funded by DfES and European Social Fund, covering areas of numeracy, reading, writing, English as a Second Language (ESOL) and Information Communication and Technology (ICT). The projects built on the *What Works Study* for Adult ESL Literacy Students directed by Larry Condelli at the American Institutes for Research in Washington DC (Condelli & Voight, 1999) which investigated the range of practices in adult ESOL classrooms, the progress made by learners, and the correlations between the two.

This paper draws on the final report of the project (Coben et al, 2007); it was written and the workshop was presented by Diana Coben on behalf of and on the basis of work by the whole team. The numeracy study was based at King's College London (Coben et al., 2007). The project team consisted of Diana Coben and Margaret Brown (joint Principal Investigators), Valerie Rhodes, Jon Swain, Katerina Ananiadou and Peter Brown. The project aimed to investigate: a range of naturally occurring variation in teaching numeracy to learners in different settings; any correlations between different practices and learners' progress; and draw out the implications for teaching, teacher training and continuing professional

⁷ Margaret Brown, Valerie Rhodes and Peter Brown are from King's College, London, UK; Jon Swain and Katerina Ananiadou are from the Institute of Education, University of London, UK, as part of the National Research and Development Centre for Adult Literacy and Numeracy (NRDC).

development (CPD). We investigated a range of approaches to the teaching of numeracy to diverse adult learners in different settings, aiming to be as inclusive as possible. We tried to ascertain the progress learners made (or did not make), and the correlation between any progress or regress and the teaching approaches used. The project took place in two phases in the academic years 2003/04 and 2004/05, and involved 472 learners (of whom we have complete data on 250) and 34 teachers in 47 classes (17 in Phase 1, 2003/04, 30 in Phase 2, 2004/05); 30 of these classes were in further education (FE) Colleges (11 in classes for 16-19 year olds and 19 in classes for adults, including ESOL, ICT, etc.), 4 in adult or neighbourhood colleges, 4 in workplaces, 2 family numeracy classes, 2 JobCentre Plus classes, 2 prison groups, 1 Army course and 1 class run by a private training provider. The classes were located in clusters around six teacher-researchers based in North Lancashire, London, Gloucester and elsewhere in England. Adult numeracy classes in the study were very diverse in terms of the range of learner ages, abilities, dispositions, purposes and aspirations. Some classes had distinctive characteristics, for example, learners with language difficulties or with poor levels of motivation. The classes took place in different settings over different time intervals, at different times of day, and had different attendance patterns.

We used a mixture of quantitative and qualitative approaches: 250 learners were assessed using a test we developed from selected items drawn from the national *Skills for Life* survey (DfES, 2003) and 243 completed attitude surveys, in both cases at two time-points in order to assess their progress. Background information was collected on teachers and learners, and semi-structured interviews were carried out with 33 teachers and 112 learners.

Many learners contrasted their (generally) more negative experience of learning mathematics at school with the positive experience of learning numeracy as an adult: over 90% of learners interviewed expressed a high level of satisfaction with their course and their teacher, and there was overall a 61% retention rate. Learners recognised that the relationship between the teacher and effective learning was critical; it was important for teachers to develop good relationships with learners and to treat, and respect them, as adults. Classroom observation indicated a high level of mutual respect.

The teachers were generally experienced and well-qualified, with many having previously taught mathematics in primary and secondary schools. The teachers' subject knowledge was generally found to be adequate. Teachers valued 'flexibility' as a key feature of effective practice. Some believed that the diversity of learners, together with the range of possible types of activity to meet different mathematical aims, meant that no one pattern of lesson activity or organizational method was optimum. Knowledge and careful preparation were felt to be important in dealing with learner diversity. We asked teachers to give us their views about effective teaching and they said that teachers need to: be flexible; plan well, with lots of variety of approach; help learners articulate what they understand; and help learners make connections.

We observed a wide range of different teaching approaches, including the way classes were organized and how resources were used. Whole class and individual work predominated, with teachers demonstrating procedures and learners working through worksheets. Most teachers gave clear explanations, an asset much valued by learners, and also broke work down into smaller steps and gave feedback to learners about their work. Learners were usually highly engaged and teachers were enthusiastic, and generous in giving praise.

We saw few higher order questions, little pair or group work, and little use of practical resources or ICT, although activities were often varied. Teachers stressed the importance of active learner participation in discussion, in order to develop conceptual links underpinning skills, to develop language skills and to enhance social relationships. There were problems

finding an assessment instrument which was both short and appropriate to sensitively and validly measure progress across such a diverse learner group. Consequently, the results do not necessarily do justice to the learning which was observed in classrooms.

Taking all the classes together, learners made significant progress in terms of test score over the duration of the numeracy courses, with an average gain of 9%. However, there was a very wide range of average gains between different classes. While eight of the 45 classes made average gains of over 15%, a few had lower average scores at the end of the course. There was very little association between the size of gains and types of learner, except that those with no previous qualifications and those wanting to become more confident, tended to make larger gains. There was no association between gains and teacher characteristics, and none between the number of teaching hours and learners' progress.

There were very few significant correlations between progress made and the extent of different classroom approaches used. The only significant positive correlations were with procedural teaching to the whole class using examples on a whiteboard (interactive or traditional). The strongest negative correlations with attainment gains tended to be with the use of resources and with a large proportion of individual work. However, none of the relations were strong and there were many counter examples; for example the use of whole class teaching and whiteboards was a distinguishing feature of the classes with the lowest as well as the highest gains. Thus it seems likely that even where there were significant correlations these factors may well not have been causative of high or low performance, but merely associated for other reasons with certain types of class.

Teaching approaches and typologies were also identified with teachers described in terms of their balance between transmission, connectionist and constructivist approaches:

- *Connectionist* teaching is concerned to develop the conceptual understanding of learners and frequently makes connections to other areas of math including moving between symbolic, visual and verbal representations.
- *Transmission* teaching is principally concerned with mastery of skills. Mathematics is seen as a series of discrete packages to be taught in small steps with an emphasis on procedures rather than conceptual understanding.
- Using the *constructivist/scaffolder* style, the teacher works alongside learners, co-constructing concepts, asking questions. They provide a series of activities to help raise learners' thinking and conceptual understanding to a higher level.

Again, there were no significant correlations with gains made; while the two teachers of classes where learners made the most progress (over 30%) used a combination of constructivist and connectionist approach, there were also transmission-style teachers with very high gains.

Learners' attitudes generally became slightly more positive at the end of the course. The changes tended to be greatest for older people, and related particularly to a perception of numeracy as less difficult. Qualitative data suggested that once learners are able to overcome their initial anxieties, both about the course and about mathematics, and when blocks and barriers are overcome, numeracy courses can have a significant and positive effect on their identities both in general, in terms of improving people's level of confidence and self-esteem, and specifically, in terms of their identity as people who can do mathematics. Some learners have been able to develop new aspirations and form new dispositions to learning. No significant correlations were identified between changes in learners' attitudes and the approaches teachers used, or any particular set of classroom characteristics.

The heterogeneous nature of adult numeracy teaching, and the number of variables amongst teachers and learners, makes it difficult to produce findings that can be generalized across the whole sector. Factors, such as learners' motivations and purposes for attending the course, their aspirations, their abilities and dispositions towards numeracy, their socio-cultural background and experiences outside the classroom, may be more influential than anything the teacher does. It may also be that approaches which work well with some learners in some settings may not work well in other contexts.

We came to feel that the underlying assumption in the research design - that the greater the rises in attainment between two points in time the better the teaching - was flawed. We found classes where researchers with many years of experience in education thought that the teaching was good but learner progress was weak, and some apparently poor teaching where progress was strong. It was thus difficult in many cases to relate the observed quality of the teaching and the measured increase in learning.

How should we define effective numeracy practice?

The question of what counts as effective practice in adult numeracy education is both complex and straightforward. It is straightforward insofar as adult numeracy provision is inspected according to standards set out in the Common Inspection Framework (ALI/OFSTED, 2001) by the Office for Standards in Education (OFSTED) and the Adult Learning Inspectorate (ALI).⁸ However, little is known about effective practice in adult numeracy education from a research perspective and the tendency has been for numeracy to be overshadowed by literacy in official reports, including Inspection reports, so that information about adult numeracy is often impossible to disaggregate from that on adult literacy. The relationship between effective teaching and successful learning in adult numeracy has yet to be established; this study represents a step towards this goal.

An example of effective practice

In this section we present a detailed description of one teacher's numeracy class as an example of what the research team considered to be 'effective' practice.

The class took place in a FE college in London, and ran for two hours on one evening each week. It consisted of about 14 learners working at Entry level 3 to Level 1, and the age range was 18–60 plus. The teacher was an experienced numeracy tutor; she has a PGCE in secondary education; her highest mathematics qualification is at 'A' level; and, at the time of the observation, she had been teaching numeracy for 21 years.

In terms of assessment results, this class achieved an average gain of over 30% with many learners making exceptional progress. In addition, learners' enthusiasm towards numeracy

⁸ A new single organisation has been created by merging the activities of OFSTED with the children's social care remit of the Commission for Social Care Inspection (CSCI), the Children and Family Court Advisory and Support Service (CAFCASS) inspection remit of HM Inspectorate of Court Administration (HMICA) and the Adult Learning Inspectorate (ALI) (http://www.ali.gov.uk/News/Talisman/issue_48/Strategy+board+appointed.htm, accessed 17 February 2006).

was noticeable both from the attitude surveys and the class observations. This was achieved by using a predominantly connectionist and constructivist approach which emphasized conceptual understanding rather than routine procedures. Mathematics was conceived as a network where the teachers and learners construct concepts together.

The teacher created a non-threatening atmosphere and learners' misconceptions were used as examples to discuss with the whole group. Learners were encouraged to discuss problems and concepts both between themselves and with the teacher, building a strong collaborative culture. Numeracy learning was viewed as social activity in which people took ownership of what they were doing, and where understanding was formed through discussion. A variety of group, individual and whole class teaching was used; however, even when learning was organized on an individual basis the learners were still encouraged to discuss problems and help each other, which helped to develop a greater understanding. The class was taught in an open style, which allowed higher order, diagnostic questioning that uncovered learners' thinking.

A range of materials and teaching resources were used, which ranged from worksheets to games, and activities including whole-class role-play. Calculators were freely available. The teacher used problem-solving activities which challenged the learners. She was also flexible and able to change direction to respond to the learners' needs.

Figure 1 below is an extract from a researcher's observation sheet. It is a narrative account that was filled in contemporaneously, and attempts to describe what was going on. For the purposes of this example, and in the interests of space, we are only taking the first hour of the session. The comments that appear in blue italics are retrospective and were not included on the original sheet; they are not intended to be exhaustive, but provide characteristics of what we believe constitute 'effective' practice. The narrative also shows how complex teaching is; how many decisions teachers have to make; and how hard they often have to work. The names of the teacher and learners have been changed.

Time	Content/focus of the session
7.00 pm	<p>Topic: Percentages</p> <p>Becky (BH) held up an individual mini-whiteboard (A4 white laminated card) with '%' hand-drawn on it. She asked the learners to tell her what it was and what it meant. In response to one learner saying it looked like a division sign she drew a division sign \div on the main (fixed) whiteboard and initiated a discussion about the relationship between percentages, fractions and decimals.</p> <p>She asks learners to call out different percentages and she writes them up on the main whiteboard. BH: "10% means divide by 10"</p>
	<p><i>The teacher asks open questions; does not give answers; initiates discussion, looks at relationships and connections and assesses learners' prior knowledge. The teaching is interactive and the teacher reinforces understanding.</i></p>
7.15 pm	<p>BH gave learners small cards with statements on 2 lines (e.g., I have 76. Who has 10% of £6500?) Learners have to read out their questions and answer if they have the right answer, otherwise keep quiet. BH: "If your neighbour is quiet they may be asleep, so you can look at your neighbour's card." At the end Becky confirmed to the class that they knew 10%.</p> <p>BH (having drawn on small whiteboard): "10% of 30. So what's 5%?" "So what's 30%?" "If I wanted 90% of 500? Greg says 'take off 10%'". BH asks for a number and Greg says "300": "50% of 300? What's 75%? Half is 50%, then halve that and add it to the 150. Notice we're talking about a half and a quarter". Learners call out the answers; Becky writes on large whiteboard. BH: "Can you see a pattern? What's 55% of 300? You can do it however</p>

8.00 pm	<p>you like.” Learners hold up their whiteboard cards as they do it. They ask each other what they’ve got. Becky helps one man (Moji). She asks (re 55% of 300) “What would be an easy percentage?” Moji: “50%”. BH “Sandra: tell Moji what to do” (she does). “One way is to use what you know here and here” (shows examples on main whiteboard).</p> <p>BH points out there are many different ways of doing percentages. In some situations one method is good, in others, another method might be better. “17 and a half percent. If you think you know what to do, write it down on your board. 10%; 5%; 2 and a half%: What have they done here? Can you work out 17 and a half% of 300?” (shows it written on mini whiteboard with figures above each other) Learners work out each element and then add them together. BH asks why they’ve added them. Learners explain. BH: “That’s VAT. It’s not too bad. Now try it with my nice number (400). Just to see how comfortable you are with it, I’ll give you an even nicer number (800).” Sandra gives the right answer BH: “Did you do that in your head: that’s impressive. So 17 and a half% doesn’t hold any threats for you. How about 63%? How will I break that down?” (learners call out different ways of breaking down 63%). BH: “Distinguish between ones you can do in your head and more tricky ones – you’d use a calculator for those.”</p> <p>BH: “Let’s try 63% of £800”. She goes around the room (using the space in the middle) helping learners as appropriate, e.g., not lining numbers up: BH: “There’s a terribly dangerous thing happening to everyone in the room and it’s all my fault! Karen, let me show what you did”: she writes 400 wrongly aligned with the other numbers to be added. BH: “Be careful that you always find percentages of the same number (800). Always refer back to the number you’re finding the percentage of.” BH: “Will 63% be more than half or less than half? Always think about doing a check. There are different ways of checking. We can learn some of those as we go along.” BH: (writing on whiteboard) when you see 25% what does it mean? ‘a quarter’, 75% three quarters; 33 and a third ‘a third’.</p>
	<p><i>The teacher uses interactive games and asks questions. She builds on, and uses, learners’ strategies, points out that there are many different strategies that can be used, highlights that some may be better than others, and shows learners which ones to use. The teacher is, again, getting learners to look for patterns. The learners work collaboratively; some assume a teaching role and explain strategies to each other. The teacher breaks maths down and works through examples. She points out that there are different ways of solving problems. The teacher assesses different ways of working and asks learners to justify what they’ve done. She breaks maths down using learners’ own methods, and encourages mental calculation. She gives praise and there is appropriate use of technology. The teacher monitors learning and identifies learners’ misconceptions. She emphasises need for checking and reinforces concepts learned with whole group.</i></p>

Figure 1: A narrative account of a numeracy session from a researcher’s observations sheet

Conclusions

It has proved very difficult to find clear associations between either teaching approaches or classroom characteristics and changes in learning or attitudes. The clearest positive correlations are with procedural teaching to the whole class using examples on a whiteboard (interactive or traditional). The strongest negative correlations with attainment gains tended to be with the use of resources, and with a large amount of individual work.

However, these characteristics do not necessarily define effective, or ineffective, teaching. Most of the factors observed to a greater extent in the classes with the highest gains were also observed to a greater extent in the classes with the lowest gains, which shows that they do not necessarily have a causal effect on class gains. Conversely, there is considerable variety in teaching approaches and classroom characteristics among both the highest and lowest

performing classes. An example is given above of part of a lesson with one of the highest attaining groups in the study. Although this style of teaching would meet current perceptions of good practice, it does not exemplify all those characteristics which correlate best with high average gains and it includes some features which correlate with low gains. It was difficult in many cases to relate the observed quality of the teaching and the measured increase in learning and we are unable on the basis of correlative data to recommend practice. Learner, rather than teacher factors seemed to be crucial in the progress made. The effect of teaching practice was dominated by other factors. Teachers' subject-specific pedagogical knowledge was found to be important in enabling them to be flexible in their teaching.

There would not seem to be clear implications for practice. However, for teacher education and CPD teachers not only need to have a firm grasp of subject and pedagogical knowledge, but also of subject-specific pedagogical knowledge (in this case of numeracy). This will enable them to be flexible in their approaches and to cater for the wide range of diversity of learners and provision in adult numeracy. The focus for research might be towards more work on learners, since learner factors seemed to be crucial in the progress made. This might include work on initial and changing identities with respect to mathematics and education. Above all, policy makers must accommodate the diversity and complexity of adult numeracy education – there is no 'one size fits all' of effective practice in adult numeracy teaching.

References

- ALI/OFSTED. (2001). The Common Inspection Framework for Inspecting Post-16 Education and Training. from <http://www.ofsted.gov.uk/public/docs01/cif.pdf>
- Coben, D., Brown, M., Rhodes, V., Swain, J., Ananiadou, K., Brown, P., Ashton, J., Holder, D., Lowe, S., Magee, C., Nieduszynska, S. & Storey, V. (2007). *Effective Practice: Numeracy*. London: NRDC.
- Condelli, L., & Voight, J. (1999). *Report on the Pilot Test of Learner Assessment for the What Works Study for Adult ESL Literacy Learners* (Internal Project Document). Washington, DC: US Department of Education, Planning and Evaluation Service.
- DfES. (2003). *The Skills for Life Survey: A national needs and impact survey of literacy, numeracy and ICT skills* (Research Report No. RR490). London: Department for Education and Skills.

Simulating Outside-the-Classroom Maths with In-Class Word Problems⁹

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Abstract

Not all word problems are designed to simulate real-life situations, but for those that are, a framework proposed by Dr. Torulf Palm can serve as a guide to developing authentic student tasks and assessments, at least as authentic as possible in the classroom. For adult students, given their life experiences and educational needs, that authenticity may be particularly important. The framework's benefits are explored.

Key words: simulation, mathematics, real life

Introduction

Math students often ask, 'When am I ever going to use this in real life'? That is a valid question, as students are interested in learning what will be relevant and helpful to them. However, word problems are often contrived and not real-life. While not all word problems are designed to simulate real life situations, it is possible to develop and evaluate word problems in terms of how closely they simulate an actual situation that students will encounter outside the classroom. This article will introduce a framework for doing just that. First, the idea of simulation will be discussed. This is followed by a framework proposed by Dr. Torulf Palm. Finally, benefits of the framework are noted as well as issues that are raised for practitioners.

Simulation

Flight Simulation Criteria

It is first necessary to talk a little about simulation. An interesting way to view simulation is by using an example of flight simulators used for training pilots. The International Civil Aviation Organization (ICAO) has a set of criteria for the qualification of flight simulation training devices (ICAO, 2009). Note how specific the criteria are for just the cockpit/flight deck layout and structure:

An enclosed, full scale replica of the cockpit/flight deck of the aeroplane being simulated including all: structure and panels; primary and secondary flight controls; engine and propeller controls, as applicable; equipment and systems with associated controls and observable indicators; circuit breakers; flight instruments; navigation, communications and

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similar use equipment; caution and warning systems and emergency equipment. The tactile feel, technique, effort, travel and direction required to manipulate the preceding, as applicable, should replicate those in the aeroplane. (p. II-App A-3)

For sound cues the criterion is that the simulator must have “significant sounds perceptible to the flight crew during flight operations to support the approved use” and with “comparable engine, airframe and environmental sounds” (ICAO, 2009, p. II-App A-19).

The atmosphere and weather section of the environment simulation includes a “fully integrated dynamic environment simulation including a representative atmosphere with weather effects to support the approved use” (ICAO, 2009, p. II-App A-45) and also:

The environment should be synchronised with appropriate aeroplane and simulation features to provide integrity. Environment simulation should include thunderstorms, windshear, turbulence, microbursts and appropriate types of precipitation. (ICAO, 2009, p. II-App A-45)

As we would hope, the criteria are very specific and we would want our pilots to have that level of simulation during training. This is especially true for simulating emergencies which would be difficult to train for while actually flying.

What level of simulation is necessary for a mathematics situation? The framework below contains a specific list of criteria. The degree to which they are followed depends on the extent that we want word problems to look like outside-the-classroom problems.

Mathematics Simulation Criteria

Palm (2006) proposes a framework that can be used when developing realistic word problems. It can also be used to evaluate the extent to which existing word problems are real-life. Table 1 is adapted from Palm’s work and contains all of the criteria in a summarized list. The reader is referred to Palm’s article for complete definitions and explanations.

Table 1. Framework for word problems as simulations of real-world situations.		
Criteria (aspects)		Defined as the degree to which:
Event	Situation	a student has a reasonable chance of encountering the described task outside of the classroom
	Question	the question in the school task is concordant with the actual event
Information / Data	Availability	the information available in the task matches the information available in the actual event
	Realism	the information in the described task is realistic relative to the actual event outside of the classroom
	Specificity	the specificity of information in the simulated task matches that of the out-of-classroom event
Presentation	Mode	the method of information delivery in the task is similar to the actual event (for example, oral vs. written or through diagrams or tables)

Table 1. Framework for word problems as simulations of real-world situations.		
	Language use	the language used does not detract from students using the same mathematics that they would in the actual event
Solution strategies	Availability	relevant solutions strategies, available to students, match those available outside of the classroom
	Experienced plausibility	the strategies experienced are plausible for solving the school task, as compared with the actual event
Circumstances	Availability of external tools	the tools available for the actual event are available in the classroom task (e.g., calculator, software)
	Guidance	explicit or implicit hints are available in the classroom task as compared with cues in the actual event
	Consultation & collaboration	there is a match, between the classroom task and the actual event, in the amount of help that is available from others
	Discussion opportunities	the student can ask questions and discuss the task in a manner similar to an actual event
	Time	time restrictions on solving are consistent between the classroom task and the actual event
	Consequences	there is similarity in the pressures and motivations of solving the task/event, based on success or failure
Solution requirements		judgments on the validity of solutions/methods are in concordance with the actual event
Purpose	Figurative context	the purpose of completing the task is in concordance with the event
	Social context	the purpose within the social context of the school task matches the purpose of the out-of-school event
Note: Adapted from Palm, T. (2006). Word problems as simulations of world situations: A proposed framework. <i>For the Learning of Mathematics</i> , 26(1), pp. 42-47.		

As can be seen in the table, the framework is quite comprehensive with 18 criteria in seven different areas. The framework, or criteria, can be useful in two ways: designing word problems and evaluating existing problems.

Designing word problems

Here is how the criteria could be used in the instructional design process. Applicable criteria from the framework are noted in brackets:

Analysis:

- What is the learning outcome?
- Is a real-world simulation appropriate and to what degree?

- What are the relevant solution strategies available to the students and do they match the expected learning outcome? [Solution strategies – availability and plausibility]

Problem Design:

- Pick a situation and a corresponding question that a student has a reasonable chance of encountering outside of the classroom. [Event – situation and question]
- Decide what information will be available to the student and the degree of concordance with the real-world situation. [Information/Data – availability, realism, and specificity]
- Describe the problem context in a way that matches the degree of concordance desired and is appropriate for the student’s level of language proficiency. [Presentation – mode and language use] State or indicate a clear purpose for completing the task. [Purpose – figurative and social context]
- Provide any hints or guidance that may be appropriate. [Circumstances – guidance]
- Design any consequences, if desired, that would match the real-world situation. [Circumstances – consequences]

Implementation Design:

Decide on the tools that will be available to the student. [Circumstances – external tools]

Determine the appropriate amount of time the student will have to solve the problem. [Circumstances – time]

Determine the extent of any degree of collaboration or discussion that will be allowed during the problem solving time. [Circumstances – consultation, collaboration, and discussion]

Evaluation Design:

- Considering the analysis questions above, decide how the student’s solution will be assessed. [Solution requirements]

Evaluating word problems

Palm and Burman (2004) used certain aspects of the framework to compare the realism of national assessments in mathematics for two countries. Their stated purpose “was to describe in what way and to what extent the ‘applied’ tasks in the assessments are in concordance with task situations in real life beyond school” (p. 5). Not all of the 18 criteria were used because some were not that useful or applicable for a large-scale assessment (for example, the criteria of *consultation and collaboration*).

Primarily, two classification categories were used by the researchers. These categories “correspond to whether the tasks were judged as simulating the specific category to a reasonable degree (Category 1) or not (Category 2)” (p. 8). Three categories were used for *specificity of information/data* where a judgment could be made that the word problem partly simulates real-life.

I believe that three categories are possible in many of the criteria. I have taken a few of what I consider to be the primary criteria and developed a rubric for which evaluation may be based on three categories of concordance. The rubric is shown in Table 2. The rubric has high, medium, and low concordance between a mathematical word problem and the out-of-classroom experience it is designed to simulate. As an example, for the *event* criterion, low

concordance refers to little or no chance that the student will experience this situation or have the same mathematical question. The *medium* classification allows for some chance that the student may one day be in that situation. The *high* classification allows for a 75% or higher probability.

Table 2. Rubric for analyzing word problems as simulating real-world mathematical situations			
Criteria	Quality of concordance		
	High	Medium	Low
Event	There is a 75% chance or higher that the student will experience this situation or have the same question.	There is some chance, between 75% and 25% that the student will experience this situation or have the same question.	There is little to no chance that the student will experience this situation or have the same question.
Purpose	The purpose of the task in the word problem is explicitly clear and matches the actual situation.	The purpose is somewhat clear and reasonably matches the actual situation.	The purpose is vague and unclear.
Information availability	The relevant information is the same as is accessible in the actual event, including whether or not it is explicitly stated or has to be retrieved.	The information is similar to what is found in the actual situation but is somewhat simplified or is missing some normally available data.	The relevant information in the word problem is clearly not the same as would be available in the actual event.
Information realism	The context of the word problem is specific and the data are realistic according to what one might actually see.	The context is not specific but the subjects/objects of the mathematical treatment are specific. The data is reasonably realistic.	The context and subjects/objects are not specific and the data is not realistic.
Guidance	The same guidance is available that would exist in the event.	The word problem contains one minor cue or hint.	The word problem contains a major cue or multiple hints.
External tools	Students are able to use the same tools (calculator, map, computer, etc.), if any, as in the actual situation.	Students are not able to use any tools but are given outputs of those tools.	There is a complete mismatch in the tools available.

Benefits

A benefit of the above framework, as shown by Palm and Burman (2004), is to conduct research on concordance with real life. They stated: “The tools of analysis, partly used in this paper, may function as an instrument for validation of the links between school tasks and real life task situations beyond school” (p. 30). Also, this operationalizing of real-life concordance provides other possibilities such as checking the correlation of student performance on assessment tasks with levels of concordance.

An additional benefit is to help curriculum designers, textbook authors, and practitioners think more deeply about “application” problems. All too often there is an assumption that contrived word problems can help students in solving real-life problems. If there is a concern about students actually being able to solve certain types of real-life problems, then we want to simulate those problems as closely as possible.

Issues

I will conclude with a few thoughts about several issues that must be considered with simulating word problems. These are described briefly below.

What is realistic?

If the goal is to have realistic problems, it may be interesting to consider a larger view of realism. Van den Heuvel-Panhuizen (2005) notes that even a fairy tale word problem may be considered realistic:

However, it must be acknowledged that the name Realistic Mathematics Education is somewhat confusing in this respect. This all has to do with the Dutch verb zich REALISE-ren that means to imagine. This implies that it is not authenticity as such, but the emphasis on making something real in your mind that gave RME its name. For the problems presented to the students, this means that the context can be one from the real world, but this is not always necessary. The fantasy world of fairy tales and even the formal world of mathematics can provide suitable contexts for a problem, as long as they are real in the students' minds and they can experience them as real for themselves. (p. 2)

Are all maths relevant for simulation?

It seems clear that basic arithmetic and pre-algebra can be made very relevant in terms of simulating potential events that students may encounter. But, the intermediate algebra level is perhaps not so easy to find real-world applications that are ripe for simulation. Palm and Burman (2004) found that the tasks in the lower courses in Swedish and Finnish school national assessments were more in concordance with actual events than the higher courses:

The high proportion of applied tasks in the tests included in this study could serve several of the purposes outlined in the introduction, such as providing good opportunities for students to experience strong links between school mathematics and real life beyond school. This is especially true for the tests used in the lower courses, which display a higher proportion of applied tasks than do the tests for the higher courses. (p. 31)

It is not desirable or necessary for all word problems, at any level, to be based on an actual event and simulated in the classroom. This leads to the next point about intent.

What is the intent?

It is very important that curriculum designers and teachers articulate the intent of tasks that students are expected to do. One of the decisions that has to be made is to decide if using a word problem in the classroom is (a) to add some relevance and interest or (b) to simulate an event that students have a good chance of encountering outside the classroom. If it is the latter, then the framework and criteria can serve as a help to developing high concordance with the actual event.

One interesting way to think about intent is to consider the concepts of horizontal and vertical mathematization. Van den Heuvel-Panhuizen (2010) interprets the earlier work of Treffers and describes vertical mathematization as “moving within the world of mathematics” while horizontal is “going from the world of real-life into the world of mathematics” (p. 4). I believe that some curriculum designers and teachers are ambiguous about this intent.

Examples of not considering intent and/or not communicating intent accurately are particularly found in intermediate algebra textbooks for developmental/remedial, tertiary prep students in the United States. The introductory information of one textbook has an applications index intended to show the relevance of this edition: “Real-world and real-data applications have been thoroughly updated and many new applications are included” (Martin-Gay, 2013, p. xvii). The index has 24 categories and over 450 sub-categories of real-world applications. Sounds great. However, the promise of real-world problems turns into contrived word problems that are more concerned with vertical mathematization. I looked at the list and picked out a problem type that sounded like it had a good chance of being real-world. In the category “home improvement” I picked the sub-category of “dimensions of a room.” I was directed to page 484 which had the following problem: “The area of a square room is 225 square feet. Find the dimensions of the room.” I cannot think of one possible reason why anyone would know the area but not the dimensions of a square room. My fear is that when we say a problem is real-life and it clearly is not, this will breed cynicism in students. This is especially important for adults who have some life experience and can clearly see when a word problem is contrived. As stated by Palm and Burman (2004):

If the irrelevance of the question is obvious to the students this may instead facilitate the forming of the belief some students have (e.g. Palm, 2002) that solving applied tasks is a game, with rules not necessarily consistent with the rules of real life problem solving. (p. 32)

Transfer of learning

Finally, a concluding thought on transfer of learning. I would not fly in a plane if it was the pilot’s first airborne flight after only training on a flight simulator. Even if the simulator was certified by ICAO to meet all of the necessary qualifications, the prospect that the pilot could successfully fly an actual airplane would be in doubt. If our goal is to simulate an event that students may experience the future, we need to realize the limitations of the classroom in simulating situations, regardless of having high concordance between a mathematical task and the actual event. As stated by Palm (2006):

It is not possible to simulate all aspects involved in a situation in real life and consequently it is not possible to simulate out-of-school situations in such a way that the conditions for the solving of the task will be exactly the same in the school situation.

References

- International Civil Aviation Organization (ICAO) (2009). *Manual of criteria for the qualification of flight simulation training devices: Volume 1—Aeroplanes* (Doc 9625, AN/938, 3rd ed.). Retrieved from mtc.gov.kz/images/stories/contents/85_Doc%209625.pdf
- Palm, T. (2006). Word problems as simulations of real-world situations: A proposed framework. *For the Learning of Mathematics*, 26(1), pp. 42-47.
- Palm, T. & Burman, L. (2004). Reality in mathematics assessment: An analysis of task-reality concordance in Finnish and Swedish national assessments. *Nordic Studies in Mathematics Education* 9.3, 1-33.
- Martin-Gay, E. (2013). *Intermediate Algebra* (6th ed.). Boston, MA: Pearson Education, Inc.
- van den Heuvel-Panhuizen, M. (2005). The role of contexts in assessment problems in mathematics. *For the Learning of Mathematics*, 2(2), 2-9.
- van den Heuvel-Panhuizen, M. (2010). Reform under attack--Forty years of working on better mathematics education thrown on the scrapheap? No way! *Mathematics Education Research Group of Australasia*.

Small-scale Educational Action Research Project: Assisting Adult Learners Develop Confidence in Mathematics – A Contemporary Approach to a Traditional Problem

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The WIT Certificate in General Studies (formerly the WIT Certificate in Foundation Studies) course is targeted towards educationally disadvantaged adults. The aim of the course is to equip the learner to successfully take part in mainstream higher education. The research targeted these return-to-education adult learners within the mathematics modules only and attempted the sensitive and equitable assessment of mathephobic adult learners of maths: equitable because assessment must be reliable and redeemable; sensitive because the majority of these cohorts present with varying degrees of mathephobia.

During my early years of engagement with these learners I experienced a so-called ‘critical incident’ causing me to research a ‘better’ way to assess such cohorts without the need for diagnostic tests and traditional entrance examinations. It was at that stage I examined the potential of learning journals in assessing adult learners of mathematics both initially and going forward; I also saw an opportunity for the learner to self-assess thus empowering the learner at the same time as building their confidence. A subsequent critical incident provided me with an idea to experiment with PowerPoint presentations as a means of confidence-building in the learners. Through collaboration and groupwork presentations on the impact of mathematics on society were delivered and peer-assessed. Subsequently this practice has been retained as an integrated part of the mathematics module continuous assessment regime.

The Learning Journal Experiment

The experiment had multiple aims:

Sensitive assessment of mathematics ability;

Early ‘diagnosis’ (self-assessment) of ‘issues’ allowing earlier intervention and, ultimately prognosis;

Instructor-learner dialogue

Building up of interpersonal relationship

The benefits of the journal were ultimately multiple and diverse and the experiment’s aims were ultimately achieved. To highlight the diverse but positive benefits I have decoupled the learner and tutor and examined the benefits from each perspective.

Learner benefits:

Indirect relationship built with the instructor – shyness or reticence overcome;

Self-image intact – ‘freer’ environment than class room to express true feelings/thoughts/opinions etc;

Learning journal becomes a ‘monument’ of learning snapshots – successes were ‘replayed’ during tougher times. One learner commented on this aspect as the ‘this too, shall pass’

benefit of the journal;

Confidence-building – firstly in mathematics, but anecdotally the benefits were visible across other subjects subsequently. Some learners commented on this aspect as being very positive for them.

Instructor benefits

Sensitive assessment of numeracy and mathematical ability of cohort. Dignity of learners respected – vulnerability and fear around mathematics and mathematics education handled appropriately;

On-going dialogue established with learners – openness increased and superficiality decreased as dialogues developed - a perception of equality established;

Early intervention by instructor was facilitated meaning an improved prognosis for course completion on the learners' behalves. This illustrated a new depth to the learner-focussed ethos of the journal;

Interpersonal relationship built facilitating an open and honest dialogue with learners – ‘the wounds of a friend are better than the kisses of an enemy’ benefit to both parties.

Although the benefits to the journal for mathematics assessment are mainly positive, the experiment highlighted several significant drawbacks to such an undertaking. From a workload perspective, the journal has the potential to become a burden. Weekly reading of entries and following-up involved much time and effort, as did the initial ramping-up period where reflexive workshops were needed ultimately to train the learners in the reflexive process; Jenny Moon's work in this area was beneficial. On- going monitoring in the early stages was time-consuming until a ‘system’ for handling this was developed.

Further significant drawbacks are:

Fabrication of ‘critical incidents’ by learners: constructing incidents in order to simply write something thus achieving a continuous assessment grade. Accounts were obviously ‘made up’ by learners, while others refused to go to any depth worth reporting preferring instead to remain at arms distance emotionally, detached and in some cases superficial. An open, honest dialogue with these learners was nearly impossible;

Learner objection to reflexivity: not to be underestimated. Learners who disengaged from the process remained disengaged despite the consequences of forfeiting part of their continuous assessment grade. In all cases, these were learners who were achieving in any examinations.

Overall, the research found that a minority of just 15% of surveyed respondents disliked using the journal; 35% were neutral. Half of all respondents liked using the journal, mostly favoured amongst low- or non-confident learners. The findings showed a higher proportion of confident/competent learners disliked using the journal.

Moreover, regularity or frequency of journaling seemed to correlate with respondents' measured confidence levels.

Half of all respondents liked writing about how they feel about mathematics; with nearly one-quarter disliking it. These data showed some low-confidence learners, who found mathematics difficult, liked using the journal. Why was this the case? Perhaps these learners identified benefits with journaling, possibly to communicate with their tutor and share their problems and difficulties with mathematics. By journaling regularly, perhaps these learners were able to ‘deal’ with their problems by making them explicit rather than allowing them to remain internalised, perhaps alleviating stress or anxiety. Findings by Selfe et al (1996) and

Lanigan (2006) may provide insights in this context. They found journals provided a written account of the students' progress as seen not only by the instructor but by the student as well. The awareness provided by this exercise thus helped improve confidence and self-esteem in the learner, helping to create a more conducive learning environment.

In the final analysis of the journal experiment, the aim of the research was to find any link between confidence and competence in mathematics. The data showed that for non-confident learners of mathematics 41% reflected more frequently due to the journal; 38% reflected more deeply (increased reflexive and decreased descriptive writing); 36% saw evidence of increased ability; and remarkably, 42% preferred quality of reflections over quantity.

Furthermore, 40% were motivated to continue using the journal out of personal motivation, moreover 27% stated they would continue using the journal regardless of whether a continuous assessment grade were applied to its use or not. Almost 40% of non-confident learners were able to identify learning strategies to assist them with learning mathematics competently. A third claimed their instructor was provided with insights into their individual learning issues through the use of the journal in mathematics.

Benefits to confident learners were as good or better in most instances.

The Collaborative PowerPoint Presentation Experiment

The overall aim of this experiment was to promote or increase the learners' awareness of and practicality of mathematics in the world around us. I hoped the learners would become aware of the everyday usefulness of mathematics and would counteract what Diana Coben termed 'invisible maths'. The experiment was influenced by the work of Howard Gardner's Multiple Intelligences theory, and would not be considered a typical kind of '3-Rs' assessment. I believed that if the experiment was a success, it would be measured by means of a paradigm shift in these learners' attitudes towards mathematics.

Benefits of the Presentation to the Adult Learner

Awareness of mathematics around them was highlighted in post-presentation reflections. For example, a portion of learners highlighted the fact that although they had learned something from doing their own presentation, they learned far more from their colleagues' presentations in some cases. This is a very useful 'surprise' to emerge from the experiment. It highlights the benefits of collaborative teaching and of having the 'fun factor' within the mathematics classroom. Learners identified they were relaxed and relieved to have presented and were therefore more open to the presentations of their colleagues. Some learners identified an increase in their assertiveness and confidence through the exercise of presenting on mathematics to their peers. Indeed some showed deep levels of reflexivity and insightfully pointed out they were, in fact 'teaching' mathematics to their peers and colleagues through this exercise. This is a very important insight to have come out of the entire experiment.

Benefits to the Instructor

One of the most surprising happenings to emerge from this experiment is that of the creation of new ideas for teaching traditional mathematics topics. Having witnessed firsthand, the uniquely creative manner in which some learners presented on topics in mathematics, I was enthused personally and professionally to approach them and seek their permission to utilise their unique perspective on the topic in my own classes in subsequent years. All those I have approached over the years have readily agreed to my requests and to say they are flattered is to understate immensely the pride they feel in themselves. Confidence is built in moments such as these.

Further benefits to the instructor include the truth about the collaboration process: the exercise of collaborating with colleagues (for better or for worse) is revealed in direct and open personal, guided, post-presentation reflections. This aids the instructor's task immensely when awarding a final grade for the team. Although the presentations are peer-assessed, I retain a veto on grades which I am very up front about with the learners prior to the exercise. Its as fair as possible under the circumstances.

Drawbacks to the Presentation

Time consuming to assess; fifteen to twenty teams presenting from anywhere from 5 to 15 minutes each;

Level of mathematics involved can be be light-weight in some due to various reasons e.g. lack of confidence, lack of motivation, lack of work-ethic, etc.;

Level of mathematics involved can be heavy in some due to over-reaching on information and attempting to cover too much too fast; not being aware of the audience's level of mathematics is a contributing factor to the failure of such presentations;

Technology: not preparing for it to fail! Poor knowledge of PC and/or data projector; poor knowledge of embedded multimedia files such as video and mp3 files etc.

Overall Presentation Findings

The findings showed that 90% of respondents learn best in practical, hands-on classes. I feel there is some potential bias in these results due to the reticence of some adult learners towards so-called 'theoretical' classes. 89% claimed their views or attitudes changed regarding their awareness of mathematics and its everyday practicality.

In the final analysis of the entire experiment as a whole, it was found that:

Journal-keeping positively benefited 'honest' learners;

Weekly journaling built confidence in the learner;

The Presentation positively impacted upon learner self-efficacy;

Overall confidence improved.

To provide some evidence for the findings discussed above, I have included some quotations from learners to highlight specific findings and provide evidence. Rather than provide multiple quotations from many learners I chose to utilise one learner's reflections to illustrate the personal nature of the journal. Some other smaller quotations are used from a selection of learners as supporting evidence for the findings.

Confidence-building...

"...now I know more maths than I have ever known in my lifetime of maths at school...I am no longer afraid of maths. I'd have more confidence in myself to tackle [maths] or apply for jobs or courses that involve maths, a road on which I would have never travelled before."

Further confidence-building

"MATHS DOESN'T SCARE ME ANYMORE...Hip hip hurrah!"

"I have heard nothing but nightmare stories on this subject",

"...It's all about understanding what's being asked (words like 'deviation'), finding averages. I'm no longer scared of starting something new (what you sow you reap)."

Instructor-learner dialogue with ‘Mary’

“...just hope I get more confident with numbers because they still frighten the life out of me...fractions! Algebra! I can’t believe I can understand them let alone do them! It’s great I really feel good about it, like I have achieved something already! Thanks Michael, I keep looking at the maths and smiling, thinking ‘I did that’ WOW!”

Ongoing dialogue example – Mary

“Had a chat with Michael today, got some stuff that was worrying me off my chest. He asked me to sit up the front of the class for maths from now on. It’s a bit scary, like I’ll have nowhere to hide anymore!!! Maybe that’s a good thing? Funny how something like my fear of numbers can have an affect on my whole life?”

Dialogue with Mary cont’d...

“I sat up at the top of the class today like Michael had asked, it’s mad but I was very nervous, felt really tense at first!!! But after a bit I calmed down, I think I even took in more, I actually managed to do the maths questions he gave us and got them right!!

Can’t believe it, and with no one beside [me] I had to trust my own judgement. I came out of maths today with a little bulb over my head and I felt very proud. I hope that tomorrow I don’t start to clam up again because there’s no nicer feeling than when you understand something. It’s put a beautiful smile on my face.

Normally when I’m in maths class I like to feel invisible but when I sat at the front today I had nowhere to hide. Thanks Michael, maybe I have taken my first step in the right direction?”

Mary’s conclusions...

“...even though numbers still frighten the hell out of me I have come to realise sitting down the back of the class won’t make them go away, that the best thing to do is sit right up the front and face them, because if you sit there and look at them, I mean really look at them and not just think ‘Oh no, I can’t do these!!’, they’re not as scary as they first seemed! I’m not going to run away any more, when the going gets tough I’m just going to get tougher. Thanks Michael for making me believe in myself.”

Concluding quotes...

“...I have achieved so much in these last few months. Facing up to my fear of maths has changed my life in so many ways!!! It’s hard to believe but it’s made me a much more confident person. I’m no longer that shy person trying to hide at the back of the class so Michael won’t see me. I can even look him in the eye now when he asks me a question instead of at the floor.”

A Happy Ending...

“When I think of maths now it does not make me feel like a total freak!!! I now know if I give myself time to sit down and look at it I can find an understanding of it now...I have a much greater understanding of the subject today than I ever dreamed possible. I know that I made the best decision of my life the day I decided to take a second chance. Its great to be able to just sit down with my kids and help them with their homework without having to say “you’re going to have to ask daddy to do that with you love” because now...I can do my kids homework with them!!”

Group dynamics and the impact on learning

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Abstract

At ALM 10 I discussed interactive whole class teaching and how materials containing a high level of participation and collaborative learning were to be piloted. During the pilot I became interested in the group dynamics within the group and how this affected the learning and teaching taking place.

Group work has been the subject of much research in educational and psycho-analytic theory fields of study. The overall conclusion is that students move from being passive to active and group work stimulates cognitive responses and enhances social growth. I would add that group work may also help contain anxiety either conscious or unconscious from previous experiences thus allowing students to learn more effectively.

Introduction

I began the workshop by discussing and explaining the context of the pilot.

At ALM 10 I presented a workshop with regards to a set of materials and discussed interactive whole class teaching and how materials containing a high level of participation and collaborative learning were to be piloted. These materials included elements of paired activities, discussion, modelling and demonstration. They represented a new pedagogical framework for adult numeracy. Understanding was taught through group work but not just group work, interactive class teaching. This, in effect means that the level of talk and discussion is high and students actively participate in the class structure often taking charge. Students do not just talk but extend their understanding by moving towards constructive dialogue in order to learn by questioning, analysing, thinking and reflecting.

The materials, 60 hours of teaching, were piloted in one class. I observed 90% of the lessons and watched with interest as the group emerged to take control of their own learning as collective. The students became confident. All passed level 1 numeracy tests but, more interestingly, 50% progressed to a General Certificate of Secondary Education intermediate class rather than a foundation General Certificate Secondary Education class which is the normal progression route. (The General Certificate of secondary education is the standard examination taken mostly but not exclusively by 16 year olds, in England in the last year of compulsory education. It implies competency in the subject and is used as a gateway tool to progress to further education.) My attention turned to why this occurred and if group dynamics may have played a part. I began to see that the very nature of groups engaged in group work moved the class forward. I needed to unpick what was going on in the maths class, e.g. why were people joining the class? Why some left? Why were some aggressive or preoccupied? Why in fact learning was taking place. I read extensively and explored psychoanalytical theories with regard to groups. These I will share with you later but first we have to look at what constitutes a group.

Definition of a group

I began by asking participants two questions: What is a group? What defines a group? These were some of the phrases used to describe groups: People working together, a shared purpose, common aims, common task, sharing an activity.

I asked:

If people are working on the same thing but individually are they a group?

If strangers are all eating in a café they may have a common purpose but are they a group?

When we all arrived at this workshop were we a group?

And finally

What will define us as a group and not a collection of individuals?

I discussed Feldman's, Mowatt's, Tuckman's and Bion's theories and definitions.

Feldman states that a group status can only be achieved when the individual members of the group see themselves as 'one' working for itself and therefore working for each other to achieve a common goal. (1992) Mowatt states that group work takes many forms, seminars, workshops, student presentations, role play and discussion however the purpose of there "is the need to help the group emerge from the collection of individual learners" (1997:95)

Tuckman suggests there are four developmental stages within groups; forming, storming, norming and performing

Forming – initial stage

Storming – a phase of confusion

Norming – shared understanding of task and positive interaction

Performing – group begins to achieve its objectives. (1965:387)

Bion sees groups and their behaviour as more complex. He states that the behaviour in groups can take one of two forms: a work group mentality which is characterised by a wish to face and work with reality and achieve its task, (whilst doing this it will reflect upon its progress and change its modes of working if necessary) or a basic assumption group mentality which is characterised by a wish to avoid reality if it proves painful or causes discord or anxiety. (Stokes 1994:21)

There was general agreement that these definitions offered clarity to how groups were defined however participants felt that in a classroom setting individuals wanted to work separately and they preferred it. Another participant who trained teachers said, despite arguing the case for group work, the trainee teachers saw individual work as the best way forward.

This reluctance may cover a fear of the situation becoming out of control and could therefore be associated with past experience and anxiety.

Experience and anxiety

We explored Roger's arguments based on Freud that no experience is truly new. All experiences build on previous ones, and we enter into no new situation without something of the past. This can be conscious and therefore accessible to thinking but more often than not it is unconscious and inadvertently distracts us from our purpose. (2001:12) If this is true then teaching adults who are deemed to have failed once, and who invariably have associated bad experiences, will always mean that anxiety is present. This coupled with Roger's statements

that anxiety is present in all learning and that if real learning is present then it invariably leads to change and change is difficult, will always make teaching adults an activity fraught with difficulties. (2001:12)

Participants agreed that anxiety was present in students and sometimes seemed to overwhelm them but that also this anxiety overwhelmed them as tutors. This is common and can helpfully be described as projection; the students fears are so great that they are projected into something or someone, this may be displayed by reluctance to work, aggression, arguing with other students or the tutor, chatting, not listening, being late, fiddling with equipment or students can simply relinquish responsibility for their own learning.

The relationship between tutor and student

Students can attach themselves to a tutor and refuse to move to other classes even if this represents progress. I have also come across students who spend time lamenting the departure of a tutor. They may say, “if only x was here I would have passed” or “I learnt so much from x but I’ve learnt nothing since she went” or “I only passed because x taught me so well before you came”. They do not take responsibility for their own achievements or, indeed failings. Students may also believe that when they get a better tutor or they move to a new class they will succeed. They can believe that their success and the way to reach it is through hope, not hard work.

This struck a chord with one participant who recognized these characteristics as commonplace. Participants who were tutors or trainers were interested in how to overcome these behaviours and how to move groups into a workgroup mentality.

A tutors’ role

The following represents suggestions I made on how tutors can become aware of the unconscious behavior of their students and the impact they have as tutors.

Salzberger-Wittenberg describes the task of the tutor as to sometimes act as a container for the anxiety felt by students – he acts as a receptacle of anxieties too great to bear. Sometimes students project their feelings, fears and anxieties just simply to get rid of them and they become frightened of the receptacle or hold it in contempt. Others use this facility to communicate their fears hoping that it may be understood and they may be helped to bear it. (1983:58)

The tutor can act as a container and bear some of the anxiety associated with learning but also can show how this anxiety acts as a trigger to continue to learn, especially by remaining curious.

This, in practice in the classroom, means that a student’s ideas and thoughts are aided by a teacher; he assists the students to make sense of these ideas and places them in some order. The tutor’s ability to reflect and think about the process of learning rather than the supply of answers helps the learner to think and give meaning to his thoughts. (Salzberger-Wittenberg 1983:60)

The skill of the tutor is paramount to balancing the needs of the individual and group process. Groups are powerful and while this can be positive it can also be “intimidating, ineffective and destructive” (Kennedy 1996:76). The tutor has to:

learn and continue to learn to achieve understanding of each member of the group,

to be aware of the group dynamics and how these may change,
to be aware of and reflect upon her own behavior and
to be aware of the connection between group and individual dynamics. (Kennedy 1996:77)

Practical suggestions

Other more practical ideas were discussed. These included:

1. Students as a group deciding their rules for how they will learn therefore taking collective responsibility for their learning. These rules could involve decisions around the following issues:

What will they do if somebody is late or absent? Will they all follow the same programme? Will they participate by coming to the front of the class? Are students allowed to opt out of activities?

Will they have homework?

These rules will differ from group to group and they are distinct from college rules.

2. “Unstuckness rules” or “try three before me” as advocated by Guy Claxton. This actively encourages students to list solutions for when they are stuck. For example:

Look back at a similar problem

Break the problem into smaller more manageable parts

Draw a diagram

Disengage for a few minutes

Ask a friend

Use practical equipment to help

Once you have tried three then you can ask me.

Students again take responsibility for their learning and more importantly realise that learning is a hard activity and that being stuck is a necessary phase of good effective learning and understanding. (Claxton 2004)

3. Classes are run in a precise order; they always have:

a group activity to begin the class;

a main teaching part and

always end with a conclusion, where time is given to reflect on the learning that has taken place

Participants’ concluding remarks

We concluded by each group member sharing their thoughts, comments and reflections on what they would take with them.

Comments included:

the importance of group work within the pursuit of understanding

the need to establish a conclusion;

students making rules;

guarding against the dependency of students on one tutor;

the realization that the anxiety felt after some classes wasn’t necessarily your own;

And finally, one participant commented that the workshop had helped clarify the importance of group work and therefore she would pursue it with added vigour and commitment.

In conclusion:

The workshop confirmed that many experienced tutors and trainers realise the importance of group work within educational settings. However the workshop also confirmed that in some institutions whilst the pursuit of individual expertise and need is often seen as paramount it is seen as unachievable through group work. Culturally and historically the emphasis on the individual and their needs cannot, for some, be reconciled within a group framework. Those tutors and trainers who believe in collective responsibility face an uphill struggle against a misplaced belief that individuals need to be cosseted against the rigors and difficulties associated with learning and understanding.

I hope the workshop and the ideas explored will equip trainers and tutors to believe in their own experience and their own conclusions, that group work leads to a greater individual experience and is therefore worth pursuing.

I apologise for any comments made and not recorded.

Literature

- Bion, W.R. *Experiences_in groups and other papers*. London. Tavistock publications, 1961. pp.145-189
- Feldman, M. Splitting and projective identification. In Anderson, Re editor *Clinical lectures on Klein and Bion*, London, Tavistock/Routledge 1992.pp.74-75
- Freud,S. *Group psychology and the analysis of the ego*. 1922 London, Hogarth press. Complete works Vol 18,p.9
- Glaxton,G. *Building learning power*. Bristol TLO Limited, 2002, p.51
- Inel,S. *Small groups in Adult literacy and basic education*. Office of educational new research and improvement, Washington DC, 1992.
- Kennedy,E. Schools, groups and the primary task. In Jennings, C and Kennedy, Eeds. *The reflective professional in education. Psychological perspectives on changing contexts*. London, Jessica Kingsley, 1996. pp.72-86
- McDonnell,A. Interactive whole class teaching. In *Learning Mathematics to live and work in our world, proceedings of the 10th international conference of Adult Learning Maths-a research forum (ALM10)Strobl, Austria 29th June to 2nd July 2003*
- Mowatt,I. Learning in small groups. In Sutherland, P editor *Adult Learning, a readerI* London. Kogan page. 1997. pp94-106
- Obholzer,A. *Managing social anxieties in public sector organizations*. In Obholzer, A. and Roberts, V.Z editors, *The unconscious at work*. London. Brunnel-Routledge. 1994. pp.169-178
- Reid, M. *Communicating successfully in groups. A practical guide for the workplace*. London, Routledge. 2000.p95
- Roberts, V.Z. The organization of work: contributions from open systems theory. In Obholzer, A and Roberts, V.Z.e ditors, *The unconscious at work*. London. Brunnel-Routledge. 1994.pp.28-38
- Salzberger-Wittenberg,I. *The Emotional experience of Learning and Teaching*. London. Routledge and Kegan Paul. 1983.pp.58-60
- Stokes, J. The unconscious at work in groups and teams: Contributions from the work of Wilfred Bion. In Obholzer, A and Roberts, V.Z editors, *The unconscious at work*. London. Brunnel-Routledge. 1994

Tuckman, B.W. *Developmental sequences in small groups*. Psychological bulletin 63, 1965.
pp. 384-399

Numeracy, Communication, and Power

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Action Aid

Abstract

Reflect is an approach that emphasizes the importance of sustained reflection and action for learning and social change, currently being used by over 350 organisations in more than 60 countries. The roots of Reflect include the theories of the Brazilian educator Paulo Freire, and participatory methodologies. Central to Reflect is the belief that people's capacity to communicate depends as much on understanding and ability to deal with power relations as it does on technical skill. Thus, by focusing on communication and power Reflect aims to improve the meaningful participation of people in decisions that affect their lives.

Mathematics is crucially important in strengthening people's capacity to communicate. Whether in oral or written form numbers affect everyone – especially in relation to money. The workshop at ALM9 enabled participants to work through some of the new resource materials developed for Reflect practitioners, thinking through how the resources might be applicable to their work contexts.

Introduction

This article briefly outlines the main principles and techniques used in the *Reflect* approach. It then considers numeracy in *Reflect*, highlighting the key elements. This is followed by extracts from the resource material, 'Communication and Power' discussed in the workshop, and brief conclusions reached by the participants.

What Is *Reflect*?

Reflect is an innovative approach to adult learning and social change, conceived by Action Aid and piloted in El Salvador, Bangladesh and Uganda in 1993-95. It fuses the theory of Paulo Freire¹⁰ with the methodologies of participatory rural appraisal.¹¹

¹⁰ **Freire, a Brazilian educator**, was one of the most influential philosophers of his generation. He believed in the power of education – which could be liberatory, or could be used to oppress – but is never neutral. (see, for example: *Pedagogy of the Oppressed*, 1972, Penguin, London).

¹¹ **Participatory Rural Appraisal**: PRA is a family of approaches and techniques that were originally developed so that development professionals could have a better understanding of the reality of the poor, and their own priorities for action. The best-known method of involving all members of the community is through participatory visualisation techniques: the construction of maps and matrices on the ground, and the subsequent agreement about the findings and their implications for development activities as well as the structural causes of poverty. Although PRA is often used to 'extract' information from target groups, *Reflect* emphasises the need for this information to remain in the control of the group. Another criticism of PRA is that it treats the community as an homogenous unit – over the last few years *Reflect* has begun to realise the complexities of power relationships in both the public and private spheres, and has adapted many of the tools to work with these issues. See the *Reflect* materials, or contact the Participation Group at the Institute of Development Studies, Sussex University, UK, for more information on PRA.

Key to the *Reflect* approach is creating a space where people feel comfortable to meet and discuss issues relevant to them and their lives. The meeting and discussions serve a dual purpose – not only does this personal and group analysis form the basis for learning and action, but people also have the opportunity to share their opinions with others – to practise communicating in different ways.

Abolishing the need for a text book, the approach enables groups to develop their own learning materials by constructing maps, calendars, matrices, diagrams or using forms of drama, story-telling and songs, which can capture social, economic, cultural and political issues from their own environment. In this process the development of literacy and other communication skills becomes closely linked to the engagement of people in wider processes of development and social change. This enables *Reflect* participants to recognise, value and strengthen the skills that they already possess. However, this does not mean local knowledge is romanticised: through this process participants may identify additional skills/knowledge that they wish to acquire.

A Reflection-action cycle is developed where participants analyse their reality, identify what they would like to change, plan how they will go about it, act – and then reflect on what they have achieved.

In the same way *Reflect* practitioners reflect and act – which means that *Reflect* is continually evolving, and innovation keeps the approach alive. Since its conception *Reflect* has been continually adapted, drawing on practitioners' experiences and insights from other research. This has resulted in the development of new materials entitled '*Communication and Power*'.

Communication and power

The basis of these materials is the understanding that no communication is neutral – the capacity to communicate and be heard is determined by power relationships that need to be analysed. By linking communication and power *Reflect* focuses beyond the technical aspects of communication and considers the various factors which influence our ability to get our voice heard. Being unable to communicate is both a cause and effect of inequitable power relationships.

The pack has been created with contributions from *Reflect* practitioners in 30 countries and aims to cover different elements of communication. For ease of reference it is divided into 6 sections: Getting Started (an outline of the key aspects to consider if planning to work with *Reflect*), Written Word (ideas for working with literacy at different levels), the Spoken Word (ways of strengthening people's oral communication), Images (use of posters, photos, television, body language etc.), numbers (explained in detail below) and examples from practice (case studies of different *Reflect* experiences). However, rather than focusing on one section, it is hoped that people will work across the categories, exploring how different forms of communication are inter-connected.

Each section is made up of A4 sheets which give a brief description of the focus of the sheet, and explain why you would want to work with the topic, when to do so and how, with examples from practice and any additional information.

Aim of the workshop

Most of the numeracy experiences included in the resource pack have been drawn from the experiences of practitioners based in developing countries, involved in a wide range of development programmes, of which numeracy only plays a very small part. I was interested

to share these materials with practitioners from the numeracy field - to find out what they thought about our materials. I was particularly concerned to see whether the ideas contained in the resource pack appealed to those involved in Adult Learning Mathematics and what sort of adaptations they thought would need to be made.

Thus I introduced the ideas and principles behind numeracy in *Reflect* and then asked the participants to look through the materials, and pick one sheet, thinking through how they might use the ideas contained on that sheet in their particular context.

Numeracy in *Reflect*

Numeracy in *Reflect* draws on ideas and understandings developed in the fields of ethno and critical mathematics. The focus is on **discovering and using prior knowledge**, linking this to the idea of **critical mathematics**.

Numeracy is understood broadly, as being about solving problems, analysing issues and expressing information clearly and concisely (spanning written, oral and mental forms). The visual representations, and the idea of graphic construction in *Reflect* are intrinsically mathematical. Further, many of the graphics, such as matrices, pie charts and bar charts (calendars) use mathematics explicitly in the analysis.

Why numeracy? Mathematics is extremely important in strengthening people's capacity to communicate and has a crucial role to play in challenging power inequities. Numbers affect everyone. The most obvious and powerful use of numbers is in relation to money. This affects every individual both directly (for instance, in relation to the price that we secure for our labour or produce, or the price of basic goods and services we rely on) and indirectly (for example, through budgetary decision making, concerning both income and expenditure, at international, national and local levels). Moreover, numbers in the form of statistics used by different agencies for planning also have a huge, but often unrecognised, influence on people's lives.

What does numeracy involve? Work with numeracy in *Reflect* includes a critical reading of existing 'texts' and the active construction of alternatives. Much of the work includes challenging traditional understandings of mathematics, highlighting and strengthening the mathematical skills that participants already have. Another focus is on using these skills within a process of analysis, challenging the power of written mathematical texts and constructing alternatives.

Key principles: The starting point is to demystify mathematics and analyse the links between the uses of numeracy and the practice of power.

...*maths in context*: Numeracy must only be introduced in context. It should not be taught mechanically, but focus on real use. Work with numbers should only take place if it is relevant to the particular topic being discussed. Calculations should be used to solve real problems and contribute to a process of analysis.

...*previous knowledge*: Participants should be supported in discovering, using and strengthening the mathematical skills that they already possess. This implies working with oral/mental mathematics. The problems encountered by adults joining a learning process are often due to formal written processes clashing with the mental way of calculating. Conversely, using participants' prior skills helps in confidence building. The participants recognise their own power and knowledge, while simultaneously enhancing their skills and understanding.

...written mathematics: This does not mean that we believe mathematics should never be written down. Participants should be able to read and write numbers. It is also important to be realistic and analyse the power of written mathematics. It is only through taking part in this analysis that participants will be able to make informed decisions about what mathematical knowledge they need. When written down mental processes look cumbersome. However, if participants have a record of their workings, and can see the complexity of what they are doing, it is likely to increase their self-confidence. Moreover, the written process can be used to show how the same mathematical processes are employed in different contexts. This understanding is crucial if people are to use mathematics to expand their opportunities.

...calculators: Where appropriate *Reflect* practitioners are encouraged to use calculators. These can be used to demystify the mathematics, so that participants can focus on the issue at hand. Further it is a way of checking mental calculations, and of illustrating how the same mathematics can be used in different situations.

...micro-macro links: A common problem with participatory tools is that they can lock people into a micro-level, local analysis, isolated from the wider context and missing the links between the local, national and international situation. Much of the work with numeracy in *Reflect* aim to provide a bridge, enabling people to place their reality in a wider context.

...external information: Further information is often necessary to make the micro-macro links. This gives rise to two issues. *Firstly*, who decides when it is appropriate to introduce external documents and how can this be done without corrupting a process which is controlled by the learners themselves? *Secondly*, how and where is this information best accessed?

There are no simple answers to either of these points. However, it is important that participants take part in these discussions. To ease problems of access, the implementing organisation can play a role in linking with other organisations, libraries and Internet centres. Further, these organisations may wish to present the information in a user-friendly format.

Working with numeracy: Although work with numeracy will have different aims and focuses at different times one way to sequence the work is the following:

Participants develop a graphic (using numeracy, either in the construction or analysis – integrated into the discussion and analysis rather than seen as a separate process).

An external text (with a numeracy element) on the same broad theme is introduced and critically read.

Participants place their previous **local analysis in wider context**

Participants identify ways in which **numerical communication** may contribute to wider action to **advance their interests**.

Limitations/assumptions: It is assumed that all participants have some level of interaction with a market economy, and that the national currency is based on a decimal system. In addition, we suppose that the participants will have devised their own mathematical systems for working within this market economy, and that there exists a developed counting system in their local language (i.e. that things can be grouped specifically according to number, and that a number sequence exists - more than just one, two, many...).

The Workshop

After outlining the principles behind numeracy in *Reflect* I asked the group to divide in two and do the following

Glance through the numeracy sheets and choose one which they thought looked interesting.

Read through the sheet and discuss:

Whether they agreed with the ideas?

Whether there was anything particularly interesting or controversial contained in the sheets?

How they thought the sheet would work in practice? – is it realistic?

How they would adapt it to their context?

What they would add to the sheet?

Any other comments

One group chose to look at a sheet which outlined ways of ‘Building on existing knowledge’ and the other examined ‘Credit, Loans and Interest Rates’. Below are some extracts from the sheets discussed.

Building on Existing Knowledge

Even where people have strong numeracy skills these may have been developed in a specific context, and may need to be ‘generalized’ in order to be able to apply them to new situations.

The key to introducing an abstract mathematical formula is to tie it in to something concrete from the participants’ life experience. This involves applying problem solving strategies that people have developed from experience to new, different contexts. Through enabling them to see that the same techniques are applicable in a variety of situations - from farming, to cooking to understanding interest rates - their existing mathematical knowledge will be strengthened and generalized. This flexibility in understanding can be hard to achieve – and it is crucial that the individual discovers links for her/ himself.

1.1.1 Choosing Examples

The first step in this process is to choose an example where participants use mathematics in their everyday life. For this different graphics, such as daily routine charts or community maps, could be used.

Participants should outline the steps they follow to make calculations in this context and then follow the same process in the new situation. Links between the two contexts should be continually highlighted.

Example from Practice: Participants in a *Reflect* group in Koffiefontein, South Africa, are involved in a small bread-making business. By generalising this experience the group were able to calculate the amount of money spent per child in school.

The mathematical formula is as follows:

Total Cost/ Total no. loaves = Individual loaf cost

Or

Total school budget/ no. of pupils = amount spent per pupil

First participants talked through the steps they go through to calculate the cost of each loaf they produce. They used photocopied money (counting out the cost of each ingredient and then counting out all the money to reach the total cost) to find the production cost of a batch of bread. By sharing the money equally between the number of loaves they arrived at a cost per loaf. Participants then generalised this experience to calculate the amount spent on each child in their local school – the total budget divided by the number of pupils.

Credits and Loans

Many people live on borrowed money. Patterns of credit and debt are similar worldwide – both for individuals and at national or governmental level. A discussion of these issues should enable participants to make links between their local situation and wider debt issues – to engage in more complex debates about the structural causes of poverty, and the relationships between money and power. The questions in the adjacent box could be asked when discussing credits and loans.

- ? **Why** do we borrow money?
- ? **How** does we feel when we borrow money?
- ? Are there **certain times** of year when we need to borrow money? Why is this?
- ? What **different ways** are there to **access** money locally? Who can you borrow money from?
- ? What are the **dis/advantages** of different sources?
- ? How does the money **repayment** work?
- ? How **often** do we need to make repayments?
- ? How and when is the **interest** calculated?
- ? What **happens** if you are **not able** to make the repayment?
- ? What **influences** the rates of interest— how are the interest rates decided, and by whom?
- ? What do we feel should **change** about the system? What **can we do** to achieve this?

Example from Practice: In Kanatalabanda community, India, participants in a Gotti constructed a graphic to analyse why they borrowed money, and from whom. They discussed how the different money lenders operated and the rates of interest they demanded, the power the money lenders had over them, and how they were made to feel when borrowing money. Participants found that the moneylender was their main source of money, and used his power to humiliate them, making them wait all day before he would lend them money. The lender would also insist that the borrowers planted cash crops from which he could make money rather than traditional crops, creating further food security problems. The lender would then insist on collecting seeds to cover the interest of the loan, reinforcing his power over the borrowers as they were pushed further into debt. Following the analysis the group decided

that they would refuse to use seeds to pay back the interest, and instead would start repaying the capital immediately. They also decided against borrowing from the moneylender in the future and began exploring alternative avenues with more favourable rates of interest.

Conclusion

A forty-five minutes workshop was not enough time to explore in detail the issues on the sheets, or think through how they could be adapted. However, it was interesting to see that our ideas did resonate with participants and that materials developed in Africa, Asia and Latin America did have a relevance in diverse European, and more formal adult learning contexts. There appears to be greater convergence between development projects and adult learning classes than I originally expected.

For example, the group which looked at issues around credit, loans and interest rates highlighted how it was not only the interest rates and repayment schedules which are important when considering which loan to take out. Questions of access are important, as is the reputation of the person/organization providing the loan. Issues of flexibility, small print and ethical basis of the organization might also be considered. These issues are applicable whether based in an industrialized western city or a rural context in a developing country.

It was also clear that adults, wherever they are based, develop their own strategies for dealing with everyday mathematical problems they encounter. And if these skills are recognised, valued and built on this can be extremely empowering. The recognition that there are a wide range of methods which can be used to solve mathematical questions can, however, give rise to difficulties when considering mathematics learning in more formal situations, especially if there is pressure on the participants (students) to achieve a particular accreditation – or if the facilitators (tutors, teachers) lack confidence in their own mathematical ability and hold a narrow definition of numeracy. This pattern is replicated around the world and although there are various strategies that have been developed to overcome these issues it could be that there is further development needed in this area.

The *Reflect* approach is a powerful way of enabling numeracy learners to engage with wider issues, to expand their focus and understand how numeracy can be used within a change process. I hope that there are more opportunities to explore how *Reflect* can be adapted and integrated with more formal processes of learning.

References

- Archer D, and Cottingham S (1996) *Action Research Report on REFLECT: Regenerated Freirean Literacy Through Empowering Community Techniques, The Experiences of Three REFLECT pilot projects in Uganda, Bangladesh, El Salvador*, Serial No 17, London: Overseas Development Agency.
- Archer D, and Cottingham S (1996) *REFLECT Mother Manual: Regenerated Freirean Literacy Through Empowering Community Techniques*, London: ActionAid.
- Archer D, & Newman K (forthcoming). *Communication and Power*, London: ActionAid *Reflect* website: www.reflect-action.org

Intervention Strategies That Foster Student Success

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Introduction

At ALM-7 in July, 2000 I presented a report about dissertations written on the topic of adult mathematics education during the previous twenty years. Over the next two years I read several of those dissertations and began to build a database that categorized the theses and captured critical identification information. The contents of that database were shared with attendees of ALM-9 in Uxbridge. In the intervening time I have read additional dissertations and by the summer of 2004 I estimate that I have read about sixty of the dissertations listed in the database.

The Interventions

One subset of those dissertations reported research on intervention programs designed to promote adult student success in mathematics classrooms. Some, but not all, were designed to allay mathematics anxiety. Others were designed to enhance study and time management skills. One was a study in retrospection where successful students were asked to describe strategies and external assistance that helped them achieve their goals. In this paper I will share the findings of five of the doctoral candidates. To organize my study of all the dissertations I used the template shown in Figure 1. My report on each dissertation will take a similar format.

DATA GATHERING DOCUMENT
AUTHOR:
TITLE:
UNIVERSITY:
YEAR:
INSTITUTIONAL ENVIRONMENT:
SUBJECTS:
INSTRUMENTS:
PROJECT DESCRIPTION:
FINDINGS:
EXTRA:

Figure 1. Template for Standardization of Dissertation Information.

Hornor, Ni Z., *The Effectiveness of an Application of Some Concepts from Andragogical Instruction as Compared With Traditional Instruction in an Introductory College Algebra Course*.

This study was conducted in a community college setting in four sections of an introductory algebra course. Two classes served as the experimental group while the other two formed the control group. The classes contained a mix of adult and traditional students and all four were taught by the researcher. The project consisted of weekly, in-class projects that were self-directed and self-paced, utilizing computer-aided instruction. Non-traditional teaching strategies such as peer-tutoring, computer-aided instruction and group presentations were employed in this study.

The findings from the study are as follows:

There was a statistically significant difference in achievement, in favor of the experimental group.

Non-traditional students in the experimental classes achieved significantly better achievement scores than non-traditional students in the control group. This was not true for traditional students.

Retention was not significantly affected for instructional method nor student classification. There was no significant difference in attitude between methodology nor student classification.

There was statistically significant interaction between the instructional method and student classification. Adult students in the experimental group had much better attitudes than those of adult students in the control group.

Berry, Andrew, *The Effects of Peer Tutoring on Adult Students in Remedial Algebra at an Urban Community College* (Community College)

At an urban community college, Andrew Berry also examined the consequences of peer tutoring. His study looked at the effect that tutoring had on student success, anxiety, and attitude towards mathematics. He also explored the differences that gender might produce. Some of the sections were concentrated in six weeks. Others met for twelve weeks.

Berry identified thirteen topics that comprised the algebra course and designed modules that reflected those topics. Material was introduced briefly by the instructor then the students, working in pairs (sometimes trios) worked through a module booklet that contained between ten and thirty questions on the new material. Tests at the end of each module generated the pairings for study of the next module. The matches worked from the ends inward. That is, the strongest student was paired with the weakest until the final pair was formed from students of similar capability. Exceptions were made for students who expressed a desire to work together permanently. The tutor/tutee roles changed dynamically within the dyad depending on the expertise of the individual students.

The instructor stayed in the background, answering questions and prompting pairs to stay on task. The modules took about four hours for students to complete, at which time the students were tested and new dyads identified and formed. Students were active partners in the research providing written commentary on their progress and dis/satisfaction with the course.

The findings reflect the role that time plays in attitudinal change:

Six-week study:

The treatment was significant at the 0.01 level in improving attitude towards mathematics. It was ambiguous whether achievement or anxiety was improved.

Twelve-week study:

Achievement, anxiety, and attitude were improved by the intervention.

Zielke, Ronald E. *The Making of a Champ: The Modification of Mathematical Self-Efficacy Beliefs of Non-traditional College Algebra Students Using Techniques Adapted from Sports and Sports Psychology* (University)

The research reported by Ronald Zielke was a case study whose subjects were three non-traditional students in credit-bearing university mathematics courses. Each was 25 or older and had been out of school for eight or more years prior to entering college. Zielke had taught the three students in an earlier course and offered to work with them to improve their self-efficacy in the current courses. An interest in sport coaching led him to hypothesize that the strategies used by athletes to succeed might also prove helpful to mathematics students.

Zielke titled the resultant intervention program CHAMP with each letter in the name indicating a particular tactic. CHAMP was designed to help mathematics students develop a winning mindset and succeed. The strategies are:

C – Cue words – Athletes use cue words as a reminder to do certain things at certain times during a game or a match

H – Focus on the here and now – set and focus on establishing controllable and achievable goals

A – arousal control – athletes know that it is important to control emotions throughout a competition and maintain a healthy level of arousal

M – Modeling and mental imaging – athlete creates mental image of good performance that can be repeated and perfected in the mind's eye

P – Praise, verbal persuasion, positive self-talk – athletes internalize praise and verbal persuasion to create positive self-talk.

Zielke detected three types of changes in the students as the study progressed. First, the students reported changes in their class participation in the way they interacted with other students and with the instructor and in the way that they compared themselves to the other students. Secondly, they disclosed changes in their preparation for class sessions and tests. An increase in ability to focus on important information and to recognize their common mistakes helped improve performance and self-efficacy. They reported that the ability to control their emotions boosted attitude towards and performance on tests. Finally, enhanced performance on word problem assignments increased confidence in their ability to solve word problems and they began to enjoy the task.

By the end of the study, the participants had internalized the CHAMP process. Zielke reported that they began to treat it as a “toolkit” instead of a lockstep procedure. Furthermore, they recognized the usefulness of the method in other non-mathematics classes and adapted it to other disciplines. Other students took note of their success and the participants found themselves in the role of coach, teaching CHAMP to other students.

Zopp, Marilyn. *Math Anxiety, the Adult Student, and the Community College* (Community College)

The subjects in Marilyn Zopp's study were eight nontraditional age students over 25 who received a high score on the Math Anxiety Rating Scale (MARS) when tested. The treatment, titled Mind Over Mathematics (MOM), consisted of three two-hour sessions at the beginning of the semester. The sessions covered student-identified barriers to learning mathematics, lack of confidence, test anxiety, study skills, time management and stress management. After the treatment period was over, tutorial assistance was available for the students. Each student was interviewed three times: at the beginning of the treatment, after completion of MOM, and at the end of the semester.

In their interviews, students confirmed the value of several of the aspects of the program. They saw benefit in the discussions about math anxiety that took place within the support group. Suggestions concerning stress management and study skills were deemed valuable. Participants felt that information about learning styles helped them modify their study methods and adjust to instructors whose teaching styles were a mismatch. They believed that the information on lifestyle issues such as diet and fitness along with time management suggestions had helped them overcome their anxiety. Students did feel that three sessions had not been enough. They thought that it should continue throughout the semester. One student felt that younger (traditional-age) students could also benefit from the program and that counselors should be informed of the existence of the program

Among the strategies that the participants reported helpful were daily work on assignments and focusing on conceptual understanding rather than memorization. Looking over a test before beginning and working on the easier problems built confidence. Like Zielke's subjects, these students practiced self-talk and found that it decreased anxiety and increased their confidence. That increase encouraged them to speak up in class and ask questions, to form study groups, to seek tutoring, and to know where to go to seek help when they felt that they needed it.

With confidence boosted and anxiety decreased, the participants felt ready to move on to the next level of their mathematics education. Students felt confident that they would pass their course or even do well (Student results were 5-A's, 2-B's, and 2-C's. One student was taking two courses at once.) All of them saw a strong connection between math and the workplace. Five of the women also saw connections to other areas of life besides work.

Parker, Sheila Latralle Blackston, *Overcoming Math Anxiety: Formerly Math-Anxious Adults Share Their Solutions*

Sheila Parker's dissertation was a study of success stories. She sought out and interviewed twelve adults who had once been math-anxious but had overcome their anxiety. Her subjects were located through referrals from both personal and professional networks. While they were a diverse group demographically, they were all high school graduates. Her criteria for inclusion in the study were that the individuals:

Identified themselves as having met Richardson and Suinn's definition of math anxiety.

Were adults at the time they overcame their math anxiety.

Were able to remember and articulate how they overcame that anxiety.

Were able to discuss how math now plays an active role in their lives.

Parker sought to understand the transition that adults made as they moved from math-anxious to math-comfortable. In her interviews she sought answers to four categories of questions:

What factors are associated with the transition of overcoming math anxiety in adulthood?
Which factors facilitate the transition? Which factors impede progress?
Do adults go through stages or phases while overcoming their math anxiety?
What kind of personal changes take place as adults overcome math anxiety?
What kind of changes take place regarding how one sees the subject of mathematics, the context of mathematical learning, himself or herself as a learner, and the role of a teacher/tutor as one overcomes math anxiety?

When she analyzed the interview data, Parker found that her subjects had passed through six transitional stages in their struggle to overcome math anxiety. She summarized these as:
Realizing that there was a need to become comfortable with math. Often the need was related to a job, current or desired.

Committing to meeting the need.

Taking Action.

Realization that a turning point had been reached.

Attaining new mathematical perspectives.

Becoming part of the math support system.

Parker concluded that there is an identifiable process of overcoming math anxiety during adulthood and it involves making a transition of major magnitude. One striking finding of her study was the critical importance of a support network for overcoming math anxiety during adulthood. Students who had succeeded identified individuals who had helped them learn the mathematics and those who had freed them for that activity by assuming their responsibilities at work and at home.

Bibliography

- Berry, A. J. (1996). *The Effects of Peer Tutoring on Adult Students in Remedial Algebra at an Urban Community College*. Unpublished doctoral dissertation, New York University.
- Honor, N. Z. (2001). *The Effectiveness of an Application of Some Concepts from Andragogical Instructions as Compared With Traditional Instruction in an Introductory College Algebra Course*. Unpublished doctoral dissertation, The University of Southern Mississippi.
- Parker, S. L. B. (1997). *Overcoming Math Anxiety: Formerly Math-Anxious Adults Share Their Solutions*. Unpublished doctoral dissertation, The University of Georgia.
- Zielke, R. E. (2000). *The Making of a Champ: The Modification of Mathematical Self-Efficacy Beliefs of Non-traditional College Algebra Students Using Techniques Adapted from Sports and Sports Psychology*. Unpublished doctoral dissertation, The Ohio State University.
- Zopp, M. (1999). *Math Anxiety, the Adult Student, and the Community College*. Unpublished doctoral dissertation, Northern Illinois University.

Classroom Materials

Post-16 Networking in England to Share Active Learning Approaches to Mathematics/Numeracy

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In England, The National Centre for Excellence in the Teaching of Mathematics (NCETM) is a government initiative to provide effective strategic leadership for mathematics-specific continuous professional development. It aims to raise the professional status of all those engaged in the teaching of mathematics so that the mathematics potential of learners will be fully realised.

This paper provides a brief overview of some of the work of this organisation from the perspective of post-16 education. This will touch particularly on subject learning coaches and networking as part of the Post-16 National Teaching and Learning Change Programme <http://www.subjectlearningcoach.net/>, the organisation of mathematics in colleges and developing a 'numeracy for employability' strategy.

<http://www.ncetm.org.uk/Default.aspx?page=13&module=res&mode=100&resid=6225> It covers the content of the workshop given at ALM 15 on 3rd July 2008.

Background

Recent developments in further education, mathematics and professional development in England have been directed by several reports including Success for All (2002) which stated that the government strategy would include

Putting teaching, training and learning at the heart of what we do by establishing a new Standards Unit to identify and disseminate best practice, which will guide learning and training programmes. (Success for All: Reforming Further Education and Training, 2002, p. 7)

In its report on 'Continuing Professional Development for Teachers of Mathematics' published in December 2002, The Advisory Committee on Mathematics Education (ACME) recommended the establishment of a National Academy for Teachers of Mathematics, to have a strategic overview of CPD at a national level and to co-ordinate its operation locally.

On 9 December 2004 Charles Clarke MP, then Secretary of State for Education and Skills, announced £15m funding over 3 years for the new National Centre for Excellence in the Teaching of Mathematics (NCETM).

The National Centre for Excellence in the Teaching of Mathematics

The National Centre for Excellence in the Teaching of Mathematics (NCETM) is a UK government initiative in England. The National Centre works collaboratively to enhance mathematics teaching across all sectors, primary, secondary and further education (post 16

education). It aims to raise the professional status of all those engaged in the teaching of mathematics so that the mathematical potential of learners will be fully realised.

The NCETM is a virtual centre, (www.ncetm.org.uk), supported in the 9 Learning and Skills Council (LSC) regions in England by Regional Coordinators. Most regions have one regional coordinator with a focus on primary and secondary schools/teachers and one with a focus on Further Education adults/teachers. It was established in 2006 by the Department for Education and Skills (DfES). In England there are now two government bodies looking after education—the Department for Children, Schools and Families (DCSF) and the Department for Innovation, Universities and Skills (DIUS).

The NCETM is a growing community. The web portal supports mathematics and numeracy teachers with news, resources, research and communities and blogs where teachers can discuss issues, ask questions etc. There is also a mathemapedia (similar to a wikipedia but for maths teaching) which is building up. There is also a self evaluation tool which is linked to national teaching standards and levels, where teachers can look at a particular level and see what topics that will involve and examples of the knowledge this requires. The latest addition to the NCETM web portal is a personal learning space, where teachers can collect their favourite items from the portal, make notes, keep their continuing professional development (CPD) record and share items with others. In Further Education in England there is now a requirement to maintain ‘Qualified to teach in the Learning and Skills sector’ (QTLS) by evidencing a minimum of 30 hours of CPD per year.

The National Teaching and Learning Change Programme

Since its establishment in 2006 part of the work of the NCETM has been to carry forward the National Teaching and Learning Change Programme in relation to mathematics. This is part of the governments programme of reform for the Learning and Skills sector, ‘Success for All (2202), which is working to support Subject Learning Coaches as champions of change within organisations by bringing together three ‘enablers’

Subject specific teaching and learning resources

Subject coaching networks

Professional training for Subject Learning Coaches

The subject specific teaching and learning resources entitled ‘Improving learning in mathematics’ were produced by the Standards Unit in 2005 and are now available in hard copy to both further education organisations and secondary schools within England. These resources and professional development sessions are also available to all for free download from the internet via the QIA Excellence Gateway at <http://excellence.qia.org.uk>.

‘Improving learning in mathematics’ is a substantial, well researched set of resources which is now being widely used. Before publication the resources were trialled with over 120 teachers working in 60 different organisations. To understand the approaches, principles and underlying research, the best starting point is the short book included within the pack by Malcolm Swan, ‘Improving learning in mathematics: challenges and strategies’. This is based on the work of Malcolm Swan published first in 2002 under the title ‘Learning mathematics through discussion and reflection’ and later in 2006 ‘Collaborative Learning in Mathematics: a challenge to our beliefs and practices.’

The pack also includes

6 professional development sessions

51 teaching and learning sessions with resources under the headings of mostly number, mostly algebra, mostly shape and space, mostly statistics and other

Software

Activity templates

The approaches are based on moving from ‘passive’ to ‘active’ learning and from ‘transmission’ to ‘challenging’ teaching. The teacher’s role in this model is to assess learners and make constructive use of prior knowledge; to choose appropriate challenges for learners; to make the purposes of activities clear; to help learners to see how they should work together in profitable ways; to encourage learners to explore and exchange ideas in an unhurried, reflective atmosphere; to remove the ‘fear of failure’ by welcoming mistakes as learning opportunities rather than problems to avoid; to challenge learners through effective, probing questions; to manage small group and whole group discussions; to draw out the important ideas in each session and to help learners to make connections between their ideas.

The resources are designed to encourage best practice through the use of some underlying principles which

- Build on the knowledge learners bring to sessions

- Expose and discuss common misconceptions

- Develop effective questioning

- Use cooperative small group work

- Emphasises methods rather than answers

- Use rich collaborative tasks

- Create connections between mathematical topics

- Use technology in appropriate ways.

There are five types of activity which are used extensively. The first is classifying mathematical objects where learners are encouraged to devise their own classifications and apply those devised by others. The second activity use is the interpretation of multiple representations. So for example learners may be asked to work in small groups to match fractions, decimals and shaded diagrams. The third strategy is evaluation mathematical statements where for example learners may have to decide whether given statements are always, sometimes or never true and to present explanations and arguments to justify their decisions. A fourth type of activity is where learners devise their own problems for other learners to solve so that learners have the opportunity to take on the role of teacher and explainer. The fifth activity type is described as analysing reasoning and solutions where learners might compare different methods for doing a problem, organise solutions or diagnose the errors in solutions.

The professional development sessions provide structured guidance to take a group of teachers working together through the opportunity to explore and reflect on the various approaches and activity types and how they can be used effectively with learners while the other sessions provide well thought out and supportive guidance for sessions to use with learners and the resources all ready to use. The sessions encourage discussion both between teacher and learners but also between learners. They provide opportunities for small group work and resources where there is scope for learners at a variety of levels of understanding to work on the same task through differentiated activity.

Many of the sessions use posters as a way of sharing, discussing and recording learning. Mini

whiteboards are used for working and for responding to questions such as ‘show me a shape that has a right angle’ or ‘show me a fraction between $\frac{1}{4}$ and $\frac{1}{2}$ ’ or ‘show me an equation of a line with a positive gradient’ to help with assessment for learning and sharing a variety of responses to open questions.

‘Improving learning in mathematics’ is a substantial resource which, as well as professional development sessions where colleagues are invited to work together to become familiar with the approaches and resources, contains a huge variety of sessions to use with learners with all the activities ready to use across several levels and many topics. They are being used in England for ages 14 years upwards to support qualifications such as GCSE and A Level. But they are not tied to particular courses, levels or qualifications. There is some indication of level of challenge— from A up to D—which in England relates to A (Level 1 Adult Numeracy and low levels of GCSE) up to D (Advanced Level, Level 3).

The sessions have been grouped into categories such as ‘Mostly number’ or ‘Mostly algebra’ for guidance, but many of the sessions contain rich activities that explore a range of mathematical topics.

The 3 enablers, the resources, the networks and the Subject Learning Coaches programme are having a very positive effect. Learners and teachers are enjoying the active approaches and assessment/results are improving and in some cases quite dramatically. Feedback from teachers and learners is very positive. Kelly Hughes, a mathematics subject learning coach at Darlington college, says

I was flying high after the GCSE results. Since I started using the ‘Improving learning in mathematics’ approaches, and taking over GCSE maths in September 2006, our success rate has increased from 56% to 73%.

Subject Learning Coaches like Kelly and other teachers and trainers attend the subject network meetings. These meetings are held at least three times per year in each of the nine Learning and Skills (LSC) regions in England and are organised and managed by the NCETM Regional Coordinators. The events provide the opportunity for teachers to work on the activities and approaches together and to share their experience and reflections. The hope and expectation is that teachers will take forward these approaches and create and share more. This is already happening at network meetings where we usually have a ‘show and tell’ session. More resources for teaching and learning are being produced and shared in the communities on the NCETM web portal as well as the QIA excellence gateway.

The Subject Learning Coaches Professional Training Programme is a course which helps teachers to become coaches within their organisations so that they can share their ideas and support and influence colleagues to use more active approaches to teaching and learning and to improve their practice.

However there are challenges to be faced. Experience is showing how important it is for Subject Learning Coaches to have the support of the managers in their organisation if the programme is to reach across the whole organisation. This can be a problem in some colleges where there may be little or no opportunity for all the staff teaching mathematics and numeracy across different levels and vocational courses to meet or spend time on subject pedagogy. These matters are thoroughly discussed in the recent NCETM report ‘The Organisation of Mathematics in Colleges’ available for download from the web site.

A recent development in 2008 has been some additional funding for practitioner action research projects and we have also just had a 2 day residential Summer School in York for over 80 teachers with teachers from all the regions and from a variety of organisations including colleges, prisons, adult and community and work based learning. Updates of these

and other related activities will be made available via the NCETM web portal.

Challenges for the Future

Can we make the networks self sustaining?

How will we persuade more providers to take a 'whole organisation approach' to the National Teaching and Learning Programme and the effective use of Subject Learning Coaches?

The recent NCETM report on 'The organisation of mathematics in colleges' (2008) makes interesting observations and recommendations.

The support of managers is critical in taking the work of the teaching and learning programme and the networks forward.

The governments ambitions for 'World Class Skills' provides challenging targets to increase the number of mathematics and numeracy learners and the number of qualified staff to teach them.

Meanwhile networking also continues via the NCETM portal. All are free to join. When you join, which only takes a few clicks you can then view more of the communities, you can add your comments and you can make use of the self-evaluation tool and the personal learning space. Or you can access the 'Subject Learning Coaches' community or the 'Thinking Through Mathematics' community where more activities are being shared for all to use.

Have a browse. There is much to engage teachers of mathematics and numeracy. Make www.ncetm.org.uk one of your favourites!

References

- ACME (2002). *Continuing professional development for teachers of mathematics*, London: The Royal Society.
- DfES (2002). *Success for all: Reforming further education and training: Our vision for the future* London: HMSO.
- Joyce, B. & Showers, B. (1993) *The coaching of teaching*.
- Smith, A. (2004). *Making mathematics count: Inquiry into post-14 mathematics education*, London: TSO.
- Standards Unit (2005) *Improving learning in mathematics* London: DfES.
- Swan, M. & Green, M (2002) *Learning mathematics Through discussion and reflection* London: LSDA.
- Swan, M (2006) *Collaborative learning in mathematics: A challenge to our beliefs and practices* Leicester: NIACE.

WEB References

Accessed July 2008

- National Centre for Excellence in the Teaching of Mathematics (2008) *Developing a 'numeracy for employability' strategy*. <http://www.ncetm.org.uk/Default.aspx?page=13&module=res&mode=100&resid=6225>
- National Centre for Excellence in the Teaching of Mathematics (2008) *The organisation of mathematics in colleges* http://www.ncetm.org.uk/files/255316/organisation_maths_report_v3.pdf
- National Teaching and Learning Change Programme <http://www.subjectlearningcoach.net/>

Evaluating an Educational Programme for Enhancing Adults’ Quantitative Problem Solving and Decision-Making

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1. Introduction

Recent research (Resnick, 1987, and Soden, 1994) confirms that higher order thinking skills are amenable to instruction and therefore can be taught and learned. It was therefore feasible and realistic to investigate the possibility of devising an educational programme that would enable adult learners improve their quantitative problem-solving skills. However, while a review of the mathematics education literature and mathematical problem solving in particular identified a number of key components that must be addressed if learners’ mathematical problem-solving skills are to be improved, the literature was predominantly focused on children and younger students. This unidimensional view was deficient regarding an adult-learner perspective in general but particularly in relation to the subset of the adult-learner population in which the researcher was interested i.e. long-term unemployed adults who had returned to education. The literature did not provide an appropriate theoretical framework for an adult-orientated programme. It was in the field of philosophy that an appropriate theoretical framework was discovered which proved central to the development of an appropriate adult-orientated educational programme.

In 1957 Bernard Lonergan published a book entitled, “*Insight: A study of human understanding*”. Lonergan was a Canadian theologian and philosopher who died in 1984. In his book he describes how ‘catching on’ or ‘getting the point’ is a frequent event in the course of our daily lives. This act, the act of insight, provides the foundation for a whole new philosophy on human understanding. Lonergan’s *Insight* develops this foundational view and also provides a number of cogent reasons why his philosophy is suitable in the context of adults solving problems:

his problem-solving ‘programme’ is adult-orientated,

he believed that a good starting point for the development of problem-solving skills is with the natural thinking process of the adult,

he provides a cognitional structure which identified the thinking processes used by adults when they solve problems.

Lonergan's cognitional structure stands at the heart of this educational programme and provides the direction and the substance of the intellectual activities addressed throughout the programme. However, other elements such as Action Learning and the uses of 'realistic' quantitative situations are synthesised in this original programme to achieve a transformation from theory to a practical process for improving adults' problem-solving skills (Colleran et al. 2000).

1.1 The educational programme

Lonergan's (1957) 'programme for life' is transformed into a concrete educational programme through six carefully designed worksheets and problem-solving activities (Colleran et al., 2000). Throughout the programme learners reflect on the resolutions they generate to quantitative problems and complete worksheets, each with a particular focus on an individual stage of Lonergan's cognitional structure. The following issues are attended to in a sequential and developmental manner:

- experience,
- common sense understanding,
- questions for intelligence,
- insight,
- formulation of insight,
- questions for reflection,
- scientific understanding,
- evaluation,
- decision,
- review.

Worksheet 6 acts as an integrator: pulling all preceding worksheets into one coherent whole. Having completed this final worksheet learners should have incrementally discovered the thinking processes they use when they resolve non-routine problems. The aim of the worksheet activity is to encourage and enable adult learners discover the manner in which they think and to practice reflective thinking which is at the heart of problem solving.

2. Research process

Action research was selected as an appropriate methodological vehicle. The programme evaluation process constituted two action research cycles – the first trial was carried out in two locations, (one group in each location) from January to June 1999 (Colleran et al, 2001). This was a convenient sample of adult learners available to the researcher. The second trial (September to December 2000) was carried out with one group specially selected to address and rectify problems that were identified during the first trial. The second evaluation employed an ethnographic approach to the collection of data and these data were then analysed using the constant comparative method (Glaser and Strauss, 1967). While there may be some references to Phase 1 evaluation this paper concentrates on Phase 2 evaluation of the educational programme.

The data used to evaluate the programme are drawn from the researcher's personal research journal and post-implementation interviews with the tutors and learners as well as primary documentary evidence from learners in the form of completed worksheets and written and spoken comments. The evaluation is reported in case study format.

Anonymity and ethical issues are addressed by referencing the sources of data as follows throughout this paper:

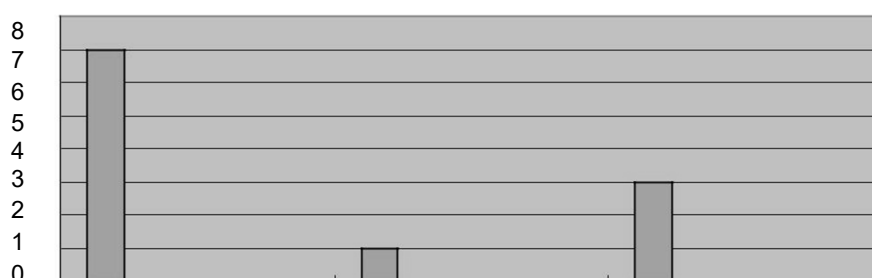
- [RJ] Researcher's research journal (which includes non-participant observation notes, tutor debriefing notes, videotape analysis and evaluation notes and the case study journal).
- [TI] Tutor's transcribed interview
- [LI] Learners' transcribed interview .
- [LJ] Learners' journal/workbook.

All other data referred to throughout this paper are described accordingly.

3. General implementation issues

3.1 Research setting, adult learners and tutor profile

The group of learners selected were participating in Adult Basic Education and had joined the Youthreach¹²² programme in September 2000. The age profile and Male/Female comparative experiences detailed in Figure 1.1 and Figure 1.2 illustrate learners' relevant details.



¹² The Youthreach programme was initiated in 1989 through a joint initiative by the National Government Departments of Education and Labour. The programme is managed at regional level by Vocational Education Committees (VEC's). Initially the programme was targeted at young people between the ages of fifteen and eighteen years who had left formal education without any recognised qualification. However, in recent years the programme's client base has broadened out to include adults over eighteen and under twenty-one years of age with no formal educational qualifications.

18 yrs.

19 yrs.

20 yrs.

Figure 1.1. Age distribution of participating learners

The tutor who carried out the implementation had been a teacher of mathematics for ten years. His teaching style was 'traditional' in nature - repetition and practice was carried out routinely while discussion and group work were not employed as part of his teaching strategy.

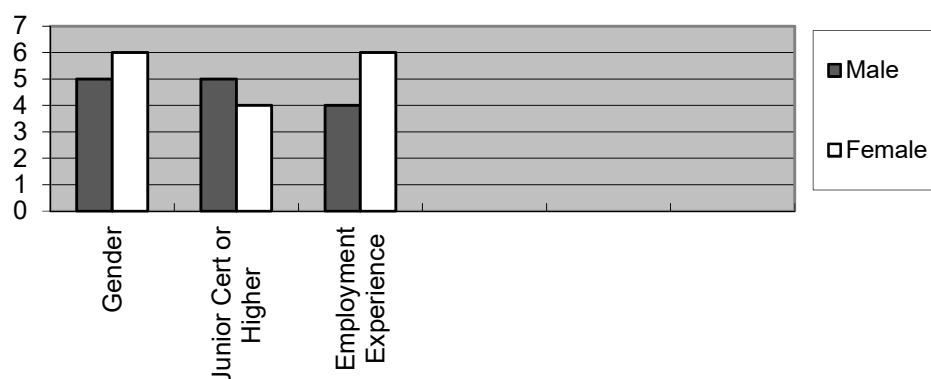


Figure 1.2. Male/female comparative chart. Blue, male; white female.

4. Case study analysis

4.1 Chronology of events

The case study was carried out over a twelve-week period between mid-September and mid-December 2000. There were twenty-two one-hour sessions throughout this period of which three were used to prepare the learners for the programme. A number of major quantitative problems were addressed throughout the case study. Figure 1.3 illustrates the point at which each quantitative problem was addressed and the amount of time spent on that problem during the programme. It also illustrates that Action Learning, Journal writing and Worksheet activity continued throughout the case study.

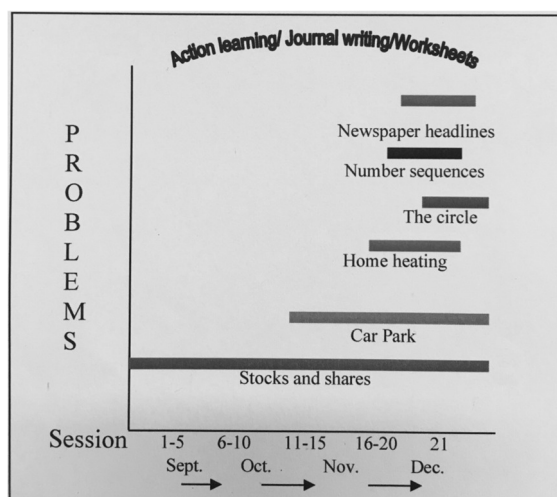


Figure 1.3. Programme timeline illustrating the time given to problems throughout the programme.

5. Quantitative problems

5.1 Stocks and shares

The pilot study suggested that ‘realistic’ quantitative problems were an effective means for discovering and developing mathematical skills. For this reason the first problem selected by the tutor from the compilation of quantitative problems developed by the researcher was the ‘Stocks-and-shares’ problem. Learners were given a hypothetical £1000.00 allocation to buy stocks and shares over the course of the programme.

This quantitative problem continued throughout the case study, however it was put aside at times to deal with other problems such as:

designing a car park,
number and letter sequences,
the circle,
newspaper headlines,
economical ways of heating a domestic house,
purchasing a mobile phone (see Figure 1.3).

There were many mathematical skills addressed and improved during the stocks and shares problem such as:

adding, subtracting, dividing and multiplying of whole numbers and decimals,
calculator work,
data tables,
percentages,
time,
estimation,
predictions.

All learners completed a number of data tables representing the names and prices of shares and the amount of money spent on each share. Totals for spending and money remaining were calculated. See Table 1.1 for example of data table produced by Learner 9.

Company	Share Price	No. of Shares	Cost
First Active	£1.69	200	£338
Tullow Oil	£.92p	500	£460
AIB Group	££9.56	21	£200.76
			£1.04 left

Table 1.1. Data table completed by L9 for the ‘Stock and shares’ problem.

The table above is typical of the manner in which the problem was presented and calculations carried out by all learners.

5.2 Designing a car park

In an effort to address mathematical skills such as measurement and geometry learners were asked to design a car park at the rear of the school/centre to accommodate as many vehicles as possible (see Figure 1.4).

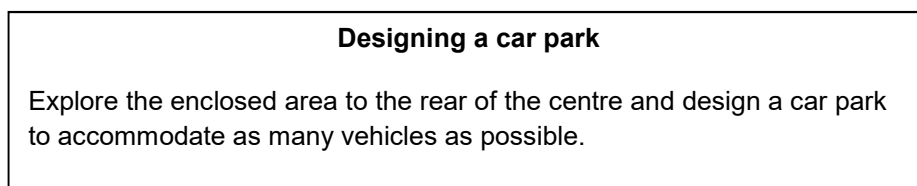


Figure 1.4. 'Designing a car park' problem statement.

In completing this project, which took four sessions, the following mathematical skills were discovered and used by learners:

linear measurements,
areas,
averages,
scales.

5.3 The circle

The tutor used the image of a circle to enable learners discover the difference between commonsense thinking and scientific thinking and to learn about the characteristics of an important mathematical shape (see Figure 1.5). Firstly the tutor displayed the shape for a few moments and asked learners what they had seen.

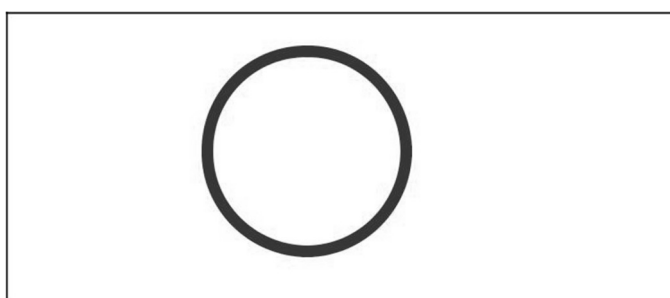


Figure 1.5. 'Looking at the shape' problem statement.

Immediately learners said it looked like a round shape, a red ring and so on. The shape was displayed again and learners were asked to write down what they see or what it is.

This was an excellent and enjoyable exploration for many learners. In his post-implementation interview L8 was astounded and satisfied with this session:

I thought it was interesting because at the start of it was just, oh, a circle right. But we kinda started talking about it and got more into it and we managed to get half an hour of talk out of it ... from a circle? which I thought was amazing... I thought well, how am I here thinking and talking about a circle for so long. I found it very interesting. [L8:LI:Dec 14th 2000]

This was an important session for many learners because they were enabled, through the gentle probing of the tutor, to uncover what they knew about the circle and develop understanding. They also discovered that new understanding is achieved by taking time to think. The researcher's journal entries about these sessions points to the importance of discussion within the classroom and the expert manner in which the tutor moved learners from commonsense thinking to scientific thinking by shifting the questioning into a mathematical and therefore scientific context:

There would be little discussion about the circle if the tutor introduced the definition first. ... It is important to emphasise the shift in context employed by the tutor to engage this form (scientific) of thinking. [RJ: Dec. 11th 2000]

Other quantitative situations such 'Heating your home' 'Buying a CD player', 'Taking a day trip', 'Newspaper Headlines' and 'Buying a car', were also addressed throughout the programme.

5.4 Developing mathematical language

The following extracts are indicative of the mathematical language used by learners during their discussions on the stocks and shares problem:

I sold my shares in Riverdeep which stood at £248.84 and invested it into three different companies. I only made a profit of £14.22 on top of my £1000.00 so I have also invested it on top of my £248.84 of Riverdeep shares. [L11:LJ: Nov. 13th 2000]

Stocks went up over the week but dropped today. I think we should do stocks more often. [L3:LJ: Nov. 21st 2000]

Clearly the mathematical development of learners was enhanced by this learning episode. This progress and development was continued as learners solved other quantitative problems. The overt use of mathematical language in discussions related to all quantitative problems is indicative of an increased use of mathematical thinking by learners throughout the programme.

5.5 Realistic problems

The programme endeavoured to enable learners engage and resolve quantitative problems that mimicked realistic situations relevant for learners. These situations provided opportunities for learners to use their common sense, their reasonableness and their mathematical skills. Learners discovered the mathematical skills that needed improvement as well as learning new mathematical skills they had not used before. Learners were also more

motivated and more participative. In his post-implementation interview the tutor was satisfied that ‘ordinary’ or ‘textbook’ problems:

Do not stimulate as much participation. Learners prefer realistic problems and a lot more thought processes occur during the problem solving... it made me aware that you have to give people a lot more time to think out problems.... [Realistic problems] motivate learners more and they are more interesting... I found the realistic problems very effective throughout. Some of the examples I wouldn't have thought of or used before, like for example, the most economical means for heating a house. I found the actual example worked itself with very little effort. [TI:Dec 18th 2000]

During observation sessions as well as videotaped sessions there was an obvious increase in the frequency and length of silences. The tutor and the learners were becoming comfortable with a thinking environment. It is clear from the journal entries of Learners 2 and 11 that they were comfortable in a thinking environment where they were expected to exercise their minds:

I thought the class was interesting. We asked more questions and there was no real meaning only what we thought about it ourselves. ...It was a thinking class. [L2:LJ: Nov. 30th 2000]

The purpose of this class I feel was to stimulate thought, ask questions, find a meaning... I exercise my mind, expanded my thoughts. It gave me a better understanding. [L11:LJ: Dec. 7th 2000]

The realistic situations encountered reinforced the need to think things out and make reasonable and deliberate decisions.

Realistic problems have many positive aspects in relation to learning and development. However they did create difficulties for the tutor. In the post-implementation interview the tutor suggested that realistic problems were not susceptible to the same type of control and planning as textbook problems:

The [realistic] problems went on for much longer than I had planned but this seems to be the nature of realistic problems. TI:Dec 18th 2000]

5.6 Worksheet activity

The six programme worksheets are designed to firstly, enable learners discover in a gradual and progressive manner the thinking processes they encounter as they solve problems, secondly to provide practice with ‘realistic’ problems in sample worksheets and thirdly to facilitate reflection on the part of learners through the completion of worksheets (Colleran et al. 2000). Each worksheet focuses on a particular phase of the problem-solving cycle beginning with some reflection on the relevant information already available to the learners. This incremental uncovering and development of the learners’ thinking skills and mathematical skills is reflected in learners’ comments throughout the programme. Learners

also confirmed that they use these thinking skills and mathematical skills not only in the context of mathematics but also in many real-life situations:

...everyday problems there's maths in it without realising ... all the maths I didn't realise I was doing ... And I actually was doing, doing the maths like and you know myself I think I've improved so I'd say I'd have a lot of confidence. [L5:LI:Dec. 13th 2000]

Yeah, in everyday life like paying bills and stuff like that, questioning where did that come in and knowing myself where I stand within money problems or mathematical problems in doing a room or a house or you know all the things I've learned in maths will help me with measurements there and paying money here and there. It will give me some sense of direction I suppose. [L11:LI:Dec. 14th 2000]

6. Summary

This summary addresses a number of significant developments such as improved thinking skills and mathematical skills, developing autonomy and the importance of context.

6.1 Improved thinking skills

Learners' improved thinking skills are corroborated from a number of sources throughout the programme. All learners agree that the cognitional cycle was an effective thinking process and was the process they themselves used everyday when they solve problems in all contexts. Many learners confirmed that they had experienced this process in contexts such as paying bills, shopping and having difficulties with relationships. This clearly supports Lonergan's view that the cognitional structure is natural and invariant.

Many learners had been insecure when it came to asking questions before they participated in this programme. They felt that questioning was an indication of stupidity and ignorance and would therefore rarely ask questions. However, as a result of this programme learners had a positive rationale for questioning and had developed sufficient confidence to ask questions when they engaged a problem situation. This altered perspective improved learners thinking skills and gave them the confidence to ask questions when they felt the need.

The classroom environment created by this programme encouraged learners to make sense of the problems they tackled. Learners brought their out-of-school or commonsense experiences into the mathematics classroom as a matter of course and this enabled learners to value their commonsense as a resource and a basis on which a solution or scientific understanding was built (Lonergan, 1957). Valuing and using one's experiences and knowledge is another indication of improved thinking skills.

6.2 Improved mathematical skills

Clearly learners who participated in this programme improved their mathematical skills. They had numerous opportunities to use mathematics in various situations. They became aware of the pervasiveness of mathematics in everyday life and how mathematics can be used to make informed decisions. They discovered the mathematical skills that needed improvement as

well as learning new mathematical skills they had not used before. The overt use of mathematical language in discussions related to all quantitative problems is indicative of an increased use of mathematical thinking by learners and a growing confidence within learners of their ability to engage and resolve quantitative situations. The self-evaluation by learners, which was facilitated by the post-implementation interview questions and journal writing, confirmed that they themselves felt they had improved their mathematical skills.

6.3 Developing autonomy and confidence

There are several indications throughout the case study to suggest that learners were developing autonomy. Learners became comfortable quite quickly with the reduced activity and contributions of the tutor in the classroom. They made decisions about the ground rules and the responsibilities of individual learners in the group. Learners were allocated or volunteered to carry out certain activities between sessions. There was clear communication between the tutor and the learners so that each knew their areas of responsibility. Many instances of Lonergan's commonsense learning occurred and learners used their relevant commonsense within each 'realistic' problem situation. This was particularly apparent in the 'home heating' and the 'car park' problems – learners began to value their out-of-school experience. Learners began to feel confident enough to contribute what they thought was relevant in a particular discussion and were willing to take help from other learners or from the tutor if other learners could not help.

As the programme progressed learners wrote more extensive and reflective notes into their journals. The following journal entries by L1 trace a growing confidence over time:

I don't think the group can function without the basic understanding of what we are doing... first class = chaos. [L1:LJ: Oct 17th 2000]

After [the tutor] has explained the mechanics of the group I feel better about the group.... Yesterday's class was good. [L1:LJ:Oct. 20th 2000]

I feel more calmer and confident in situations... I am more confident within myself and within the group... It [the cognitional cycle] is as [L11] says 'a coping mechanism'. [L1:LJ: Nov. 6th 2000]

Learners' growing confidence is also corroborated by the following:

setting ground rules,
taking more responsibility for the learning process,
using the commonsense they already have about realistic problems at hand,
using the mathematics they already have,
willing to act on the problems,
using the cognitional cycle in many contexts,
explaining or helping other learners,
calmly approaching quantitative problems as well as other problems,
writing journals at the end of each session – giving their honest opinions even though they had been told that the researcher would read them later, extended thinking times or silences, making decisions.

While individually each of the above may not be convincing, however when taken together there is compelling evidence that learners' confidence and therefore their ability to deal effectively with quantitative situations had improved.

6.4 Engaging 'realistic' problems

'Realistic' problems were not susceptible to the same type of control and planning as textbook problems however they did have many positive aspects regarding learning and development. Learners found that they could readily address and engage the quantitative problems presented to them throughout the programme. Evidence that learners were committed and found the problems engaging is corroborated by a number of sources:

learners found the quantitative situations relevant and realistic,
many learners were sourcing information outside the classroom,
learners spent time struggling to identify or name the problem or sub-problem,
there was serious and extended mathematical discussions throughout the programme,
many times the group asked for more time to complete discussions,
learners took it upon themselves to allocate and carry out particular actions between sessions,
learners employed their reasonableness and decision-making skills particularly with the 'Stocks-and-shares', the 'Car park' and 'Heating the home' problems.

That learners were sufficiently motivated to engage the quantitative situations presented throughout the programme is confirmed by the following mutually supporting statements:

I think today's class was a lot harder than yesterday's because we had a decision to make whether to sell our shares or to buy more and to work out how much profit we made on other shares... I was going to put last week's profits in with this week's but what I didn't know is when those stocks go down so does my profit from last week. [L4:LJ: Nov 7th 2000]

I feel very confident in this class because I was trying to solve a real problem and because if I made a mistake I knew it would affect my money situation. [L9:LJ: Nov 6th 2000]

I'm still unsure about the measurement of the car. So I am going to measure a car park space today out in the Parkway [shopping centre]. [L10:LJ: Nov. 30th 2000]

Making the decisions and taking the actions mentioned above indicate learners' engagement and commitment to the 'real' quantitative situation at hand. Solving 'realistic' problems also convinced the tutor to change his approach from one that valued the amount of problems solved to one that emphasised an improvement in learners' mathematical thinking skills.

6.5 The importance of context

Realistic problems created a context in which learners were enabled and encouraged to draw on relevant personal experiences. The Action Learning process created a context in which learners were encouraged and required to participate in discussions which led to the construction of solutions to particular problems. The problem-solving environment created by both the Action Learning process and the quantitative situations

enabled learners to engage, own and resolve quantitative situations, and take appropriate actions.

The importance of a thinking context i.e. the angle or stance from which one approaches a problem, was convincingly illustrated when the tutor gently changed learners approach to the exploration of a round shape by asking: “If this shape was put on the board in a maths class what would you be looking for?” Immediately learners began talking about circles, radius, circumference and so on. This is very significant for three reasons:

Learners, who may have been conditioned to see and think through a ‘mathematics-class lens’ before they participated in this programme, had broadened out their scope beyond an exclusively mathematical perspective. While this was not helpful in the context of an abstract mathematics problem like discovering why a particular shape is a circle it was illustrative of a mathematics class that encouraged contributions from many angles including mathematics,

The change in context provided the catalyst for the insights that led to an explanation,

This is an indication of the discovery-constructivist approach employed by the tutor throughout the programme. His gentle probing and questioning enabled learners to discover and construct their own meaning.

6.6 Transfer’ of skills

Learners reported the use of the cognitional structure in many real-life contexts. They also believed that the mathematical skills they had used during the programme could also be used to tackle problems outside the classroom. ‘Transfer’ of learning was not directly addressed throughout the programme, however it seems that the use of many ‘realistic’ situations coupled with the cognitional structure provided a bridge for many learners to translate (Evans, 2000) their mathematical skills and their thinking skills outside the classroom.

6.7 Tutor’s changing attitude

The tutor’s objective, in the early stages, was to address and resolve a number of quantitative problems – the number of problems resolved being the criterion for success. However, towards the end of the programme the criterion for success had shifted to the improvement of learners’ thinking skills and mathematical skills. The achievement of these criteria was much slower and less obvious than the completion of lots of problems. This fundamental shift in focus meant that the tutor now used the quantitative problems as a means to incrementally develop learners’ thinking and mathematical skills and not as an end in themselves. This changing attitude was reinforced through the joint evaluation sessions. This marked change in the tutor’s objectives signalled the transition from a teaching approach that valued the number of problems solved to one that emphasised an improvement in learners mathematical thinking skills.

7. Conclusions

The overall conclusions from this evaluation are as follows:

Lonergan’s view that the cognitional structure is natural and invariant is confirmed,
learners improved their quantitative problem-solving skills,
learners improved their mathematical skills,
the importance of ‘realistic’ problems for the process was confirmed,

the Action Learning process provided a context in which learners developed a social learning environment that was comfortable, satisfying and enabling.

References

- Colleran, N., J. O'Donoghue and E. Murphy (2001). 'An Adult Educational Programme for Enhancing Adults' Quantitative Problem Solving and Decision Making.' In: FitzSimons, G. E., J. O'Donoghue and D. Coben (Eds.) *Adult and Lifelong Education in Mathematics: Papers from Working Group for Action 6, 9th International Congress of Mathematical Education, ICME9*. Sydney: ALM/Language Australia.
- Colleran, N., J. O'Donoghue and E. Murphy (2000). 'Improving Adults' Quantitative Problem-Solving Skills: A new Approach.' In: Johnson, S. and D. Coben, (Eds.) *Proceedings of the sixth international conference of 'Adults Learning Maths – a Research Forum'*. London: Goldsmiths College,
- Evans, J. (2000). 'Adult Mathematics and Everyday Life: Building Bridges and Facilitating Learning 'Transfer''. In: Coben, D., O'Donoghue, J. and Fitzsimons, G. (Eds) *Perspectives on Adults Learning Mathematics*. Dordrecht: Kluwer Academic Publishers.
- Glaser, B. G., and A. L. Strauss (1967). *The Discovery of Grounded Theory: Strategies for Qualitative Research*. Chicago: Aldine Publishing Company.
- Loneragan, B. J. F. (1957). *Insight - A Study Of Human Understanding*. London: Longmans, Green, and Co.
- Resnick, L. B. (1987). *Education and Learning to Think*. National Research Council. Washington, D.C.: National Academy Press.
- Soden, R. (1994). *Teaching Problem Solving in Vocational Education*, London: Routledge.

Using Dialogue Scenes with Adult Numeracy Learners: Towards a Framework of Analysis

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Abstract

The importance of language and communication in learning mathematics has been recognised for some time (e.g. Hoyles 1985). This paper will outline plans for a study into what happens when adults read aloud and discuss a short scene of dialogue about a percentage problem (developed for the government by a team led by Malcolm Swan). This study will utilise a participational paradigm around mathematical activity rather than focusing of acquisitional notions on learning (Sfard 1998). As such, two broad aspects will be analysed when looking how adults respond to such scenes of dialogue. One aspect will consider the linguistic tools that are used in learner-learner discussion along with the associated meta-rules of communication (Kieran et al 2002, Sfard 2008). Secondly, it will be important to consider a range of sociocultural issues that are involved when such discussion occurs (Oughton 2009). The analysis will assist in understanding the role of such scenes of dialogue as a mathematical activity in the adult mathematics classroom.

Key words: dialogue, scenes, discussion, adult, numeracy, framework, analysis

Introduction

This work involves the investigation of an intervention with adult mathematics learners in which the participants read aloud a scene of dialogue about a mathematical situation and then discuss the meaning. The notion of encouraging learners to talk aloud about mathematics stems from the interest in the use of discussion in mathematics classrooms.

The idea for the research came from attending a seminar in which the audience were asked to participate in the reading aloud of sections of dialogue between adult learners (Tomlin 2000). This struck me as an interesting approach and one that might be employed in teaching and training. At the time that this work was conceived, I had involved the use of scenes in teacher training but I had not had the opportunity to use such scenes in teaching adult mathematics learners. So, to gain some practical experience to inform further developments, it was decided to explore the use of scenes with some real adult learners. The result of this exploration was outlined in Griffiths (2014) and the scene, task and one transcript of a discussion can be found in the Appendix to this paper. After some thinking about this data, and a break from the research in order to become a parent, I return to the work and consider how such data may be analysed.

Some background

The significance of language and communication in education has been understood for some time. Studies of feral children (see Candland 1993) noted that, along with their lack of language, the individuals appeared to have not developed cognitively as would normally be expected. For example, the 19th century foundling Kaspar Hauser was a young man when discovered by the authorities but behaved like a small child. Of course, this raised (and still raises) a question of what is meant by ‘cognitive development’ and what the link of such development is to language and communication.

For many years, educational research was dominated by cognitivist perspectives. In the move from behaviourist forms of analysis to cognitivist thinking, educational thinkers developed theoretical models in which a structure, internal to the mind, is created and developed. The notion of an internal schema has been used by Piaget (1952) as a representation of what is meant by understanding. This schema is built through an individual coming into contact with a variety of experiences; these experiences would include a range of everyday activities as well as explicitly pedagogical activity.

Within such a cognitivist paradigm, the job of the educationalist was to investigate the ways in which such internal schemata are developed. An example of such a process was described by Piaget and this informed the original development of the scene used in this study (see Swan 2008 for the principles and DfES 2005 for the activity). Piaget proposed the two processes of assimilation and accommodation when an individual is presented with new information. Assimilation occurs when new information does not conflict with existing information and is added to the schema. Accommodation is the process occurring when new information conflicts with existing information and the schema would need to be changed in order to ‘accommodate’ the new information. The notion of ‘cognitive conflict’ has been used widely in mathematics education to discuss the development of mathematical ideas that involve the process of accommodation.

Piaget, and other cognitivists, have seen the process involved in learning to be an essentially individual experience in which the mind alters its internal structures in response to external stimuli. Parallel to educational developments in the west, Vygotsky, along with others in the Soviet Union, developed a notion of conceptual development that was less individualised and more social. For Vygotsky language was not an independent feature that can be used to communicate thinking but rather an inextricably linked aspect of cognition. Within this school of thought, the social element is seen as key to educational development with communication being central. Vygotsky assumed that concept development occurred through a number of stages in which notions are tested and honed through to a true concept.

Key to Vygotsky, is that these concepts are interrogated and amended through problem solving activity with communication being central. These ideas of the importance of communication and social aspects of learning began to take hold in the mathematics education community (see, for example, Hoyles 1985).

In the post 16 sector, Swan (2008) argued for a systematised teaching practice in which collaboration and discussion is emphasised. Swan worked with young adults who had failed their GCSE examination at 16 and were revisiting mathematics. He identified a series of activity types that encourage discussion which he hoped would assist conceptual development. Swan employed quantitative tools and collected data from pre and post intervention tests that indicated some learning gains through the use of such collaborative activities. The author argues that it is useful to consider the theoretical positions of those such as Piaget and Vygotsky as metaphors to interpret what may be happening when the

activity types are employed. Swan accepts that it may be impossible to reconcile different theoretical perspectives but that some of the component elements of these theories may be useful to help describe what may be happening in a learning environment. As such, Swan finds it useful to use the idea of 'cognitive conflict' as part of activity design, while using Vygotsky's social elements of discussion as a tool to analyse what happens with the activities. Such reconciliation is less useful to me as I am not interested in such quantitative analysis and am more interested in the detail of interactions. Nevertheless, using an already developed activity will help to compare perspectives as well as utilising an already tested resource.

This section has noted some of the background ideas that have informed this work. Having identified the use of scenes to read aloud and wanting to analyse the discussions that follow, I will need to identify potential frameworks that will be helpful for the analysis I wish to undertake.

Learning as participation – some potential perspectives and frameworks

Sfard (1998) has argued that there two different metaphors for learning: the acquisition metaphor and the participation metaphor. She notes that much of the early work in education focused on learning as 'acquisition' and argues that "while the acquisition metaphor is likely to be more prominent in older writings, more recent studies are often dominated by the participation metaphor Sfard" (1998 p5).

As noted above, for Piaget, learning is the building of an internal structure (the schema) and would be seen as employing an acquisition metaphor. On the other hand, Vygotsky focuses more on the social activity of learning and therefore might be seen as participational.

Since noting these metaphors for learning, Sfard has continued to work on how thinking can be understood as an aspect of communication rather than the acquisition of internal structures (Sfard 2008). Sfard (2002) argued that, from a structural perspective, communication involves two key elements. The first element consists of the linguistic tools that are employed in communication with the second element being a set of meta-discursive rules that guide the communication. The authors argue that these two elements are a space for analysis those interested in learning. In the case of the meta-discursive rules that govern the discourse, Sfard points out that by definition such rules will not be explicit and will need to be interpreted from discourse. Such interpretations will need their own perspectives and framework for analysis.

Indeed, Lerman (2001) points out that the analysis of discourse should involve consideration of a number of socio cultural issues. He argues that the role that individuals play and aspect of power and position in society will have an impact on discourse and need to be considered. The significance of gender and mathematics has been studied (for example, see Mendick 2005), and social class and mathematics texts was central to Dowling (1998). However, there may also be less obvious sociological considerations. For example, Oughton (2009) has looked at adults involved in collaborative learning and identified a number of issues that seem significant in collaborative learning. One such feature of significance that she noted was the multiple roles that humour played while undertaking collaborative activities.

If we consider the literature that looks at mathematical discourse, then there are existing frameworks that may support the type of analysis I am looking for. For example, Alshwaikh and Morgan (2014) have developed a framework for analysing texts, which they employ to discuss how abstract mathematical reasoning is being developed differently in texts from two countries (Palestine and England). The framework considers (a) the nature of the mathematics and how mathematical activity is construed and (b) how the

learners and their relationship to mathematics is construed. The authors argued that the Palestinian texts position the reader to undertake thinking and reasoning in ways that the English texts do not. It seems that this type of positioning of the learner – to the activity and the mathematics involved – could be important in discussing the response to reading aloud.

It is worth noting that there are at least three ‘texts’ that can be analysed within the study. Within the activity constructed by Swan, there are two separate texts: the task itself and the scene of passengers discussing the prices (see Appendix). Then for my study, there are the resulting discussions of adult learners. While the focus for me is on the latter, it will be important to understand the relationship between the first two texts and the resultant discussion.

At present, the perspectives outlined in this section suggest some possible ingredients to a framework for analysis but need working on for a more systematic approach. Nevertheless, I offer a few thoughts in relation to using these ideas in relation to the task, scene and resultant discussion from the exploratory data.

Discussion on the data utilizing some of the ideas in the previous section

Following Sfard (2002), it is noted that there will be sets of linguistic tools that will be employed by learners when discussing the scenes. For example, it is hardly a surprise to hear direct extracts, or paraphrasing, from the scenes being used by participants (e.g. Caron “it went up by 20%”). It is also noted that the mathematical examples used within the text might be mirrored back in discussion. For example, Annie uses the proposed ticket price from the dialogue scene, “let’s say it was 100 pounds originally”, whereas in Griffiths (2014) it was noted smaller values of ticket prices were used when the task without the scene was presented to some learners. It is the intention that further investigation of this type of textual use will be employed.

Of course, the text itself is not the only resource that the learners will draw upon. There will be other tools that will most likely be brought in to play. These will include the use of mathematical discourse that draws upon prior experiences (either from the group of learners working together or from their earlier educational experiences) as well as non-mathematical discourse. From a research angle, this discourse will not be explicit and will need to be interpreted. In the case of our scene, which involves a discussion around the price of rail tickets, it would not be surprising for participants to contribute with discourse around their own use of the rail system (which may help our understanding of the participants positioning in relation to the task).

What of the meta-discursive rules of communication? It was noted in Griffiths (2014), that the participants in the scene appear to come to the ‘correct’ answer before continuing to discuss the mathematics involved. It appears that the text has clues which suggest the answer without the participants feeling that they understand (or at least understand enough). Then after further discussion, the conversation is brought to an end with the use of humour (with Annie suggesting that, whatever happens, the passengers would be “ripped off”) (see Oughton 2009 for how humour is used in undertaking mathematical activities). We can certainly see some sub sections to the discussion and that there is something (a meta-discursive rule?) which is guiding the discussion. Investigating the ways in which such discussions occur, and considering rules which mediate discussion, may offer some insights into the process of learning mathematics.

Conclusion

The work involved in my research started with the consideration of a particular intervention, i.e. adult learners ‘playing’ a scene of dialogue, with a focus on the discussion generated in response to a particular task. As noted, there has been a broadening of the number of ‘texts’ which will require analysis, i.e. the task, the scene as well as the resulting discussions. As part of the analysis, it will be important to consider how the participants relate to the text. There are a number of questions that will be of interest. For example, in what ways does the context of the scene distract or support mathematical discussion? In what ways does the reading aloud distract or support mathematical discussion? What evidence do we see? The frameworks will need developing in order to undertake such analysis.

References

- DfES (2005) Improving Learning in Mathematics Department for Education and Skills Standards Unit. Sheffield: UK.
- Dowling P (1998) The Sociology of Mathematics Education: Mathematical Myths / Pedagogic Texts. London : Falmer Press
- Griffiths G (2014) Using dialogue scenes with adult learners: some exploratory data in A Hector-Mason, J Gibney and G Griffiths (eds) Critical moments in adult mathematics. Proceedings of the 20th international conference of Adults Learning Mathematics - A Research Forum. Caerleon: University of South Wales / ALM. pp.165-172
- Hoyles, C (1985) What is the point of group discussion in mathematics? Educational Studies in Mathematics, 16 (pp 205 – 214)
- Lakatos (1976). Proofs and Refutations. Cambridge: Cambridge University Press
- Mendick H (2005) A beautiful myth? The gendering of being/doing ‘good at maths’ Gender and Education Vol. 17, Iss. 2, 2005
- Oughton, H. (2009) ‘A willing suspension of disbelief? “Contexts” and recontextualisation in adult numeracy classrooms’, Adults Learning Mathematics: An International Journal 4(1) pp.16-31
- Sfard A (1998) On Two Metaphors for Learning and the Dangers of Choosing Just One Educational Researcher, Vol. 27, No. 2 (Mar., 1998), pp. 4-13
- Sfard A (2002) There is More to Discourse than Meets the Ears: Learning from mathematical communication things that we have not known before. In C Kieran, E Forman and A Sfard Learning Discourse Discursive approaches to research in mathematics education. Dordrecht Kluwer.
- Sfard A (2008) Thinking as communicating. Cambridge: Cambridge University Press. Swan, M (2006) Collaborative Learning in Mathematics Leicester: NIACE and NRDC Vygotsky L (1986) Thought and language. MIT.

Appendix

A scene about reverse percentages

This scene, a discussion of 'reverse percentage', was produced as part of a package for teachers (Swan 2007). It involves some pre amble text in which the task is introduced followed by the scene of adult passengers discussing the rail fare issue.

The task

In January, fares went up by 20%. In August, they went down by 20%. Sue claims that: "The fares are now back to what they were before the January increase" Do you agree?

If not, what has she done wrong?

Figure 1: Rail prices task (from Swan 2007)

The scene

Harriet: That's wrong, because ... they went up by 20%, say you had £100 that's 5, no 10.

Andy: Yes, £10 so its 90 quid, no 20% so that's £80. 20% of 100 is 80, ... no, 20.

Harriet: Five twenties are in a hundred.

Dan: Say the fare was 100 and it went up by 20%, that's 120. Sara: then it went back down, so that's the same.

Harriet: No, because 20% of £120 is more than 20% of £100. It will go down by more so it will be less. Are you with me?

Andy: Would it go down by more?

Harriet: Yes because 20% of 120 is more than 20% of 100.

Andy: What is 20% of 120?

Dan: 96...

Harriet: It will go down more so it will be less than 100.

Dan: it will go down to 96.

(Swan 2007)

Using the task and scene

The resulting discussion by one group of adult learners (see Griffiths 2014).

Group A

After reading out the scene, the following discussion took place.

Annie: Did anyone understand that, because I didn't understand that at all.

Ben: It's a discussion about the 20% and whether it's more or less.

Caron: In January it went up by 20%.

Annie: So it went up to 120.

Caron: Then in August it goes down 20%.

Annie: So 20% of 120 is ... 24 pounds isn't it?

Diana: Yes.

Caron: Yes.

Annie: So it won't be the same.

Caron: So we don't agree.

Annie: No.

[pause]

Caron: That discussion doesn't make sense to me.

Annie: What they're saying is in the beginning it went up 20%.

Caron: How much was the original price?

Annie: Let's say it was 100 pounds originally. It went up by 20% that makes it 120. Now it's gone back down to 20%.

Ben: That makes it 100.

Annie: No ... it'll be less.

Caron: It will be less because it goes to 96...

Annie: So... you still get ripped off which ever way.

Asking Good Questions in the Numeracy Classroom

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Abstract

How do we know when a student really understands something? Many learners answer questions routinely without fully understanding the mathematical concepts. Open-ended questions, or “good questions”, require students to communicate their mathematical thinking, thereby providing teachers with valuable information that can inform their teaching. “Good” questions require more than mere recall and allow for a variety of creative responses, permitting the student to answer correctly at any level while stimulating higher-order thinking about the mathematical content. Even closed questions, calling for a single answer, have diverse forms and can be framed in a “good” way that demands a different level of thinking. When students confront these better questions they apply the skills of problem-solving and become more aware of what they need to know.

If 42 is the answer, what is the question?

(With apologies to Douglas Adams and *The Hitchhiker’s Guide to the Galaxy*)

Ask a room full of adult educators why we ask questions of our students, and the answers are likely to cover a logical range: to check their understanding, to see if they’ve learnt what we wanted them to, or to build the field for a new concept. Ask the same group whether there are different types of questions and you will be told that there are *open questions* and *closed questions*. Your audience will readily tell you that open questions are valuable for numerous reasons. Open questions give students an opportunity to “be right”—an important confidence builder. Open questions give an insight into our students’ thought processes, helping us to clarify their misunderstandings. Open questions encourage students to explore multiple possibilities rather than thinking in limited ways. Teachers will also tell you that open questions are a wonderful aid to inclusive teaching practice, allowing students to answer according to their own realities and experiences, rather than being silenced by a dominant paradigm. And open questions are fundamental in fostering critical numeracy, looking at the ‘why’ and the ‘who’ in numeracy, not just the “what”.

Yet research suggests that in classrooms—numeracy classrooms especially— 93% of questions are lower-order “closed” questions (Dains 1986). Why is this? As classroom teachers working with adult numeracy students in metropolitan Sydney, we have been experimenting with open-ended questions in our maths classes. We thought that by redressing the balance of question types we might learn something about our students’ mathematical processes.

Closed Questions

It is important first to pay our respects up front to the usefulness of traditional questions in adult numeracy. Our students often come to us with an expectation that learning will consist of top-down “received” knowledge from the teacher-as-expert. These students, particularly in numeracy, expect questions to have fixed and specific answers and are therefore quite at home with the limiting nature of closed questioning. There are times, too, when the teacher, for practical reasons, may wish to limit the response of the student: when building the field or propelling the lesson towards a neighbouring concept. Closed questions are quick and easy to invent and for many concepts they are a reliable indicator of understanding.

The major drawback with closed questions is that they discourage disclosure. When a student “just gives the answer”, a teacher may wrongly assume that learning has taken place. Hall and Wright (1994) suggest that mathematics teachers’ primary focus during classroom discussion is “keeping control of the discourse”, while at the same time appearing to give students the chance to discover mathematics for themselves:

We found few instances where teachers asked questions whose principal purpose appeared to be to encourage problem solving or to help students form and articulate arguments. That is, there were few questions where students were encouraged to reflect, and where their responses were relatively unanticipated, creative and divergent.

Students can develop their own methods for getting right answers and can mislead us into thinking they understand something. Sometimes their methods are mathematically sound, other times they are not. Students, for instance, can come to believe that “average” means “middle” in all cases, especially if all the examples to which they have been exposed do not vary. Similarly, students can react to questions on perimeter by “knowing” that they must total the numbers on each side of an object, often unaware of the meaning of “length” or of “distance around”. Open questions on both these concepts will reveal much about learners’ real thinking.

What are “Good Questions”?

For this reason Sullivan & Clarke (1991) refer to open questions as “good questions”. Open-ended questions require students to communicate their mathematical thinking, thereby providing teachers with valuable information that can inform their teaching. They have the potential to promote critical thinking and encourage creativity; they stimulate student interaction and are inclusive of all learning styles; and they help students to think independently and critically.

Once students become accustomed to the unfettered form of open-ended questions, they tend to give increasingly unrestrained or free responses. But a word of caution: these questions can be time-consuming for the teacher to create and deliver; they may produce torrents of unnecessary information, and may require more effort on the part of the student.

Let's have a look at what exactly we mean by "good" adult numeracy questions. The cards in figure 1 each have a typical closed question first, followed by an open version

<p>What is 5% of \$3.50?</p> <p>or</p> <p>My bus fare went up by 5%. How much might I have paid before the increase?</p> <p>What is my new fare?</p>	<p>Construct a graph from the given data.</p> <p>or</p> <p>What could this be a graph of?</p>
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Figure 1. Cards with both closed and open questions

Designing good questions

In discussing this type of question with teachers, we have found that most practitioners find it quite tricky initially to learn how to formulate an open question in numeracy. This is surprising given that many also teach literacy and frequently pose open questions to their literacy students without thinking twice about it. For some, it may be that we don't feel as confident in our methodology teaching maths, and so we shelter behind the more traditional approaches that we ourselves experienced as students.

Our experience is that once we had turned a couple of questions on their heads and let them loose upon our students, with a little practice we were readily able to come up with Good Questions to challenge them across a range of topics. Sullivan and Clarke describe two main approaches to formulating a Good Question:

Take a traditional closed question and adapt it.

Start with an answer and think back to what questions might have generated it.

We have found that both methods work well. The most significant term in the creation of good questions, we found, was the term "might". *Might* (or *could*) implies maybe, possibly, who knows exactly, uncertainty, multiplicity—terms not commonly heard in traditional numeracy classrooms. The appendix shows some examples relevant to adult numeracy learners, using each of the methods.

Using Bloom

In creating the above examples, we also found it very helpful to look back at Bloom's taxonomy of questions, familiar to most of us from teacher education studies. Bloom's work generates a range of terms—from lower order to higher order—which teachers can use in creating numeracy questions that avoid the familiar "list, find, give, rank" imperatives and point us towards the more relaxed "tell me all you know; why does ...? in your own words; how would you use ...?"

Tell me all you know about ... (a rectangle, a pie chart)

Why does ...? (a leap year have 366 days, a straight line = 180°)

In your own words, how would you describe ...? (average, perimeter) How would you use ...? (division in a restaurant, fractions in cooking)

Using open questions in adult numeracy lessons

Many experienced adult numeracy teachers will protest that our less confident numeracy learners NEED the predictability, precision, and clarity of traditional questions – and we would agree entirely. We also use a lot of closed questions in our classes, and we have found that open questions have to be used at the right time, and for the right reasons. When is the right time?

After you are satisfied that your students have a grasp of the concept and have some autonomy in their approach to a topic or a type of question.

After they have practiced a good number of traditional questions and have experienced reasonable success.

After there has been lots of discussion and explanation.

It is also important that Good Questions are premeditated and thought through, rather than thrown into a lesson *ad hoc*. This is so that we, as teachers, can take time to think laterally and prepare ourselves for the many possible directions in which students may take an open question: it is so easy to paint yourself into a corner! Having said that, it is fascinating to “hang on and go for the ride”, seeing what paths your students take and how they end up dealing with questions as individuals or in groups.

Introducing good questions to a class for the first time can lead to uproar in the short term. Be prepared for this: the questions can generate a LOT of classroom talk as students try to figure out what is being asked and whether their responses are going to be okay. For this reason we’ve found that open questions in numeracy are really great for classes with non-English speaking background students. As well as generating a lot more discussion and requiring active use of the language of maths, open questions give wonderful opportunities for peer teaching. Differing levels of numerical ability are often levelled by differing levels of language ability, changing familiar classroom dynamics in a productive way.

Evaluating some examples

Nina’s Chocolate Problems

Nina received two different boxes of chocolates for her birthday. How many might be in each box?

How many chocolates did she have in total?

She decided she wanted to make them last for two weeks. How many could she eat each day? Were there any left over for her flatmate to eat?

Then she changed her mind. She thought she would bring one box to TAFE to share with her Maths class. How many chocolates would each of us get?



Example 1 - Jill’s comments

I made this worksheet for my beginner level Maths Workshop group at the end of a couple of lessons work on basic operations, focussing on division. The class is a group of 8 very unconfident students working on everyday maths. Half the group is NESB and a couple of the other students have a physical or intellectual disability. One of the students, Nina, was having a birthday, and after we celebrated with lollies and other treats, I suggested they work in pairs on this sheet. This was the first time the group had encountered open questions of this type; and the first time they had an exercise in maths containing the word “might”.

Surprisingly, they were relatively matter of fact about the unusual questions. Several of the students tried counting the sweets in the actual bags we were eating in class – counting rather than guessing. A couple of the students put the same number of lollies into each box, missing the word “different”. Working in pairs rather than individually seemed to be a good way to take the pressure off

Unusual Questions

1. During a nation-wide sale, Woolworths sold \$350,000 worth of Tiffany toasters. How much might each toaster have cost, and how many customers bought them?
2. Find two fractions which total to $1\frac{1}{4}$.
3. A garden has a perimeter of 18 metres. What shape might it be?
4. A woman wishes to plant a garden with 45% corn, 25% tomatoes, 15% beans, 10% peas and 5% herbs. Show how she might divide her garden bed.

Example 2 - Brian’s comments

I presented these questions to a numeracy class as a form of assessment. I had hoped that the open nature of the questions would lead to “infinite” answers and was dismayed to find that nearly all students gave the same responses. The problem, I discovered, lay in my framing of the questions: although the questions were reversals of standard numeracy questions, they did not compel students to use higher-level thinking. The final question about the vegetable garden was quite unrealistic, and students were unable to see it as a “real life” situation. I was able to rewrite the questions to boost the problem-solving aspects, thereby learning how difficult it can be to make “good questions”.

Do-It-Yourself Maths

1. Write a Subtraction Problem

19 70 83 3,795 12,000

Choose any two of these numbers and use them to write a subtraction problem about:

- x a wedding
- x a home loan
- x a low-fat diet

2. Write a Multiplication Problem

8 42 80 255 30,000

Choose any two of these numbers and use them to write a multiplication problem about:

- x a barbecue

Example 3 – Brian’s comments

This teaching resource was extremely well received. The students dived into it and really enjoyed working together. Providing a choice of “situation” made it much easier for them come to terms with the question.

References

- Dains, D. (1986). Are teachers asking the right questions? *Education 1*, (4) 368-374.
- Hall, N., & Wright, J. (1994). *The roles of question/answer techniques in secondary school mathematics*. Paper presented at AARE Conference.
- Sullivan, P., & Clarke, D. (1991). *Communication in the Classroom: the Importance of Good Questioning*. Geelong: Deakin University.

Appendix: Creating Open-Ended Questions 1

Think of a topic	Think of a standard question	Adapt it to make a “good” question
Rounding	Round 97 to the nearest ten. Answer: 100	Give a number that might round to 100? Answers: 95, 96, 97, 98, 99, 101, 102, 103, 104
Division	Nine people “split the bill” at a coffee shop. If the bill came to \$36, what did each person pay? Answer: \$4 each	Some friends “split the bill” at a coffee shop. The bill came to \$36. How many people could have shared the bill and what did each person pay? Answers: 2 people—\$18 each; 3 people—\$12 each 4 people—\$9 each; 5 people—\$7.20 each 6 people—\$6 each; . . . and on to 18 people—\$2 each
Temperature	The maximum and minimum temperatures for the day were 9 ⁰ C and 25 ⁰ C. What was the range? Answer: 16 ⁰ C.	If the temperature range for the day was 16 ⁰ C, what might the maximum and minimum temperatures have been? Answer: All reasonable pairs of temperature readings with a difference of 16 ⁰ C.
Discount	What is \$15 less 20%? Answer: \$12	During a sale I bought a pair of shoes discounted by 20%. How much might they have cost me before the sale? How much did I save? Answers: many, based on shoe price

Creating Open-Ended Questions 2

Think of a topic	Think of an answer	Make up a question which includes the answer
Multiplication	72	What numbers will multiply to give an answer of 72?
Distance	100 kilometres	If I travelled 100 km from Newcastle “as the crow flies”, where might I end up? (Use a pair of compasses and a scaled map)
Percentage	26%	I spend 26% of my weekly income on rent. What is my income and how much rent do I pay?
Time	1450 hours	If you leave home at 1450 hrs, when might you arrive home? If your plane departs at 1450 hrs, when might you need to leave home?
Money	\$1.15	I spent money in a gift shop and received \$1.15 change. What did I buy and how much money did I hand the cashier?
Fractions	$2\frac{1}{2}$	A cake recipe calls for $2\frac{1}{2}$ cups of dry ingredients. How much sugar, flour and coconut might be needed?
Shape	Obtuse angle	The angle made by the two hands of a clock is obtuse. What time could it be?
Volume	200 cubic metres	My swimming pool holds 200 cubic metres of water. How long, wide and deep might it be?

Modelling Workshop: Which Is My Favorite Phone Tariff?

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Abstract

We think that is very useful for the acceptance of mathematics in the eyes of students if they have a clear and good recognizable benefit of learning mathematics. Choosing the best tariff is a very good example for this. Students learn (or repeat) two types of competences: Linear functions and graphs of them on one hand and reading und understanding mathematics containing text on the other hand.

Starting Point: This was the announcement of our workshop at the ALM 9 conference in Uxbridge in the year 2002: "Many people use phones. Many companies offer different payment systems: Most of these tariffs are linear functions from a mathematical point of view. One step to answer the question "Which is the best offer?" is to plot these functions and to compare them. Another step is more important: What are my ideas to use the phone? Do I want to phone some minutes per week or some hours per day?

Our workshop has two parts: Firstly we work as a course to find models for some different users and offers. Then we will discuss about our experiences in this simulation of a course and the possibilities to transfer it into our practice as teachers."

Our plan for this workshop has some **background ideas** that we want to summarize before we report some results of the workshop.

1. In many countries in the world governmental services are privatized: Transport, energy support, electronic communication and media are now offered by several companies. People have the chance to select between different offers of taking a bus, a railroad, transporting a mail, paying for telephone and mobile phone, buying electric energy or oil or gas for heating and so on. Whether we like the privatizing trend or not: This trend opens a chance to save money if people find out the best offer for them. But: What is the best offer for a person? The answer depends on the structure of the scale of charges and on the individual needs. Mathematics is very useful to analyze and to understand the structure of tariff offers. This process of understanding leads to many questions about the intended individual use of the offered services. Answering these questions helps to find out what are the individual really needs. Therefore mathematics is helpful to save money and to act more rational using this service.

2. If this view on the situation is correct (we think so) it is useful to talk about scale of charges structures in mathematics courses for adults for at least two reasons: It is helpful for them (for all consumers) to know more about that for their daily live and

is helpful for them to have the experience that learning mathematics and using it to understand the world has really a practical use.

3. The practical use of mathematics is a magic word in many mathematics courses for adults. At school many of them learn a lot of mathematics as a theory, a long list of mathematical (and this means: abstract) algorithms were learned step by step (sometimes it seems: only) for the next test. With this history of learning mathematics in their mind many adults do not like to learn mathematics. They often ask: “What is the practical use of the mathematics we should learn now?” Sometimes this question is a sign for a general dissatisfaction. Practical use is sometimes a metaphor for “easier to understand” or “please not so difficult”. We as mathematicians know that more abstract concepts are often easier to understand but this is what we have to teach before it is understood by everyone.

These three points are concentrated on the content: They are keywords for reasons why we think that the type of content we wanted to offer in the workshop is as an example for teaching mathematics with practical use. The next two points show our **didactical intention** realized in the way we presented it in the workshop.

1. At conferences a lot of people want to present their work of years in a few minutes. This often leads to the result that the presentation is reduced to a short information about the content: what could be taught or has been done. Presentations about teaching are often reduced to explanations about the content that has been taught. We wanted to give a chance to discuss the methods of teaching. We know that there are presentations about such methods but if someone is talking about methods it often tends to be theoretical (for example research results about the psychological effects of learning in groups or of listening to presentations). Sometimes research to compare different methods is done and reported. This all is very important, but we think that it is not enough.

2. Our question was: How is it possible to talk about the practice of teaching? One idea is showing a video or a transcript (a written document of a classroom situation). One other way happened in our workshop: a simulation of teaching was used as common basic experience. We initiated a simulation of a course with adults learning mathematics with us as teachers and the participants of the workshop at the conference as learning adults.

Report about the workshop:

Preparing the simulation we found a lot of information about telephone scales of charges in Austria in the internet. The internet part of an Austrian newspaper gave an overview with a long list of companies offering the telephone service. We do not want to show here the long list (a printout of internet pages) that we gave the participants in the workshop. It had a structure like this:

Region information	Austria	International	Online	Additional
Company A:	4,6/2,1	18,17	5	4
Company B:	5,0/2,0	20	4	5
Company C:	6,0/2,5	10,7	3	7
....				

There are about 20 telephone companies within this list and some of them have up to 5 scales of charge offers for special groups of customers like business or - one the other end of the spectrum - for people who seldom use their phone. To understand this information a little explanation is useful: Company A offers to connect a customer to someone in his region (for example 50 km around his home) or in Austria for 5,01 Cents per minute at daytime and for 2,4 Cents per minute at night. Company C makes a difference between region or Austria. Some companies offer a service as internet provider and some not. Special offers are for example a cheap tariff for connections to the USA or Arabia. Some companies have a basic connection charge (for example 15 Euro per month) and some not. Other additional information says something about additional offers like a (free or paid) list of phone calls made in the last months and their costs.

We started the workshop with a short introduction in our planning and the background for the workshop as written at the beginning of this text. Then we gave some information about the role which they and we should play: The course we simulate has learned a lot about linear functions, graphs of linear function, solving systems of two linear equations (numerically and graphically) in the weeks before this lesson. This gave the mathematical frame of what is needed to work in this simulated lesson. We did not explain the mathematics (linear functions and equations,...) because we were sure that the participants in the workshop know enough about the mathematical background to enjoy the workshop.

To start the course we asked the participants to go into subgroups of 3 or 4 members and work on the following task written on a paper that we gave to the groups:

Please help!

My neighbor has got a new job in Austria. He is working as a commercial representative for his company. At his new home he needs a phone with an answering machine to be reachable at any time.

Looking for the best (cheapest) telephone company in Austria he found a lot of information. But he has no time to analyse it. Could you please help him? Please have a look at the attached information from the internet and work out an overview (use graphics).

We distributed the information from the internet about the offers of the telephone companies. The groups started to read it and asked some questions. We (as teachers) helped them to understand the texts. Some of the additional information had to be translated and explained.

The groups started to understand the information. Some groups decided to work with the first aim to reach more detailed assumptions about the person (neighbour) who wants help. They "modelled" this person with assumptions like this: He will phone a lot in Austria (business for his company). He will often phone home to GB (family and friends). He will seldom phone in his region - he is a stranger there. From this more detailed description of the person it is easier to select an offer of a company.

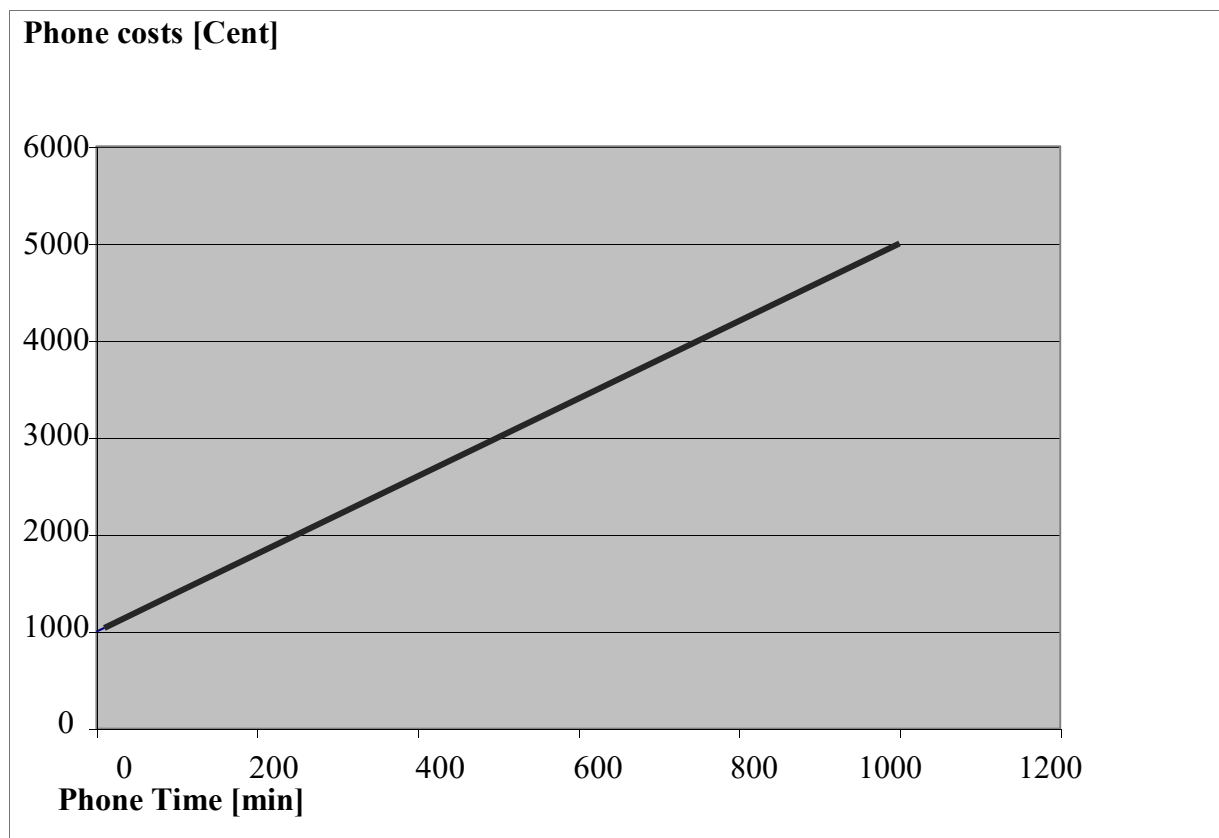
Some other groups started to sort out high price offers and in this way they reduced the number of offers they want to analyse. After finishing this sorting out they started to look closer at the information about three companies. They intended to graph the linear functions and calculate the intersection points.

We did not have enough time to bring the results together. After about 45 minutes working in subgroups we finished the course simulation and started a reflection phase.

Results and Reflections

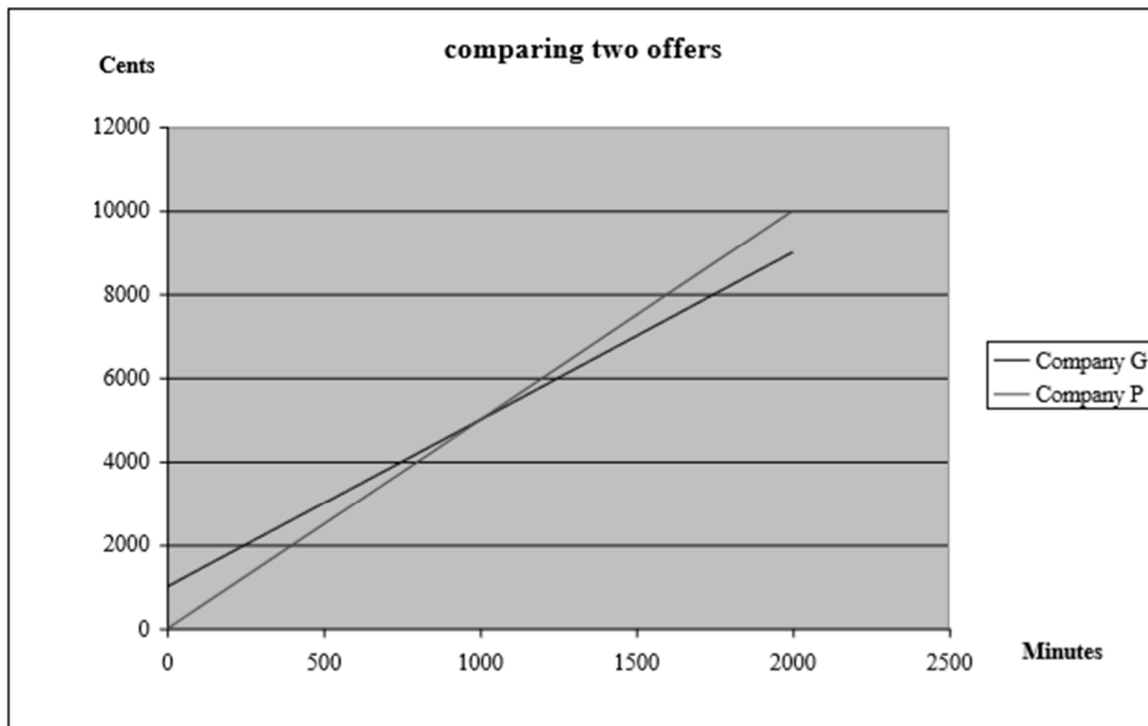
The first feedback of all participants was positive: They liked to be students and to work in subgroups. But before the participants agreed to debate in more detail about the method we simulated they wanted to know "the solution": What is the best tariff? How should we present our solution to the neighbor?

We presented this simplified way to graph two different offers as examples and as a starting point for more complex graphs. Company G offers a basic charge of 10 Euros per month and costs of 4 Cents per minute at daytime for phoning in Austria (and the region). This is the graph 1:



Graph 1: First draft

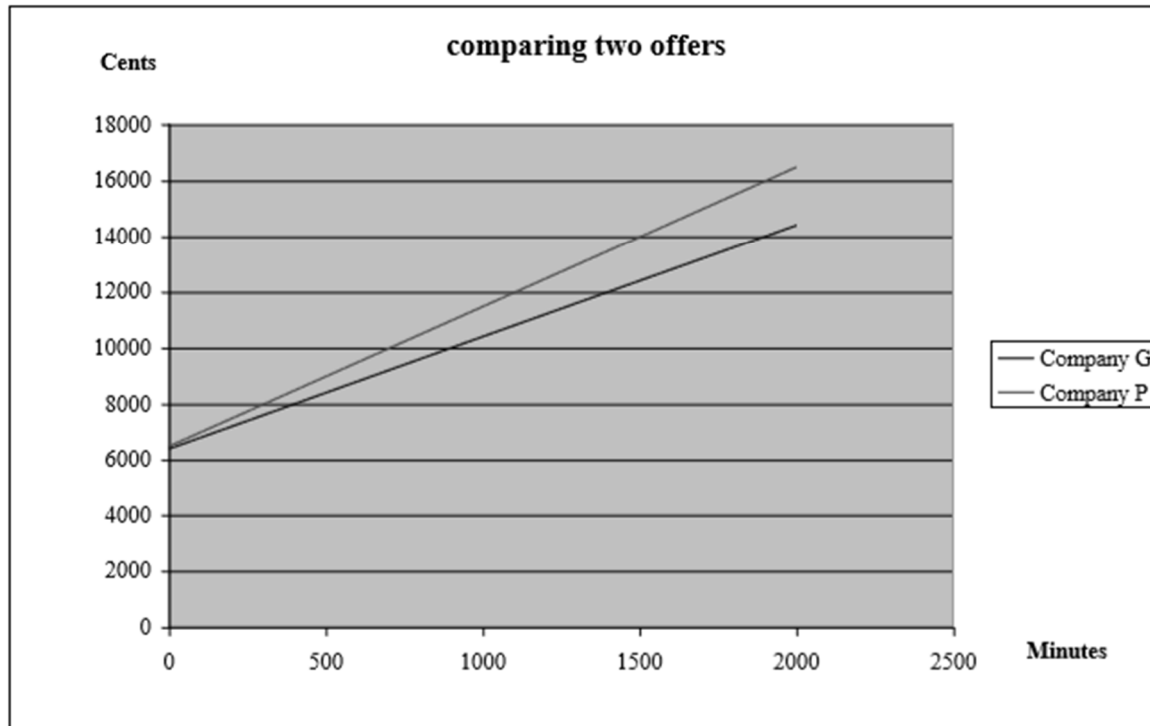
Company P offers costs of 5 Cents per minute at daytime for phoning in Austria (and the region) and no basic charge. We add this offer to the first graph and get graph 2:



Graph 2: Comparing two companies

If the neighbor is phoning less than 1000 minutes (16 h 40 min) he should select company P and if is phoning more than 1000 minutes company G is the better offer for him.

Perhaps the neighbor would not accept this help: "I will phone home to GB about one hour a week! Will this influence my decision?" Yes! We must look at the tariffs for phoning to GB. Company G offers 15 Cent per minute and company P 18 Cents per minute. We calculate the effect: 1 hour a week is 4 hours a month. The costs are 5400 Cents per month (company G) and 6480 Cents (company P). Let us graph it!



Graph 3: Comparing two companies

The result is clear: Company G should be selected (the neighbor will phone more than some minutes).

Other additional information may change the result in a similar way: We have to calculate the resulting costs (or the reduction of costs) and change the graph. We do not show the solution of the linear equation system here - it is easy for the readers of this text.

Discussion about the method (working in subgroups)

The participants of the workshop had the feeling that the given task was too difficult to be solved in 45 minutes time. This is a real life problem with very complex concrete information, many offers, many conditions for special groups of users etc. But they had the experience that this is a typical situation for real life problems. Some of such problems are much more complex and difficult to understand for us. But if we want to teach mathematics for adults not only in an abstract way it is necessary to work with such examples. The motivation to understand them is very high. An additional effect often seen in such learning situations is that in this case the motivation is very high and it seems to be very easy for learners to understand the mathematics they need. A hint for all teachers who want to use this idea for their course is that it seems to be a good idea to prepare this part of the course by cutting the whole question into several steps, as shown with the graphs in this text. It is an important task for the method of solving real world problems to cut down complexity in a first step, solve a simple part of problem and reintroduce the real complexity step by step. Working in subgroups in parallel, with the same task, or in a working cooperation with different tasks, is a good method to handle real world problems in the classroom.

“Miss Moneybags here”: Meaning and materials in mathematical learning

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Abstract

Card activities are increasingly chosen by adult numeracy teachers as alternatives to traditional worksheets and word-problems. Their effectiveness is usually theorised in cognitive terms, focusing either on the affordances of tasks such as sorting, categorising and matching for promoting understanding of mathematical concepts (Swan 2006), or on the benefits of multi-sensory approaches for reinforcing learning (Henderson 2012).

In this paper, I argue that card activities also bring benefits within affective, critical and social domains of learning. I draw on social practice and multimodal literacy theories – which suggest that the style and physical format of written texts are associated with certain settings and practices – and apply these to learning materials. Thus while conventional worksheets of word-problems are strongly associated with an accepted set of responses in the mathematics classroom, alternatives such as card activities can disrupt learners’ prior expectations of how they should respond, allowing them to engage more critically with the meaning and relevance of their learning to everyday life.

Introduction

An on-going challenge in adult mathematics classrooms is making mathematics relevant to learners’ lives. Traditionally, word-problems have been used in an attempt to present learners with realistic contexts from everyday life. However, research shows that learners rarely engage with the contexts provided (e.g. Cooper and Harries 2002; Wyndham and Saljo 1997; Verschaffel, De Corte and Lasure 1994; Mukhopadhyay and Greer 2001).

In this paper I examine possible barriers to learners’ engagement with these contexts. I present transcripts of classroom discussion to illustrate how adult learners responded to two different classroom activities, both of which presented them with socio-critical contexts. Using social practice theories as a framework, I analyse the extent to which the features of the mathematical learning materials used in each activity served to encourage greater learner engagement with context. I conclude by discussing implications for practice, including the tensions involved in preparing learners for formal assessment.

Background

The Problem with Word-problems

The mismatch between classroom mathematics and everyday numeracy practices has been well-documented through ethnographic studies of how children and adults use numeracy in their lives. These have included studies of: children selling water-melons and sweets in Brazil (Carragher, Carragher and Schliemann 1985; Saxe 1988); adults grocery shopping and weight-watching in the USA (Lave 1988); adults working as market traders, fishermen, builders, carpenters and farmers in Brazil (Nunes, Schliemann and Carragher 1993); and young unemployed adults in Australia (Johnston, Baynham, Kelly, Barlow and Marks 1997). The common theme emerging from these studies was that the situated numeracy practices undertaken by participants were fundamentally different from classroom mathematics. The real-life problems to be solved were generated by participants themselves, and were structured in terms of goals to be achieved, rather than mathematics, with social relationships central to many practices. Participants who struggled with written problems in the classroom were found to perform competently within these meaningful situations.

Attempts to embed classroom learning into contexts more relevant to learners' lives have often proved unsuccessful, particularly the word-problems which dominate many mathematical learning and assessment activities, and which aim to present realistic situations such as those associated with money, measurement or employment. The limitations of traditional word-problems as a genre are explored, for example, by Gerofsky (1996; 1999), who critiques their typical three-part structure, generally consisting of: a "set-up" to establish a minimal story-line; items of numerical information; and one or more question(s). The problem is normally expected to contain two numbers to be combined using addition, subtraction, multiplication or division. As Evans and Tsatsaroni (2000:59) point out:

A student doing a calculation in shopping has different purposes and constraints than when they are doing it in the mathematics classroom. The calculations have to be more accurate in the classroom, because that is what is required, or what it takes to keep the teacher happy, and because this is what is a valid answer in school assessment practices.

Social Practice in the Mathematics Classroom

Social practice perspectives recognise literacy and numeracy as embedded in people's lives in domains of practice such as home, work or the community. The ideas were first developed through studies of literacy in use in a variety of communities, for example by Scribner and Cole (1981), Heath (1983) and Street (1984). Literacy is not regarded as a set of autonomous skills, to be learned in school and transferred unproblematically to other contexts, but as an ideological practice, which "encompasses the knowledge, feelings, embodied social purposes, values and capabilities that are brought into play through the reading and writing of texts" (Mannion and Ivanič 2007:16). Literacy is seen to be practiced differently in different domains, only one of which is the domain of formal schooling (Gee 1996; Barton and Hamilton 1998; Crowther, Hamilton and Tett 2001; Papen 2005).

Many of these ideas have since been applied to numeracy (Baker 1998; Johnston and Yasukawa 2001), including the recognition that numeracy is embedded in social practice and that certain domains of numeracy and mathematics are more highly-valued by dominant discourses than others (Coben 2002). According to Street, Baker and Tomlin (2005:20) numeracy practices involve "the conceptualisations, the discourse, the values and beliefs, and the social relations that surround numeracy events as well as the contexts in which they are located".

Such perspectives have tended to focus on the *mismatch* between classroom mathematics and everyday numeracy practices, and there has been a tendency to categorise classroom activity as conforming to an "autonomous" or "skills" model (Street 1984; Green and Howard 2007). In this paper I recognise the *classroom itself* as a site of social practice, and examine the literacy and numeracy practices which take place within it.

Multimodal Literacies

Studies of literacy as *multimodal* recognise that readers and writers of texts will respond differently to the affordances offered by the modality and materiality of a text; for example to texts on paper, computer screens, or cards (Jewitt and Kress 2003; Pahl and Rowsell 2006; Gillen and Hall 2009). Kress, Charalampos, Jewitt and Ogborn (2001) stress the additional scope for semiosis through bodily interaction with physical written materials as they are handled, owned and manipulated by learners.

Learning materials in adult numeracy classrooms in England frequently take the physical form of A4 (21.0mm x 29.7mm) photocopied worksheets – to such an extent that the phrase

“death by worksheet” will be recognised by many practicing teachers, though scholarly research on its ubiquity is hard to find. In recent years, however, teachers have been encouraged to use alternative formats to supplement, if not displace, the A4 worksheet, such as cards¹³ and mini-whiteboards. Although a few teachers have been using card activities for much longer, the practice became widespread following the dissemination of learning materials and training (DfES 2005; 2007), based largely on research by Swan (2000; 2006). Swan’s research focused on the *cognitive* affordances of cards, which allow learners to develop a deeper understanding through activities such as sorting, ordering and matching representations of mathematical concepts.

In this paper I suggest that card activities not only bring benefits in the cognitive domain, but also in social, critical and affective domains of learning, through their potential to disrupt traditional classroom expectations and practices.

Methodology

This paper presents data collected in two adult numeracy classrooms in England, in which learners’ naturally-occurring discussions were audio-recorded as they worked together in small groups to solve mathematical problems (Oughton 2009; 2012). The audio-recordings offer close insights into how the learners responded to the fictional contexts in which classroom mathematics problems were set.

The Classroom, the Learners and the Teacher

The participating classes (both taught by the same teacher) took place in two adult community education centres which offered free literacy and numeracy provision to any adults lacking qualifications in those subjects. The classes each comprised between eight and twelve learners, aged between 20 and 55 years old. As a requirement of the provision, all learners were working towards the *National Certificate in Adult Numeracy* at either Level 1 or 2¹⁴.

Working in small groups, the learners undertook a variety of mathematical learning tasks, ranging from conventional worksheets of word-problems (in preparation for formal examinations) to innovative card activities. The learners supported each other during all these activities, calling on the teacher’s help only as a last resort. Participants were audio-recorded (with their informed consent) during their usual classroom activities and no intervention was requested for research purposes.

While I would resist the notion of a “typical” classroom, experience as a teacher-educator in this sector leads me to suggest that these classes were by no means *untypical*. Broader

¹³ I do not refer here to playing cards (though these can have their place in mathematics classrooms), but to any learning material or activity presented on cards small enough to be handled and manipulated by learners. This may include, for example: question and answer pairs; representations of mathematical concepts; or quantities for manipulation. Uses might include matching, sorting, ordering, or – as in the example presented in this paper – random selection and allocation.

¹⁴ Level 2 is equivalent to the target level for 16-year-olds completing compulsory schooling in England. The *National Certificate in Adult Numeracy* has now been superseded by the Functional Skills Mathematics qualification, discussed below.

ethnographic accounts of similar classrooms may be found in Appleby and Barton (2008), Rhys Warner and Vorhaus (2008) and Cara et al (2008).

Data Collection and Approaches to Analysis

Mobile phones were used as audio-recording devices, placed unobtrusively on classroom tables. Since learners usually brought their own mobile phones to classes, they had become “part of the furniture” and participants tended to ignore them. Labov (1972) furthermore suggests that speakers’ discourse becomes more natural when they are intensely engaged, as the learners were in their mathematical problem-solving. Learners seemed quickly to forget that they were being recorded, and their talk appeared to become naturalistic within a few minutes.

Eleven hours of recorded discussion was collected over two terms. The audio recordings were then transcribed for analysis, using field notes to enrich the transcription where relevant.

Analysis focused on how learners related classroom activities to their everyday numeracy practices, and the relevance that these seemed to have in their lives. Codes were drawn up to identify and categorise ways in which learners responded to the contexts in which mathematical problems were set.

Samples of the learning materials used in each activity were also collected, and analysed to examine the structure, modality, tenor and materiality through which the contexts of mathematical problems were presented to the learners.

Data from the Classroom

Two illustrative episodes have been selected from the data. The first is representative of the learners’ response to a traditional word-problem, although the context provided is one which might be expected to produce a more a socio-critical response than most. The second is a rarer example, in which the learners responded very differently to the problem they were posed. It may be regarded as a “telling case”, which allows “small facts get in the way of large issues”, and has the potential to disrupt generalisations by showing that alternatives are possible (Mitchell 1984; Hannerz 1987:556).

Episode 1: Percentages Word-problem

Five learners, Ruth, Dawn, Gemma, Jackie and Charlotte¹⁵, were working together to solve the traditional word-problem shown in Figure 1. This was one of ten similar percentages word-problems on an A4 worksheet, intended as preparation for examination at Level 2. Answers to all questions were included on the reverse of the sheet, but the learners assiduously avoided looking at the answers provided until they found their own solution and were ready to check it.

¹⁵ Names of all participants are pseudonyms.

4. Over 2004 there were 220 fatal accidents in the Construction Industry.
55 of them were on building sites.
What percentage of the total fatal accidents was on building sites?

Figure 1. Extract from *Skillsworkshop* worksheet on percentages (Skillsworkshop 2007, original emphasis)

Episode 1:

- 1 Ruth [paraphrasing from worksheet] Right, 220 fatal accidents
- 2 55 of 'em on building sites.
- 3 [reading from worksheet] "What percentage of the total fatal accidents?"
- 4 Dawn Right, so you've got 220
- 5 Over 55
- 6 No, no, it's the other way round isn't it?
- 7 Gemma Is it the other way round?
- 8 Dawn I don't know
- 9 Ruth [reading from worksheet] "What percentage of the total fatal accidents?"
- 10 Dawn Yeah, it's – I think it's that way
- 11 Jackie Which way?
- 12 Gemma Is it the other way round, yeah?
- 13 Dawn I think so
- 14 Ruth 55, yeah, over 220, and then
- 15 Dawn Yeah, so you've got to cancel that down
- 16 Ruth And how d'you do that then?
- 17 Dawn Five'll go in to it, won't it?
- 18 [whispered] Five, ten, fifteen, twenty
- 19 Ruth (...) is eleven
- 20 Dawn Yeah. How many fives into 220?
- 21 Ruth Well fifty's ten, a hundred is twenty
- 22 [laughing] 150 is what, thirty?
- 23 Forty, forty-four
- 24 Forty-four?
- 25 Charlotte Yes
- 26 Ruth So it's eleven forty-fourths? [laughs]
- 27 Jackie Oh, no
- 28 Ruth We surely can get lower than that
- 29 Dawn Yeah, so
- 30 Charlotte Because eleven
- 31 Dawn Yes, eleven'll go into forty-four
- 32 So it'll go in one, and four, so it's a quarter
- 33 Charlotte Yeah
- 34 Ruth Hang on a minute, whoa, whoa, whoa
- 35 Now you've got me now

36 Gemma Do you know your eleven times table?
 37 Ruth Eleven, twenty-two, thirty-three, forty-four.
 38 Dawn So eleven'll go in –
 39 Ruth Hang on, hang on
 40 How do we suddenly –
 41 Because I would have been thinking, what does that, and that, go
 into?
 42 What goes into both of them?
 43 Dawn Yeah, yeah
 44 Gemma Eleven
 45 Jackie So it'll just be one, won't it? Because only one eleven goes into
 eleven.
 46 Ruth Yeah, I'm with you, I'm with you
 47 The bottom is forty
 48 Dawn Eleven'll go into itself once
 49 And it'll go into the bottom (...)
 50 It's a quarter
 51 Ruth Yeah I'm with you
 52 So you've got your job back now
 53 Dawn Right, okay, I'm happy now
 54 Ruth [laughing] Good
 55 Charlotte So that's twenty-five percent
 56 Ruth So that's one quarter
 57 Right
 58 [The learners then move on to the next problem on the sheet.]

I have included the learners' entire discussion on this problem in order to illustrate a pattern which recurred throughout the data I collected; that learners extracted numerical information from the word-problem and carried out their calculations, without appearing to respond at any point to the context in which this problem is set. This is particularly striking when that context might be expected to provoke concern, shock, or at least a query as to its validity.

Lines 1-14 are concerned with reading the word-problem to extract the relevant numerical data and mathematical relationships. Although Ruth reads aloud the potentially emotive words "fatal accident" three times (lines 1, 3 and 9), the learners focus only on the numbers, grappling with the difficulties of which is the denominator. Ruth's paraphrasing of the question in line 1-2 suggests her understanding of the narrative, but she does not reflect aloud on its significance.

From line 15 onwards, the learners respond to the problem merely as an instruction to find 55 as a percentage of 220. Once Gemma has made her case that 220 should be the denominator rather than the numerator, the learners' discussion is solely of arithmetic through to the correct solution at lines 50 and 55, centering largely on the identification of eleven as a common factor.

Rather than responding to the construction industry fatalities context, the learners respond to the expectations of the word-problem genre. For example, they show familiarity with conventions of simplified numerical relationships; in line 27, Ruth recognises that 11/44 is unlikely to be the correct answer, even though she has not yet spotted the equivalence to a quarter. On obtaining the correct answer, checked using the answers provided, the learners show satisfaction, a sense of resolution, and mutual congratulation (lines 50-57).

Episode 2: Wages Cards

The second episode was part of an activity to demonstrate how the mean of a data set can be distorted by outlying values; in this case how the mean salary in a small company might be distorted by one very high salary. The teacher asked each learner in the group to select and keep a card at random from the set of shown in Figure 2.



Figure 2. Wages cards used in Episode 2.

No other details (for example, roles or job titles) were provided on the cards or by the teacher. Nonetheless, the learners spontaneously seized the opportunity to role-play, using their knowledge of typical salaries to match the cards to employee roles:

Episode 2a:

- 1 Teacher If I give you each one of these
:
2 They're meant to be, er, your wage
3 We're all working, all working in a factory or something
4 Okay?
5 [walking round class offering each learner a card to pick at random]
Take one of those
6 Take one of those
7 ...
8 Donna: [seeing Judith's card] That's alright [laughing]
9 Judith: Thank you
10 Donna: Ooh, that's even better
11 You're supervisor
12 What've you got?
13 Teacher How many of us are there? One, two, three, four, five
:
14 [overlapping talk]
15 Donna: Have you? Oh she's the director.
16 Sally: Now that's just not fair
17 [laughter]
18 Donna: [receiving her own card] I'm the cleaner.
19 [laughter]
20 Teacher Right, so we've all got different wages
:
21 Judith: You've picked a big one as well

22 Teacher I'll write on the board what we've all got
 :
 23 And we're going to calculate what the typical wage from our (...)
 24 So what have you got there?
 25 [overlapping talk and chairs banging]
 26 Donna?
 27 Donna: Ten thousand [pounds]
 28 Sally: Twelve
 29 Teacher Twelve thousand
 :
 30 Judith: Miss Moneybags here
 31 [laughter]
 32 Abigail: [in "posh" sing-song voice] I have a hundred thousand
 33 Donna: The director. You're the director.
 34 Abigail: [quietly] I wouldn't know what to do with it anyway

Their spontaneous role-play suggests that the learners felt unconstrained by the expectations of more traditional classroom activities. They demonstrated a playful creative knowingness about wage distribution and a satirically mocking and critical attitude to wealth. Judith's mocking remark in line 30 is not, of course, directed at Abigail, but at individuals who earn high wages, and the shared laughter allows all learners to show solidarity in their relationship to higher earners. In line 32 Abigail ostensibly imitates a happy complacency which those on high wages might be supposed to feel, while at the same time the overtly stylised tone distances her from those she is imitating. She reverts to a more natural and very quiet tone as she dismisses the personal value of wealth in line 34.

Significantly, the role-play enables them to make more critical sense of the eventual conclusion about how the mean has been distorted (Episode 2b below). Note the learners' knowing laughter in response to the teachers comment in line 15, as they identify themselves and each other with the wages on their cards:

Episode 2b:

1 Teacher: How would we calculate the mean?
 2 Abigail: Add them up
 3 Teacher: Yeah, so, add all these values up. What does that actually add up to?
 4 Learners: (...) A hundred and eighty-seven
 5 Abigail: Divide by seven
 6 Teacher: Seven, we've got. Right, divide that by seven.
 7 What do we get?
 8 It's not going in exactly, is it
 9 ...
 10 But it will be over 26,000, it's saying
 11 Well looking at those wages, how many people are actually over £26,000?
 12 Learners: One
 13 Teacher: Only the one
 14 So that mean's been distorted, by an extreme value,
 15 Somebody getting a lot more than everyone else
 16 Learners: [laughter]
 17 Teacher: So that's the problem with the mean
 18 If there are very small numbers or very big numbers it distorts it
 19 And makes it look that the, um, the typical wage is bigger than it actually is

20 Judith: Mm, it's a lot bigger

Note also how this activity does not have a pre-defined “right” answer; it depends on the number of learners participating and the wages they randomly select. The teacher must therefore work together with the learners to calculate the mean value (lines 1-10), and the mean value is not an exact figure but the result of a slightly messy calculation, as are solutions in real-life.

Analysis from a Social Practice Perspective

The Social Context of the Classroom

The numeracy problems in *both* the above episodes were presented in contexts which might be supposed to be of socio-critical significance to adult learners: in the first case, employee safety in the construction industry; and in the second, wealth distribution and the distortion of statistical measures. Yet only in the second did the learners respond to the context provided.

It becomes important at this point to interrogate what is meant by “context”. Dowling (2001:20) describes how attempts to set classroom mathematics problems in supposedly real-life contexts merely “mythologise” the practices they are supposed to represent, while Evans and Tsatsaroni warn of the dangers of:

an overly simplified notion of context as a ‘thin veneer’ of applicability, that only *seemed* to make ‘word-problems’ in the classroom different from abstract calculations (2000:56 original emphasis)

I argue that the context in which the learners are practicing numeracy is *not* that of the construction industry and its fatalities, nor of wage distribution in small companies, but the adult numeracy classroom together with the discourses and structures which regulate it (Papen 2005; Oughton 2007). The classroom should be seen as a site of social practice in order to examine the literacy and numeracy practices which take place within it.

Thus in my analysis below of how learning materials are used in the classroom, I take into account features of the classroom as summarised in Table 1 below, drawing on Chouliaraki and Fairclough (1999); Street, Baker and Tomlin (2005); and Barton and Hamilton (1998; 2000).

Table 1. The Social Context of the Adult Numeracy Classroom: Key Elements

Physical setting:

A classroom in a dedicated adult community education setting
Surrounded by educational resources such as whiteboards, textbooks and educational posters
The materiality, modality and mediation of the learning materials

Historically and socially situated:

The *Skills for Life* infrastructure – classroom activity is planned around the curriculum, qualifications and targets
Relationships with other learners and with the teacher
The cultural capital associated with success in mathematics
Mathematics qualifications as a gateway to employment
Learners’ own history of schooling

Learners' social purposes – long term:

To obtain a qualification needed for work or further study

To help own children with their schoolwork

For personal fulfilment

Learners' social purposes – short term:

To obtain the “correct” answer – usually the one given on the answer sheet

To support each other in solving the problem


The classroom context can be seen to be strongly associated with the curriculum, accreditation and school mathematics. Thus any artificial “context” provided by the learning materials needs to engage the learners sufficiently to disrupt these dominant discourses of the classroom.

Literacies and the Percentages Worksheet

This worksheet was downloaded by the teacher from the *Skillsworkshop* website, through which teaching resources are contributed by volunteers and made freely available to teachers. The following is therefore *not* intended as a critique of the website, the worksheet nor its author, but a comment on the curriculum and accreditation structures and discourses of which it is part.

Although this was a word-problem, it was less conventional than some. It referred to a specific year (most word-problems use, anomalously, the present indicative tense) and presented numerical data which, if accurate, might be regarded as a cause for social concern. On first listening to the audio-recording, I was initially surprised (and, admittedly, shocked) at the casual way the learners repeated the phrase “fatal accidents” without responding to it. Their response appears to conform to what Street (1984) would describe as an “autonomous” model.

August 2007. To print your own copies of this document visit <http://www.skillsworkshop.org>

 **Percentages – Level 2**

1. A jacket normally costs £79.50.
In a sale there is 30% off the normal price.
What is the sale price of the jacket?
2. Organisers want to raise a total of £250 for new equipment.
The sponsored walk raises £150.
What % of the total does the sponsored walk raise?
3. The cost of a new driveway is £2400 before VAT.
VAT is 17.5%.
What is the total cost of the job including VAT?
4. Over 2004 there were 220 fatal accidents in the Construction Industry.
55 of them were on building sites.
What percentage of the total fatal accidents was on building sites?
5. A DVD player normally costs £239.50.
The price increases by 25%.
How much is the DVD player after the increase?
6. A family pays £1200 per month on bills.
They pay British Gas £720 per year.
What percentage of the monthly bill, goes to British Gas?
7. A survey shows that there were a total of 4250 crimes last year.
25% of crime is burglary.
How many crimes were due to burglary?
8. A weekly shop costs £85.00.
A man gets a 15% off with some coupons.
How much does he pay?
9. 9870 people go on 10km Run.
Only 60% complete the race.
How many complete the race?
10. A shop reduces a table lamp from £19.99 to £17.99.
The shopper rounds the prices to the nearest pound to work out the percentage reduction.
What is the % reduction?

This resource kindly contributed by Ian Saunders, Darlington Borough Council Training Unit Ian.Saunders@darlington.gov.uk
NZ12.7 Order and compare percentages and understand percentage increase and decrease. NZ12.8 Find percentage parts of quantities and measurements. NZ12.9 Evaluate one number as a percentage of another.

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Percentages – Answers

As calculators are not permitted in the Level 2 exam – quick mental methods have been shown where possible (but these are not the only way of answering the questions!).

1. 10% of £79.50 = £7.95 (move the decimal point one place to the left)
So 30% = (£8 x 3) = 15p = £23.85
Sale price of jacket = (£79.50 - 23.85) = £55.65
2. $150/250 = 3/5 = 60\%$ of total needed for new equipment
3. VAT = $240 \times 120 \div 60$ (i.e. $10\% \times 5\% + 2.5\%$) = £420
So total cost of the job = £2400 + £420 = £2820
4. $55/220 = 11/44 = \frac{1}{4} = 25\%$ fatal accidents were on building sites
5. 25% of £239.50 = £59.875 (divide by four or halve then halve again)
Round to £59.88
So DVD player is (£239.50 + £59.88) after increase = £299.38
6. A family pays £1200 per month on bills.
£720 ÷ 12 = £60 per month on gas.
 $60/1200 = 1/20 = 5\%$ monthly bill goes to British Gas
7. $4250 \div 4 = 1062.5$ (halve, then halve again)
Rounds up to 1063 crimes due to burglary
8. A weekly shop costs £85.00.
15% of £85 = £8.50 (10%) + £4.25 (5%) = £12.75
So man pays £85.00 - £12.75 = £72.25
9. 50% of 9870 = 4935 (divide by 2)
10% of 9870 = 987 (move decimal point one place left)
So 60% = 4935 + 987 = 5922
5922 complete the race
10. A shop reduces a table lamp from £19.99 to £17.99
£20 - £18 = £2 reduction
 $2/20 = 1/10 = 10\%$ reduction of original price

This resource kindly contributed by Ian Saunders, Darlington Borough Council Training Unit Ian.Saunders@darlington.gov.uk
NZ12.7 Order and compare percentages and understand percentage increase and decrease. NZ12.8 Find percentage parts of quantities and measurements. NZ12.9 Evaluate one number as a percentage of another.

Figure 3. Percentages Level 2 worksheet, front and reverse (Skillsworkshop 2007)

The percentages worksheet itself was a single A4 sheet, photocopied in black and white, and presenting ten mathematical word-problems. The A4 format is very commonly used in classrooms throughout all educational stages in England, and learners would have encountered similar materials both in their own schooling and in the homework their children bring home from school. The title of the worksheet, *Percentages – Level 2*, is strongly classified and framed by curriculum area and level (Bernstein 2000). The word-problems on the sheet conform to the genre commonly found in classrooms and assessment materials. Most make anomalous use of the present indicative tense, and present the typical three-part structure: a “set-up” to establish an arbitrary scenario; items of numerical information; and a question, reinforced using bold type. Answers are on the reverse, together with hints on carrying out the necessary calculations. Advice is given on examination preparation, emphasising that learners need to work out answers without a calculator; a constraint which would not apply to real-life numeracy problems. The answers are neat and familiar percentages (unlike the intermediate answer of 11/44 obtained by the participants in Episode 1), and they are understood to be exact, correct and non-negotiable. Thus this format will be firmly associated with the classroom and formal learning¹⁶.

The teacher inevitably had a mediating role as she distributed the worksheets, which reinforces this classification and framing. Her introduction was couched in terms of a forthcoming examination, the *National Certificate in Adult Numeracy* at Level 2 (the “it” in line 6 refers the examination itself):

Episode 3

- 1 Teacher: Right, the sheets I’ve got
- 2 I’ve got quite a lot of different ones really here
- 3 ...
- 4 Do level two if you think
- 5 Have a go at level two questions
- 6 Cos it’s coming up in two weeks time

Literacies and the Wages Cards

Given this analysis of how the worksheet reinforced the classroom context, which features of the cards used in the second activity might have encouraged the learners towards a more socio-critical response?

The activity was closely mediated by the teacher, who created a subjunctive mood by inviting the learners to *imagine* themselves in a different situation: “They’re meant to be, er, your wage. We’re all working, all working in a factory or something,” (Episode 2a, lines 2-3). The first and second person pronouns (*your*, *we’re*) serve further to engage the learners in identifying with the position of employees in the factory.

The cards (Figure 2) were laminated and tactile, and only slightly smaller than a bank note or a wage slip. The tactile quality seemed to give each participant a sense of ownership (c.f.

¹⁶ Note that this is not due to the intrinsic nature of the format, but its association with school classrooms. Colleagues who were teaching in the sector at the time recall worksheets first being introduced as a revolutionary and flexible alternative to textbooks (my thanks to David Kaye for this insight).

Kress et al 2001) and their random distribution may be seen as mirroring perceived randomness in wealth distribution.

In contrast to non-negotiable “correct” answers on the back of the percentages worksheet, the result of calculating the mean wage was not known, even by the teacher, at the outset. The answer depended on the number of learners participating, and their random selections from the cards. The teacher encouraged them to calculate the mean without an electronic calculator, and was obliged to do the arithmetic along with her learners. The result involves division by seven and, in contrast to the round, tidy answers on the percentages worksheet, is a messy, inexact number, as are solutions to numerical problems in real-life. In another parallel with real-life problems, the teacher concludes that they do not need the exact answer in order to draw their conclusion; a value accurate to the nearest £1,000 is sufficient to see that the mean has been distorted by one very high wage, and is not representative of the other values.

Discussion

As a former adult numeracy teacher myself, I am aware that mathematics does not have to be functional to be meaningful to learners. Learners may study mathematics for its own sake, to help their children or for a qualification (Swain 2005). Nonetheless, since functionality is, at present, central to policy and accreditation in England, it is worth considering whether this enforced emphasis succeeds in making classroom mathematics more relevant to learners’ lives.

I have outlined how the real and present social context for the classroom is dominated by curriculum and accreditation constraints and expectations. Thus for learning materials to be effective in engaging learners with the fictional contexts they present, they must be powerful enough to disrupt the dominant expectations of the classroom.

In Episode 1, the textual features of the percentages worksheet: its mediation by the teacher in terms of exam preparation; its A4 format; its layout, tenor and content; are strongly associated with the discourses of the mathematics classroom and serve to *reinforce* learners’ expectations about how they should respond. They ignore the construction industry context and strive to obtain the correct answer.

By contrast, in Episode 2 the random distribution of the wages cards by the teacher; their physical possession by the learners; and lack of known “right answer” appear to *disrupt* the classroom context, and the learners’ expectations about how they should respond. They thus identify themselves with the wages indicated by their cards; assign themselves roles; simulate mockery or complacency; and laugh in resignation at the apparent injustice of the unequal wages. The wage card activity is not only effective cognitively in helping learners understand the concept of the mean, but it also brings benefits within critical and social domains of learning.

This paper presents just two examples of numeracy classroom activity, and does not attempt to make specific recommendations for practice. Nonetheless, single cases have the potential to disrupt over-generalisation and to demonstrate the ways in which superficially similar situations can differ from each other.

The following table summarises the characteristics which appeared to make the learners respond differently to these two learning activities and the texts associated with them. By developing learning materials which feature more of the characteristics apparent in Episode 2 (wages cards), teachers may be able provide classroom activities which are more meaningful to their learners and relevant to practices outside the classroom. Some of these characteristics have previously been found effective in encouraging learners to relate classroom mathematics

to everyday numeracy practices, by changing the subject of word-problems from the third person to the second person (Palm 2008), or the conditional “would” instead of the present indicative typical of traditional word-problems (Oughton 2009).

Table 2: Key characteristics of learning materials	
Word problems	Alternatives
A4 photocopied sheet – associated with schooling	Physical resources – can be handled, shared and “owned”
Introduces fictional third person	Addresses learner directly (first or second person)
Present tense, indicative mood - states a “fact”	Subjunctive mood - invites learner to imagine a situation
One pre-defined “right” answer	Several possible answers, or answer not pre-defined
Answer is “tidy” (e.g. 25%)	Answer is “messy” (e.g. division may not be exact or fractions may not terminate)
Exact answer required	Approximate answer may suffice
<i>Reinforces the classroom context</i>	<i>Disrupts the classroom context</i>

Final word: The issue of formal assessment

While Table 2 offers tentative recommendations for developing learning materials, one issue cannot be ignored. The achievement of formal qualifications remains a primary aim in many adult mathematics classrooms. This impetus comes partly from the learners themselves, who may need a qualification to progress in work or further study, and is also imposed by policy; funding is dependent on the successful achievement of qualifications.

At the time of writing, the most common qualification taken by adult numeracy learners in England was *Functional Skills Mathematics* at Level 1 or 2. The assessment for this qualification does indeed represent an improvement on its multiple choice predecessor, the *National Certificate in Adult Numeracy*. Many of the questions have several stages, and are designed to assess reasoning and problem-solving skills rather than a single right answer. Some awarding bodies (for example AQA and OCR) provide a separate data booklet in which candidates must look up numerical information in order to solve the problems.

Nonetheless, barriers remain which reinforce the classroom context. The assessment materials still take the ubiquitous form of word-problems presented on A4 sheets, and where data booklets are provided they take the same format. The word-problems themselves generally introduce a superficial scenario, an arbitrary fictional third person, and are written in the present indicative tense.

The question of whether formal written examinations are the best way to assess adult learners’ numeracy skills is beyond the scope of this paper. Nonetheless, I argue, so long as this form of assessment prevails, it will present a barrier to learners’ full engagement with context and relevance in their numeracy learning.

References

Appleby, Y. and Barton, D. (2008) *Responding to People’s Lives*. Leicester: NIACE

- Baker, D. (1998) 'Numeracy as Social Practice; an adult education context in South Africa' *Journal of Literacy and Numeracy Studies* 8(1) pp. 37-50
- Barton, D. and Hamilton, M. (1998) *Local Literacies: Reading and writing in one community* London: Routledge
- Barton, D. and Hamilton, M. (2000) 'Literacy Practices' in D. Barton, M. Hamilton and R. Ivanič *Situated literacies: reading and writing in context*, London: Routledge, pp. 7-15.
- Bernstein, B. (2000) *Pedagogy, symbolic control and identity* Revised edition. Lanham MD: Rowman and Littlefield.
- Cara, O., Litster, J., Swain, J. and Vorhaus, J. (2008) *The Teacher Study: The impact of the Skills for Life strategy on teachers, Summary Report* London: NRDC.
- Carraher, T., Carraher, D., and A. Schliemann (1985) 'Mathematics in the streets and in schools.' *British Journal of Developmental Psychology*, 3 pp.21-29.
- Chouliaraki, L., & Fairclough, N. (1999). *Discourse in late modernity : rethinking critical discourse analysis*. Edinburgh: Edinburgh University Press.
- Coben, D. (2002) 'Use Value and Exchange Value in Discursive Domains of Adult Numeracy Teaching' *Literacy and Numeracy Studies* 11(2) pp. 25-35
- Cooper, B. and Harries, A. (2002) 'Children's Responses To Contrasting 'Realistic' Mathematics Problems: Just How Realistic Are Children Ready To Be?' *Educational Studies in Mathematics* 49 pp.1-23
- Crowther, J., Hamilton, M. and Tett, L. (2001) *Powerful Literacies* Leicester: NIACE
- DfES (2005) *Improving Learning in Mathematics*. London: DfES
- DfES (2007) *Thinking Through Mathematics: Strategies for Teaching and Learning*. London:DfES (Maths4Life)
- Dowling, P. (2001) 'Mathematics Education in Late Modernity: Beyond Myths and Fragmentation' in B. Atweh, H. Forgasz and B. Nebres (Eds) *Sociocultural Research on Mathematics Education: An International Perspective*. Mahwah NJ: Lawrence Erlbaum Associates, pp.19-36
- Evans, J. and Tsatsaroni, A. (2000) 'Mathematics and its publics: texts, contexts and users' *Social Epistemology* 14(1) 55-68.
- Gee, J. (1996). *Social linguistics and literacies: Ideology in discourses* (2nd Edition.). London: Falmer.
- Gerofsky, S. (1996) 'A linguistic and narrative view of word problems in mathematics education, *For the Learning of Mathematics*, 16(2) 36-45.
- Gerofsky, S. (1999) 'Genre analysis as a way of understanding pedagogy in mathematics education, *For the Learning of Mathematics*, 19(3) 36-46.
- Gillen, J. and Hall, N. (2009) *The Edwardian postcard: a revolutionary moment in rapid multimodal communications*. Paper presented at British Educational Research Association Annual Conference, University of Manchester
- Green, A. and Howard, U. (2007) *Skills and social practices: making a common cause*. London: National Research and Development Centre for Adult Literacy and Numeracy (NRDC)
- Hannerz, U. (1987) 'The World in Creolisation' *Africa: Journal of the International African Institute*, 57 (4) pp. 546-559
- Heath S. B. (1983) *Ways with words: language, life, and work in communities and classrooms*. Cambridge: Cambridge University Press
- Henderson, A. (2012). *Dyslexia, Dyscalculia and Mathematics: A Practical Guide for Teachers*. Abingdon: Routledge
- HSE (2005) 220 *Work Deaths Last Year* Health and Safety Executive press release C019:05. www.hse.gov.uk/press/2005/c05019.htm [accessed 19/09/13]
- Jewitt, C. and Kress, G. (2003). *Multimodal literacy*. New York: Peter Lang.
- Johnston, B. and Yasukawa, K. (2001) 'Numeracy: Negotiating the World Through Mathematics' in B. Atweh, H. Forgasz and B. Nebres (Eds) *Sociocultural Research on Mathematics Education: An International Perspective*. Mahwah NJ: Lawrence Erlbaum Associates, pp. 279-294
- Johnston, B., Baynham, M., Kelly, S. Barlow, K. and Marks, G (1997) *Numeracy in Practice: Effective Pedagogy in Numeracy for Unemployed Young People – Research Report* Sydney: University of Technology Centre for Language and Literacy / Australian National Training Authority, DEET
- Kress, G., Charalampos, T., Jewitt, C., and Ogborn, J. (2001) *Multimodal Teaching and Learning: The Rhetorics of the Science Classroom*, London: Continuum
- Labov, W. (1972) *Sociolinguistic Patterns* Philadelphia, PA: University of Pennsylvania Press
- Lave, J. (1988) *Cognition in Practice*. Cambridge: Cambridge University Press
- Mannion, G. and Ivanič, R. (2007) 'Mapping literacy practices: theory, methodology, methods', *International Journal of Qualitative Studies in Education* 20(1) pp. 15-30
- Mitchell, J. C. (1984) 'Typicality and the Case Study' in Ellen R.F. (Ed) *Ethnographic Research: A Guide to General Conduct*. London: Academic Press pp. 238-241

- Mukhopadhyay, S. and Greer, B. (2001) 'Modeling with Purpose: Mathematics as a Critical Tool' in B. Atweh, H. Forgasz and B. Nebres (Eds) *Sociocultural Research on Mathematics Education: An International Perspective*. Mahwah NJ: Lawrence Erlbaum Associates, pp. 295-312
- Nunes, T., Schliemann, A. and Carraher, D. (1993) *Street Mathematics and School Mathematics*. Cambridge: Cambridge University Press.
- Oughton, H. (2007) 'Constructing the "Ideal Learner": A Critical Discourse Analysis of the Adult Numeracy Core Curriculum' *Research in Post-Compulsory Education* 12 (2) pp.259-275
- Oughton, H. (2009) 'A Willing Suspension of Disbelief? "Contexts" and Recontextualisation in Adult Numeracy Classrooms', *Adults Learning Mathematics: An International Journal* 4(1) pp.16-31.
- Oughton, H. (2012) 'Whose funds? Whose knowledge? Conformative and disruptive discourse in Skills for Life classrooms', *Studies in the Education of Adults*, 44 (2) pp. 204-224
- Pahl, K. and Rowsell, J. (2006) *Travel Notes from the New Literacy Studies: Instances of Practice (New Perspectives on Language and Education)* Clevedon, England: Multilingual Matters
- Palm, T. (2008) 'Impact of authenticity on sense making in word problem solving' *Educational Studies in Mathematics* 67 pp.37–58
- Papen, U. (2005) *Adult Literacy as Social Practice: More than Skills (New Approaches to Adult Language, Literacy and Numeracy)*. Oxford: Routledge.
- Rhys Warner, J. and Vorhaus, J. (Eds) 2008. *The Learner Study: The impact of the Skills for Life strategy on adult literacy, language and numeracy learners. Summary Report* London: NRDC
- Saxe, G. (1988). 'Candy selling and math learning.' *Educational Researcher*, 17(6), pp.14-21
- Scribner, S., and Cole, M. (1981). *The psychology of literacy: A case study among the Vai*. Boston: Harvard University Press
- Skillsworkshop (2007) *Percentages – Level 2* <http://www.skillsworkshop.org/resources/percentage-problems> [accessed 19/09/13]
- Street, B. (1984). *Literacy in Theory and Practice*. Cambridge: Cambridge University Press.
- Street, B., Baker, D. and Tomlin, A. (2005) *Navigating Numeracies: Home/School Numeracy Practices*, Dordrecht: Springer
- Swain, J. (2005) "'Beyond the Daily Application": motivations for adults attending numeracy classes', *Research in Post-Compulsory Education*, 10(3) pp. 305-323
- Swan, M. (2000) 'GCSE mathematics in further education: challenging beliefs and practices', *The Curriculum Journal*, 11(2) pp.199–233.
- Swan, M. (2006) *Collaborative Learning in Mathematics: A Challenge to our Beliefs and Practices*. London: National Institute for Adult and Continuing Education (NIACE); National Research and Development Centre for adult literacy and numeracy (NRDC).
- Verschaffel, L., De Corte, E. and Lasure, S. (1994) 'Realistic Considerations In Mathematical Modeling Of School Arithmetic Word Problems' *Learning and Instruction*, 4 (4) pp.273-294.
- Wyndham, J. And Saljo, R. (1997) 'Word Problems And Mathematical Reasoning – A Study Of Children's Mastery Of Reference And Meaning In Textual Realities' *Learning and Instruction* 7 (4). pp. 361-382.

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Biographical Summary

Helen Oughton has 18 years' experience in teaching and teacher-education in adult numeracy. She is currently a researcher and doctoral supervisor at the University of Bolton, UK. Her research interests include discourse and power in adult numeracy, and making classroom numeracy relevant to adult learners' lives.

Having Some Fun with Maths – The Aussie Way

Dave Tout

Language Australia

This practical and hands-on workshop presented a number of examples of student activities that illustrate different strategies that can be used to support successful adult numeracy teaching. They are based upon the following approaches to the teaching of adult numeracy.

Maths Language

Encourage and use familiar and relevant language in the classroom. “Talking maths” and sharing words and meanings between students, and with the teacher, is the best way to overcome difficulties in comprehending maths language. Activities that support and encourage this should be used.

Co-operative Work

Get students to work co-operatively together to encourage interaction and discussion, and hence help learn from each other. This can be in pairs, small groups or even as a whole class group. This approach encourages learning through communication from a number of sources, not just the teacher or textbooks. Students will learn by having to explain their ideas to each other.

Enjoyment and Success

Many adult students have had negative experiences of maths teaching and may suffer from maths anxiety. It is important therefore to provide an exciting classroom atmosphere with a range of activities and teaching experiences which stimulate interest, discussion, and active learning. Most of all learning should be fun. Maths activities, including games, can demonstrate concepts whilst providing an opportunity for students to interact in a relaxed and enjoyable way.

There is nothing better than getting students in the group to experience success in solving problems, either in a group or individually. This will build confidence and help overcome any maths anxiety they may have.

Practical and “Hands-on” Materials

Remember that most adults were taught maths by traditional pen and paper methods and were expected to remember rules and formulas by memory without ever really understanding them. Concrete materials are a great way of explaining to adults why things really do work. If you don’t understand a concept properly to start with then consequent learning becomes an almost impossible task.

Relevant Contexts

Try to place maths learning in a context relevant to the students, drawing on students' backgrounds, interests, and experiences. This includes placing mathematical ideas into an historical and social context. It could take into account such maths related activities as, for example, shopping and banking, measuring, cooking, the weather, reading timetables and street directories, following directions, and sports activities.

Students can learn about relevant topics while doing mathematics. Many areas of knowledge require an understanding of maths concepts and skills. Areas such as the environment, health and diet, geography, statistics, and public and political processes can all be used as content areas of mathematics.

Teaching by Understanding

Teach by understanding, not simply by a reliance on memorization. Often people have learnt the “wrong” way to solve a problem and until they are shown why their way is wrong and *why* your way works they will have difficulty in coming to terms with the correct methods. Although the ultimate aim may be to know some facts off by heart (e.g., times tables) the way to achieve that is to ensure that initially the reasons and understanding behind what is happening and why you do something is clearly explained and understood. The memorization can then follow through practice and reinforcement.

Sample Activities

Activities selected from the following list will be demonstrated during the session.

Data and graphs

- average wages
- pumping petrol

Cooperative logic problems

- the flats
- the supermarket
- cities
- what's the number
- where is grandma's house?

Decimals

- it was one of those days
- double zero three and nine
- target 100
- decimal dilemma

Dice games and place value

- multi-digit
- double digit
- dicing with decimals

The above activities are taken from the following Australian adult numeracy resources:

References

- Goddard, R., Marr B., & Marin, J. (1991). *Strength in numbers*. Melbourne: Adult, Community and Further Education Board.
- Johnston, B., & Tout, D. (1995). *Adult Numeracy teaching—Making meaning in maths*. Melbourne: National Staff Development Committee for Vocational Education and Training.
- Marr, B., Anderson, C., & Tout, D. (1994). *Numeracy on the line*. Melbourne: National Automotive Industry Training Board.
- Marr, B., & Helme, S. (1987). *Mathematics: A new beginning*. Melbourne: Norther Metropolitan College of TAFE.
- Marr, B., Helme, S., & Tout, D. (1991). *Breaking the maths barrier*. Canberra: Department of Employment Education and Training.

Institutional Support

Bridging the Gap for First Year Students with the Use of Peer Tutoring*

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Abstract

First year mathematics and statistics classes generally do not accommodate to the range of technical abilities which students have when they enter into first year university. UTS offers extra support for first year students by the use of a Mathematics and ICT Study Centre. This resource is available for students from a variety of faculties, who study or use Mathematics in their degree. A peer tutoring service has been implemented in the study centre with volunteer students from third year mathematics and statistics classes being rostered, simultaneously bridging the gap in the budget and the students' education. This paper describes a study which used qualitative methods to examine the motives and benefits that have opened up for the student tutors through peer tutoring.

Peer tutoring is defined as students taking on the role of a teacher to other students of either the same academic level or lower. It is important because it has great potential in the development of students' skills both educationally and personally.

Much work has been done on looking at the benefits of peer education as a whole and of different aspects of peer education. Damon and Phelps (1989) distinguish three approaches to peer education namely peer tutoring, cooperative learning and peer collaboration.

Cooperative learning is generally a team based learning approach where students pool their resources together on a particular topic and in peer collaboration simultaneously approach all aspects of a topic, constantly working together. With these two aspects of peer education, the students are generally on an equal level to one another whereas in peer tutoring, one student takes on the role of a tutor and the other(s) take the role of a tutee. Griffin & Griffin (1998) investigated the positive effects of reciprocal peer tutoring on various educational levels.

They found that peer tutoring is effective at increasing student achievement for both the tutor and the tutee with the tutor often benefiting more than the tutee. Peer tutoring schemes have been implemented in a variety of subjects and educational levels. Carroll (1996) discusses the effectiveness of senior medical students acting as co-tutors working in tandem to the academic tutor for first year biology students and Bush (1985) describes a peer tutoring program used for introductory accounting courses as a possible suitable substitute to current laboratory classes. In both of these cases the senior students were paid as academic tutors and were able to relieve full time academic staff while at the same time providing quality tuition to first year tertiary students.

Peer tutoring has benefits not only for the tutee but also for the tutor as a means of promoting educational and personal development. Both Hopkin (1988) and Houston & Lazenbatt (1999) investigate the use of reciprocal peer tutoring within a class environment at a tertiary level

where each student (or group of students) in the class is responsible for a particular topic and then take on the role of the tutor, teaching that particular topic to the other students in the class. They found that this type of peer tutoring fostered independent and responsible learning, promoted greater levels of communication, student participation and a deeper understanding of the work involved (mathematically in Hopkin and educationally in Houston & Lazenbatt) for the tutors.

This paper describes a study of a peer tutoring program where senior students voluntarily take on the role of an academic tutor for first year mathematics students. Unlike the other papers discussed, this study used qualitative analysis of the use of peer tutoring from senior students who were volunteering their time and efforts, not for financial gain or means of passing a subject, but rather as a genuine commitment to the further development of mathematics for themselves and others. We shall describe the different approaches the tutors took in teaching mathematics, their reasons for volunteering, accomplishments achieved and any connections between them, how their views on learning mathematics and mathematics in general have changed since tutoring, and look into the different approaches to and depths of their own mathematics and how the peer tutors' responses reflect these. This paper is important because it confirms the belief that positive benefits are gained through peer tutoring while exploring a new realm of what is possible within this tutoring system.

Background

The transition for students from secondary school to tertiary education involves major changes not only academically, but also socially and environmentally. At the University of Technology, Sydney (UTS) there is a mathematics study 'drop-in' centre where students come and receive support from tutors. It is also commonly used as a meeting place for collaborative learning where students do group work and often receive support from other students. This form of social academic interaction has helped make the transition less stressful for the first year students.

Mathematical support for first year students is available in universities through a variety of means. There are standard tutorial and laboratory classes, some subjects implement extra support tutorials and students can also visit lecturers for extra support when needed. At UTS there is also the operation of a Mathematics and ICT (information and communications technology) study centre. The operations of the study centre include the running of support tutorials, workshops, bridging courses as well as the drop in centre where a tutor is available to give support for first year students requiring help in mathematics, statistics and introductory computing. There are four computers in the drop in centre which enable students to receive assistance in using the mathematical and statistical programs. Throughout the past few years the study centre has been a popular environment for mathematics and statistics students from a range of faculties. They visit the centre for assistance and extra support. It also became popular in its use as a meeting place for collaborative learning.

Peer tutoring program

Traditionally at UTS, honours and doctorate students have been paid to act as peer tutors in the Mathematics and ICT drop in study centre. Last year the implementation of a peer tutoring service, in which third year volunteer students were added onto the tutoring roster, was incorporated into the drop-in study centre. In the semester just passed (Autumn, 2005) the tutors consisted of one doctorate student, three honours students, one full-time staff member/researcher as supervisor and 18 third year volunteer students. This has shown to be highly effective among the first year students, the tutors (third year volunteer students and honours students) and also on the budget, making it a win-win situation for all involved.

The requirement for the volunteer tutors was that they were each to tutor in the drop in study centre for one hour every lecturing week, with their choice of tutoring during the non-lecturing weeks, whereas the paid tutors were in the centre every week. The volunteer tutors could either do this hour on their own or pair up with another volunteer and share a two-hour shift between them. There was no minimum academic achievement result required for the students to volunteer as a tutor as there were always other resources available for the tutors if they did not feel confident in answering a particular question asked by a student. Emphasis was placed on the tutors' responsibility and communication for the successful operation of the drop in study centre. If a tutor could not make his or her shift for any reason it was requested that they attempt to swap a shift with another student for that week and if they were unable to do so that they be in communication with the supervisor who would try and find a replacement or post a notice of their absence.

Towards the end of semester the tutors were asked to complete a survey on their learning experiences while assisting in the study centre. The survey was designed to address the nature of teaching in the drop in study centre, the experience gained by the tutors and to gain an insight into peer tutors' different points of views for the learning and teaching of mathematics at an introductory tertiary level. In addition to demographic questions, the following questions were asked:

- Why did you originally volunteer or sign on for peer tutoring?
- How do you go about teaching mathematics in the maths study centre?
- What are your views of mathematics and how have they changed since tutoring?
- What are your views on learning mathematics and how have they changed since tutoring?
- What sense of accomplishment do you feel you have gained through peer tutoring?

The survey was not compulsory and out of the 22 peer tutors in the study centre (other than myself) there were 12 responses. Of those students, nine were third year volunteers and three were paid honours students. The details of the students participating in the survey are shown in Table 1.

Student Reference	Degree	Year
A	B. Maths and Finance	3rd
B	B. Maths and Finance	3rd
C	B. Maths and Finance (Honours)	4th
D	B. Maths and Finance	3rd
E	B. Maths and Finance	3rd
F	B. Science in Maths	3rd
G	B. Maths and Finance	3rd
H	B. Maths and Finance	3rd
I	B. Maths and Finance	3rd
J	B. Science in Maths (Honours)	4th
K	B. Science in Maths (Honours)	4th
L	B. Maths and Finance	3rd

Table 1. Students in Survey

Methods of teaching

The tutors used a wide variety of methods and resources when teaching in the maths study centre. Some of the tutors explained the theory and concepts involved behind the questions, as seen by [G] and [H] (below) taking a deeper approach to teaching, while others used examples and explained the steps involved to show how to tackle problems in a more systematic manner as seen by [L]. Sometimes these differences in approach were due to how much the tutors felt the students understood or wanted to know, for example [J] and [K].

[G] ...I try to give them an overview of the basics to ensure that they aren't just learning how to do the question.

[H] I tried to clarify what sort of explanation/help they really require and explain things in the simplest way I can.

[L] I attempt the question by myself first and when I get an answer I show the student how I reached that answer. I clearly explain what I did in each step and make sure that they understand the method.

[J] ...more times than not its just that they want help in doing a single problem or it's an assignment problem. It's not often at all that someone will walk up and say 'hey I don't get integration by parts can you help me review it?'

[K] I read the question and ask them what it is they don't understand about it, then I teach them the concept or the topic they are confused with, and after that if they still have trouble doing that particular question, I give them an example similar to that question, and usually at this stage they can do the question on their own

Many questions required a unique modelling approach rather than a systematic manner of teaching, such as some science or worded questions. Some peer tutors found it challenging to get these types of questions across to the students, for example [C], [I] and [J] (below). When difficulties were encountered in explaining the work the tutors used a variety of different resources. All of the tutors incorporated other resources into their teaching in some way or another, such as looking up topics from the students' textbook or textbooks in the centre, incorporating the assistance of fellow peers in the centre at the time and when all else fails, requesting the assistance of a lecturer [F] was always an option. The peer tutors also asked the students requiring assistance what they knew about the topic as both a resource for the tutors, and a solidification of the student's knowledge, as seen by [I] and [J]. There was a great emphasis on communication between the peer tutors and students while teaching. Much of the communication from the students to the tutors seemed just as important for their understanding of the work [J] as from the tutors to the students, helping to solidify the students' knowledge at the same time as having them feel comfortable to ask the questions [B].

[C] ...it is not very easy to explain maths to students. Well I do what I can, drawing a diagram or graphs is often helpful...

[I] We may have never seen the application of maths with regard to some engineering and finance students, or it may have been years since we had dealt with a topic. So in working with the student we need them to tell us everything they know and maybe to isolate textbook material. This process is often times as important to the solidification

of the student's understanding as are the links and explanations that the tutor can offer.

[J] I Try to think about the best way to explain what it is happening in the question in a pseudo mathematical way (important for word questions). If I don't know I ask to see the textbook and work over that section with them. Or I try to think though the problems from scratch with some help from them ... I then try to walk them though it with them prompting me on what to do or I ask them questions on what should happen next

[F] If I can answer it straight away, then I do, if not I look for a friend in the MSC at that time who could help. I have also called a friend on the phone to help me. If I feel there is no way that I can help, I suggest the student visit their lecturer, or direct them to a lecturer that I know will be able to help.

[B] I would also try to develop a friendship between students so as for them to feel comfortable asking questions and being honest with what they do and don't understand.

Views on mathematics and learning mathematics

Different views of mathematics are apparent through the peer tutors' responses. We have defined three main levels in which people view mathematics. A lower level view of mathematics is that it is a tool used for calculating things. A mid level view starts to see mathematics in an abstract or analytical way, as well as seeing how it can be used in modelling real life situations. The highest level of viewing mathematics is that everything in life can be described by mathematics. Students who view mathematics on a lower level often take a surface approach to learning how to do maths, often by rote learning. Students who view mathematics on a mid and higher level usually foster a deeper approach to learning mathematics as they go beyond seeing it as just a tool and consider how it relates to life in a modelling sense, and analytically how it starts to relate to itself, as an abstract language. Students who didn't understand the work on a deeper level would be more likely to struggle to follow topics [B-1] (see below). Most of the peer tutors had a mid or high level view of mathematics. A high level view can be seen in [B-2] and [L]. In [D] it is seen in the practical sense but not very deep in the analytical sense. Most of the tutors viewed mathematics and learning mathematics as challenging [B-2], [D], and interesting [G].

[B-1] ...there is a core requirement for understanding mathematics which is basic understanding and this is one thing I discovered whilst tutoring. Students who missed one core fact of a topic would have trouble understanding everything related.

[B-2] I have always viewed Mathematics as a fundamental subject to education which can be very challenging and thus requiring logical and analytical thinking. I believe maths exists in all areas of life and even in more subtle situations like decision making which seems maths free but also requires analytical thinking.

[L] I believe that mathematics is one of the most important subjects. It is incorporated into almost everything around us.

[D] ... it is helpful in the real world, but the depths of it are just useful if you are an insightful person or just love maths. It is very challenging and logical...

[G] Mathematics is an exciting area of study with a lot of interesting applications. Tutoring in the MSC has made me even more appreciative of these applications and areas of study.

At times, some of the tutors disliked maths or found it boring. It can be seen in the following two quotes that at those times a surface approach to learning was most probably taken, they have not fully understood what was going on and have more than likely made it through those situations by rote learning the components.

[A] There are topics I feel are boring and complicated and confusing, these I find hardest to learn because of inability to understand fully. I feel that some of those topics are 'pointless', but still try my best to learn them because I don't want to fail.

[D] Maths is fun when it is easy, but I dislike it when it gets into physics and very scientific.

The comments below indicate that some of the tutors have developed a higher level view of mathematics, through a greater understanding, since they started the peer tutoring program. However, a few of the tutors' views of mathematics have not changed since tutoring. By teaching other students, some have developed a greater appreciation of the practicality of maths [D], a greater understanding of the methodology used in applications [B] and insights in seeing how mathematics plays a fundamental role in real life situations [K]. Since teaching, some tutors also gained a greater analytical understanding of the underlying theory [J]. Overall, the peer tutors have fostered a deeper level of understanding of mathematics.

[D] ... since tutoring I noticed how [much] more practical maths is.

[B] ...through tutoring I have come to understand that more importance should be placed on methodology and understanding than purely repetition.

[K] By tutoring other students, you can get the overall picture of the topic in mathematics that you have taught, and clearly you can see what sort of problem you are solving, and [how] they might be used in the workplace.

[J] When you first do it, you are more thinking about the mechanics of the problem as in we do this then that when it looks like this. But now when I look back at that work it seems to make more sense.

It is said in Damon (1984) that the students have more to gain in learning from their own age group in addition to that of the school teachers, as they are direct in their communication and on a level easily understood. A similar view is shared by some of the peer tutors, seen by [F] for example, thus indicating that the peer tutoring program is a win- win situation for all involved [I].

[F] A lecturer knows their stuff inside out, back to front, and has the answer before you have read the question. A student tutor, however, has a quality that the lecturer has lost and will never regain. The student tutor can teach short cuts to learning and understanding concepts on a much more basic and willingly absorbable level.

[I] If you can teach something then it's a sure way to learn it good. I see a lot of students in the study centre get onto this fact and jump at the opportunity to get involved in tutoring.

Reasons for volunteering and accomplishments achieved.

The reasons why students volunteered or signed on for peer tutoring can be divided in two categories, extrinsic and intrinsic. Extrinsic reasons include the opportunity to help younger students, share knowledge, to take the pressure off other tutors, to put something back into the mathematics department and into the university community.

[K] When I was in 1st year I noticed the Maths Study Centre was helpful to me. That is why I wanted to help students with the extra little bit of help they might need in order to do well in their subject.

Different intrinsic reasons include filling time in between classes, gain some teaching experience, a nice addition to the resume, solidify knowledge, increase communication and interpersonal skills, increase confidence, meet more people, learn from the students and the self satisfaction of teaching enjoyment. This last point could possibly be viewed as extrinsic in its origination, as it is more than likely the tutor gains such enjoyment through making a difference for other students. We shall look at this again through the peer tutors' accomplishments.

After the peer tutoring program all of the tutors who responded to the survey said that they felt a sense of accomplishment. Extrinsic accomplishments were all based around being able to help contribute to other students' knowledge and understanding of mathematics and inspire them for future study. Intrinsic accomplishments gained by the tutors included having developed some new friendships, a greater sense of responsibility, an increase in or grounding of their own mathematical knowledge and the development of other skills such as interpersonal skills and the ability to convey knowledge to others. Some also said it had increased their levels of self confidence, in tackling unknown problems and they had rediscovered their overall confidence in mathematics. The most prevalent of the accomplishments is intrinsic in nature with extrinsic origins in that the tutors felt good about themselves by helping make a difference to others. This accomplishment is often a direct link to the reason why people have an enjoyment in teaching. It also plays a significant role in the outcome of the other intrinsic achievements, as seen by example in the following two quotes.

[F] When I have helped just only a hint, enough for them to recognize the rest of the problem on their own, it gives me such a boost of confidence, motivation and inspiration, that my own study becomes a pleasure.

[H] ...the satisfaction of successfully helping others impacted greatly on my self-confidence.

Only a few of the peer tutors knew that they enjoyed teaching when they originally signed on for volunteering in the centre, however, nearly all of the respondents expressed experiencing pride and enjoyment from their teaching in the peer tutoring program. This is illustrated by the following quotes.

[A] I feel good when I am able to provide help to other students. ... Seeing students who have a better understanding after my help is most rewarding.

[C] Well, it always feels good when you find that you've been helpful to some

people. And yeah, it kinda gives me pride to hear that students have got good marks thanks to my teaching.

[D] It feels great when you successfully help a student with their maths. It's the concept of 'giving yourself a pat on the back'.

[G] It is great to see that you can make a difference to a student. When they go away with extra knowledge and a smile on their face, you are left with a feeling of achievement.

[I] I get a real kick out of helping students out.

[L] I've always found it very rewarding to help others in need.

An overall accomplishment brought forth through the achievements of communication and interpersonal skills, friendships developed and responsibilities gained, is that a greater sense of community was realised among the peer tutors.

Conclusion

This paper has described a peer tutoring program to support first year students who study mathematics or statistics in their degree. Research has shown that peer tutoring has much to offer students, benefiting both the tutors and the tutees. Volunteer students have worked as peer tutors in the Mathematics and ICT study centre, bridging the gap financially and educationally, in what can be described as a "successful" operation.

Many first year students frequented the drop in study centre to utilize the tuition of the peer tutors. A survey was completed by the tutors and a qualitative analysis of their responses was performed. Of the nine from eighteen volunteer students who completed the survey, all claimed the program had fostered a deeper level of understanding of mathematics and many of them gained confidence in themselves personally, or in mathematics over the course of the program. It could be argued that the nine volunteers who did not complete the survey did not benefit from the operation. The survey however was not compulsory and was distributed only a few weeks before the commencement of exams and they may simply have chosen not to complete it due to being busy in the preparation of their final exams. Overall, we are assured that at least 50 percent of the volunteer tutors did have a positive experience in the program.

The third year student volunteers did not have anything to gain financially over the course of the program. This gives a sense of reassurance with the quality of teaching as the volunteers were not simply going through the motions of a job but rather they held a genuine commitment to furthering the development of mathematics for themselves and others.

This study has confirmed our belief of positive benefits gained through peer tutoring and explored the possibilities of its potential. The Mathematics and ICT study centre will continue to incorporate senior student volunteers into the peer tutoring program in the future while taking on board the tutors' suggestions for improvement to ensure the continuation of a "successful" operation. With the close to negligible operational costs, positive benefits gained by the tutors and quality reassurance in teaching to visiting first year students, this peer tutoring program has shown to be a win-win situation for all responding tutors.

References

- Bush Jr, J. L. (1985). A Peer Tutoring Program for Introductory Accounting Courses. *Journal of Accounting Education*, 3 (2), 171-177.
- Carroll, M. (1996). Peer Tutoring: Can Medical Students Teach Biochemistry? *Journal of Biochemical Education*, 24 (1), 13-15.
- Damon, W. (1984). Peer Education: The Untapped Potential. *Journal of Applied Developmental Psychology*, 5, 331-343.
- Damon, W., & Phelps, E. (1989). Critical Distinctions among Three Approaches to Peer Education. *Peer Interaction, Problem Solving, and Cognition* (pp. 9-19). NM: Webb.
- Griffin, M. M., & Griffin B. W. (1998). An Investigation of the Effects of Reciprocal Peer Tutoring on Achievement, Self-Efficacy, and Test Anxiety. *Contemporary Educational Psychology*, 23, 298-311.
- Hopkin, A. G. (1988). Promotion of Experimental Learning through Peer Group Seminars. *International Journal of Educational Development*, 8 (2), 109-116.
- Houston, S. K. & Lazenbatt, A. (1999). Peer Tutoring in a Modelling Course. *Innovations and Education Training International*, 36, 71-79.

Mature Students in HE: Academic Maths Support

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Academic Maths Support at Sheffield Hallam University (SHU)

At SHU there was a demand from staff and students alike to provide some form of support for students who seemed to be struggling in mathematics. SHU's academic maths support facility, Maths Help, was in response to this demand. This seems to be a need which is common to many institutions and this need is frequently met by the setting up of some form of support. (Beveridge and Bhanot (1994:13) report that 76% of FHE institutions have some form of maths support.) Maths Help offers a daily open access drop-in session and specially tailored short courses. It serves education students at the Mathematics Education Center and students who are maths specialists and who are studying other serviced subjects at City Center sites.

An analysis of who uses Maths Help has revealed that the need for support in mathematics is widespread and not, as might be assumed, only for those with weak or non-traditional maths backgrounds. Students with a range of previous qualifications, on a range of courses with a range of attainment and performance levels come with a range of problems.

Methodology

The initial purpose of our study was to evaluate the effectiveness of the Math Help system with a view to improving the level and nature of the support offered. We are involved in an ongoing project to review who comes forward for help, why they come, why others don't come, what is causing students' difficulty and to investigate whether Maths Help is supporting those needing support, if it is actually helping students and if it is the most appropriate form of support.

In this research we have focused our investigations on the students themselves through direct observation, questionnaires and in-depth interviews. We have also undertaken an analysis of Maths Help users and an analysis of assessment performance.

Theoretical Context

We have chosen to set our work in the context of general research findings on teaching and learning within HE rather than the more specific research on maths learning most of which has been conducted at school level. A predominant concern of research has been to investigate student's' experiences of studying and learning in HE with a view to improving that experience. Marton and Sajlo (1984:36-55) focused attention on deep and surface approaches to learning. These concepts are now established within the research literature and embedded in many of the subsequent curriculum developments. (Gibbs 1992:24-161)

More recent work has focused on a variety of factors which encourage deep or surface learning. A number of studies have investigated the relationship between students' approaches to learning and the context in which they are working. Biggs (1989:17) reports that

Teaching that gave evidence of deep learning contained in sharp form one or more of the following:

- an appropriate motivational context
- a high degree of learner activity
- interaction with others, both peers and teachers
- a well- structures knowledge base

The student complements these four elements with her/his intrinsic motivation. We are particularly interested in the motivational factors and the acquisition of a well- structured knowledge base.

There is an assumption in the literature that deep approaches to learning are always “best.” At first we questioned this; for many of the students we help, maths is not the main focus of their study; there is increased availability of IT maths tools which, for example, enable students to solve equations without “understanding.” However, preliminary findings suggest that students are keen to understand their maths and that a lack of deep understanding of some “basic” concepts is creating problems for a significant number of students. Staff want their students to have “lasting learning” and “transferable learning.” Gibbs (1992:4) examines the notion that a full understanding is not always required and that an ability to memorize without understanding is sometimes enough. He reports that.

Studies have shown that a surface approach does tend to produce marginally higher scores on tests of factual recall immediately after studying. However this small advantage is quickly lost. A surface approach leads to rapid forgetting and as little as a week later students who have taken a deep approach will score far higher than those who have taken a surface approach, even on tests of factual recall.

Maths Help in This Theoretical Context

We have considered our initial quantitative findings (who comes to Maths Help, what they ask for help with etc) within this theoretical context.

By providing the drop-in facility we are sending out signals to students. For some, this reinforces the validity of their anxiety and can create an expectation of a “quick fix.” For others, this provides a non-embarrassing, non-threatening initial opportunity to ask for help.

Does the drop in facility encourage surface approaches to learning? If it does, do we need to be worried? Or, is it important that students have at least the opportunity to admit their difficulties in the belief that this is the first step towards being able to do something about them?

Do we provide the four key elements described earlier in a drop-in situation? Out conjecture is that this is very difficult to do. Is the students’ motivation intrinsic? Out evidence suggests that the students’ need for Maths Help is assessment driven, that is, their motivation is extrinsic.

Can the specially tailored short courses do better? WE can create an environment that encourages learner activity and interaction, we can work towards a well structure knowledge base. Much depends in the students’ motivation. The majority of students who request courses and who come to short courses are mature students. The provision of short courses

can provide an opportunity for mature students to bring their wealth of experience to the learning situation; the issue for us then is about **reconstruction** of their knowledge base—not “topping-up” or “filling the gaps.”

Our data suggests that significantly large numbers of students, including those with formal mathematics qualifications, feel that they do not have a well structured knowledge base in mathematics. Some students without this nevertheless appear to have a deep approach to learning, often culled from other life or work experience. Other have clearly neither a well-structured knowledge base nor a deep approach to learning. The following excerpts from in-depth semi-structured interviews with two students illustrate this:

Student A says

I got 90% in my algebra assignment.

I don't understand algebra.

You can learn by simply accepting but it does not allow you to put it into other contexts.

At work someone would come to you with a problem. What you saw on the surface wasn't real, the real problem. You had to look deeper.

Student B says

I only got 20% in the algebra assignment.

I can do basic algebra.

I'm that frightened and I know I can't do it.

I understand the help I've been given but I still don't know how to start.

I don't know what you're supposed to do.

I liked it on my HND course. A topic was introduced, questions were set, there were lots of assignments. I need deadlines. Basic explanations.

I failed the maths on my HND course.

Our interpretation is that Student A appears to be “alright,” this student can take a deep approach and more importantly perhaps can identify problems and take action. For A, Maths Help's role is that of facilitating the reconstruction of the student's knowledge base.

Student B, however, insists that a surface approach is right albeit acknowledging that it has not worked. B may be able to do basic algebra but does not have a well-structured knowledge base. B does not have the connections and the ability to relate new work to old which would enable a starting point for solving problems to be found.

How can Maths Help help student B? Student B does not have a well structured knowledge base and is not taking a deep approach. Our conjecture is that it is easier to help a student to reconstruct their knowledge base if the student is able to take a deep approach. Can we instruct students in the use of deep approaches? Ramsden, Beswick and Bowden's (1987:168-176) research shows that attempts to do this have resulted in an effect opposite to that desired.

The theoretical context of deep and surface approaches to learning whilst providing a helpful framework for the categorization of students does not help us to formulate appropriate support strategies for some students. The students we see have already learned, or rather failed to learn, algebra. Students deep-rooted difficulties think they have learned algebra; this is one of the major problems. How can we help students restructure their knowledge in such a

way that they can use it? We need to investigate the meaning of algebra to individuals and so it may be more helpful to interpret our findings in the context of hermeneutics.

References

- Beveridge, I. and Bhanot, R. (1994) "An examination of Maths Support in Further and Higher Education" in *Mathematics Support Newsletter I*.
- Biggs, J.B. (1989) "Approaches to the Enhancements of Tertiary Teaching" in *Higher Education Research and Development*, Vol 8, No 1
- Gibbs, G. (1992) *Improving the Quality of Student Learning*. U.K. Oxford Technical and Educational Services.
- Marton, F. and Saljo, R. (1984) "Approaches to Learning" in Marton, F., Hounsell, D.J. and Entwistle, N.J. (eds.) *The Experience of Learning*. Edinburg: Scottish Academic Press.
- Ramsden, P., Beswick, D. and Bowden, J. (1987) "Learning Processes and Learning Skills" in Richardson, J.T.E., Eyesenck, M.W. and Warren Piper, D. (eds.) *Student Learning*. U.K.: Open University Press.

Adult Learners and Mathematics Learning Support

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Abstract

The provision of some level of Mathematics Learning Support is now standard in the majority of Higher Education Institutions in Ireland and the UK, and it is also available in many other countries. This provision is, in part, a response to the large numbers of students entering Higher Education who do not have the mathematical skills required and this cohort includes a significant number of Adult Learners. Research indicates that these students have different motivations and approaches to learning than traditional age learners. This paper considers the analysis of a large-scale student evaluation of Mathematics Learning Support in Ireland. In particular, it presents the responses and engagement levels of Adult Learners and compares these to those of traditional students. The findings are key to ensuring best practice in the provision of Mathematics Learning Supports for the wide variety of students who avail of it.

Key words: Adult learners, engagement, evaluation, mathematics learning support

Introduction

The availability of some form of Mathematics Learning Support (MLS) is now what students can expect to find in the majority of Higher Education Institutions (HEIs) in Ireland and the UK. MLS is also available in HEIs internationally, for example in Switzerland, Canada and

Australia (Gill et al., 2008; Perkin et al., 2012). MLS has been defined as a facility offered to students which is surplus to their traditional lectures and tutorials, the purpose of which is to offer non-judgemental and non-threatening one-to-one support with mathematics (Ní Fhloinn, 2007; Lawson et al., 2003; Elliot and Johnson, 1994).

The main reason for the establishment and significant growth of MLS is as an approach to addressing the well documented ‘Maths Problem’. One of the ways O’Donoghue (2004) defines the ‘Maths Problem’ refers to the mathematical preparedness of incoming students in terms of their mathematical shortcomings or deficiencies at the university interface. Significant numbers of students entering HEIs are deemed as at-risk of failing or dropping out because they do not appear to be appropriately prepared for mathematics in HE and they often exhibit very weak mathematical backgrounds. This ‘Maths Problem’ is common place in HEIs in Ireland, the UK and internationally (Gill et al., 2010; Lawson et al., 2012). These at-risk students are main target of MLS.

One benefit of the economic downturn has been the welcome increase in Adult Learners returning to HE (Golding and O’Donoghue, 2005). In the Dublin Institute of Technology (DIT), Adult Learners constituted one fifth of the attendants at the Mathematics Learning Support Centre (MLSC) in its opening year (Ní Fhloinn, 2007). In 2012 Adult Learners accounted for 15.3% of full time students enrolled in HE in Ireland and 21% of full and part time students. In 2015/16 10% of all full time new undergraduate students and 85% of part time undergraduate entrants were Adult Learners (Higher Education Authority, 2016). Faulkner et al. (2010) stated that the presence of so many Adult Learners is one contributing factor to the increased numbers of at-risk students in first year courses.

In order to establish best practice in the successful provision of MLS, it is essential that it is comprehensively evaluated on a regular basis (Matthews et al., 2012). For example, quantitative research suggests that appropriate engagement with MLS can have a positive impact on student retention and progression (Lee et al., 2008; Mac an Bhaird et al., 2009). One of the initial aims of the Irish Mathematics Learning Support Network (IMLSN), which was established in 2009, was to conduct a large scale survey of student opinion on MLS.

Given the increasing proportion of Adult Learners in mathematics in first year courses, it was considered key that they should be identifiable in the survey so that their responses regarding the evaluation of MLS could be studied in detail. In particular in this paper, after a description of relevant literature and the methodology of the survey, we focus on Adult Learner responses, their backgrounds and we compare them to the overall cohort.

The main research questions we are trying to address are:

1. What are the motivational factors of Adult Learners who seek MLS?
2. Why do some Adult Learners of mathematics not seek MLS?

Literature review

There is a concern that a lack of preparation in mathematics can lead to increased failure rates and low self esteem (Symonds et al., 2007) in HEIs. Aligned with that is a worry of impeding students in the study of other disciplines, e.g. engineering, science, etc. (Pell and Croft, 2008; Gill, 2006). Many students arrive in their HEI having chosen mathematics-intensive courses

unbeknownst to themselves (FitzSimons and Godden, 2000). Most degree programmes, even non-specialist mathematics degrees, contain some mathematics and/or statistics component, as prospective employers require graduates to be proficient in mathematics, with some even setting numeracy tests as part of their selection process (Lawson et al., 2003). The mismatch between the knowledge of many students and the expectations of HEI teachers is one contributory factor to the problem and this mismatch arises partly through the increase in diversity of the backgrounds of students (Lawson et al., 2003; Faulkner et al., 2010). Diversity in the standards of teaching and class size in HEIs tend to exacerbate the situation (Lawson et al., 2003; Gill, 2006).

One of the key responses to the 'Maths Problem' was the opening of Mathematics Learning Support Centres (MLSCs) to attempt to deal with the mathematical shortcomings of students (Pell and Croft, 2008; Gill, 2006). In 2004 in the UK it was reported that 62.3% of 106 surveyed universities offered some form of MLS (Pell and Croft, 2008, p168). In 2012, this number had jumped to 85% (Perkin et al., 2012). In 2008, an audit carried out by the Regional Centre for Excellence in Mathematics Teaching and Learning (CEMTL) in Ireland demonstrated that 13 out of 20 HEIs provided MLS in some form (Gill et al., 2008). In 2016 Cronin et al. conducted a study on MLS in Ireland which showed that 83% of 30 institutions surveyed offered some form of MLS. Most MLSCs are committed to servicing the needs of traditional and non-traditional (i.e. international and Adult Learners) students (Ní Fhloinn, 2007; Gill and O'Donoghue 2006). Carmody and Wood (2005) reported on the benefits of a drop-in MLSC for easing the transition to HE for first-year students. The drop-in centre caters for students from all faculties and has become a meeting place for collaborative learning. Tutors use a variety of teaching methods and resources, which is easier to do in a one-to-one situation than in front of a large class. Engagement with MLS has been shown (through mostly quantitative research) to impact positively on mathematics performance and grades and retention (Burke et al, 2012; Mac an Bhaird et al, 2009; Pell and Croft, 2008; Symonds et al., 2007). Pell and Croft (2008) state that while MLS is provided first and foremost for 'at-risk' students, it is more often the case that users tend to be high achievers working to attain high grades, a view supported by Mac an Bhaird et al. (2009) who have also shown that many 'at-risk' students still do not engage with MLS.

An Adult Learner, or Mature Student, is classified in the Republic of Ireland as a student that is 23 years of age or older on 1st January of the year of registration to HE (Ní Fhloinn, 2007). Entry for Adult Learners who have not got the minimum requirement for entry to their chosen course of study is usually gained via interview and is based on a number of factors including life experience and motivation, in addition to prior qualifications. Faulkner et al. (2010) studied the student profile in service mathematics programmes at the University of Limerick (UL) since diagnostic testing began there in 1997. The increase in Adult Learners of mathematics in these modules was quite pronounced. In 1997 there was one registered Adult Learner in Science and Technology Mathematics, two of the biggest service mathematics modules provided by this university; in 2008, there were at least 55 Adult Learners. This statistic is supported by Gill (2010) who states that in 2009/10, Adult Learners in UL constituted 14% of the entire cohort, a jump of 49% on the previous year. In 1997, 30% of students in service mathematics modules at UL were deemed to be at-risk. Fast forward to 2012 and 61% of students in the same modules are categorised as at-risk.

Adult Learners in mathematics who return to education constitute a heterogeneous cohort. For example, participants on the 'Head Start Maths' bridging programme at UL range from 23 to over 45 years of age. A significant number of the students on the programme in 2008 had not studied mathematics in any formal sense for up to 20 years and 30% of participants had not taken the Leaving Certificate (LC) examination (Gill, 2010). The LC is the terminal examination taken by pupils at the end of secondary school in Ireland. Mathematics is compulsory for students and can be taken at three levels: Higher (HL), Ordinary (OL) and Foundation (FL). In DIT, Ní Fhloinn (2007) outlines how Adult Learners fall into the full-time, part-time or apprenticeship categories, with each type of student presenting with different characteristics and issues relating to their preparation, their approach to learning mathematics and confidence issues. For many adults returning to HE, mathematics presents an obstacle. Many find the idea of studying mathematics intimidating and this can have a potential negative impact on their mathematics confidence and subsequent performance (Golding and O'Donoghue, 2005). Diez-Palomar et al. (2005) acknowledge the difficulty for adult mathematics education in efficiently addressing the needs of diverse cohorts. It can be very difficult for students to catch up with forgotten fundamentals and keep up with current studies simultaneously (Gill, 2010; Lawson et al., 2003).

Under-preparation of adults in mathematics is a grave issue in HE (FitzSimons and Godden, 2000) as students with an array of previous qualifications, on vastly different courses with a series of attainment and performance levels often present with a range of problems (Elliot and Johnson, 1994). Research tells us that many Adult Learners of mathematics exhibit maths anxiety when faced with mathematical tasks and can lack confidence in their mathematical abilities (Gill, 2010; Ní Fhloinn, 2007). This anxiety may impact adversely on their participation and performance in mathematics activities (Ashcraft, 2002). In fact Gill (2010) reported that mathematics is often the main worry/concern of students returning to university. Singh (1993) attributes this anxiety on the part of Adult Learners partly to examinations and a fear of failure. It has been well documented that mathematics learning is related to student confidence in their abilities (Coben, 2003). Many adults who are well capable of learning mathematics are inhibited from doing so because of their fear of the subject (Benn, 2000).

Diez-Palomar et al. (2005) and O'Donoghue (2000) acknowledge the difference between Adult Learners of mathematics and traditional learners. Adult Learners carry with them an abundance of experiences that need to be considered in pedagogical practices. This view is supported by Tusting and Barton (2003) who add that Adult Learners have different motivations for studying than traditional learners and are more inclined to be autonomous and reflective learners. The decision to return to education has generally been their own decision and a deliberate one (FitzSimons and Godden, 2000). Though Adult Learners may lack confidence in their own abilities, they tend to be highly motivated (Ní Fhloinn, 2007; FitzSimons and Godden, 2000). Traditional lectures and assessments are not conducive to learning for many Adult Learners (Gordon, 1993 cited in FitzSimons and Godden, 2000) so many rely on MLSCs for support. In 2009/10 Adult Learners of mathematics at UL constituted 54% of the attendance at the drop in centre, even though they represented just 14% of the entire student population (Gill, 2010).

While the importance of research in the teaching and learning of mathematics among Adult Learners has been duly recognised in recent years (Coben, 2003) it remains an ‘under theorised and under researched’ area (Galligan and Taylor, 2008, p99). Furthermore, research conducted on the teaching and learning within MLSCs is sparse (Galligan and Taylor, 2008).

Methodology

The IMLSN was established in 2009, and its guiding principles are similar, on a smaller scale, to the leading experts in the provision of MLS, the **sigma** (The Centre of Excellence in Mathematics and Statistics Support) network (<http://sigma-network.ac.uk/>) based in England and Wales. The IMLSN aims to support individuals and HEIs involved in the provision of MLS in Ireland. Once set up, the network decided it should promote the benefits of MLS to both staff and students on an institutional, national and international basis and agreed that a student survey was the best approach initially. The IMLSN asked the panel of researchers listed on this paper to undertake this student survey.

Student questionnaires are commonly used in the evaluation of MLS services (Lawson et al., 2003) in individual HEIs, so it was decided to create a student survey that could be used in all HEIs which provide MLS. HEIs who already distributed questionnaires on MLS were invited to submit them to the committee; these were amalgamated and a communal questionnaire was formed as a result. This questionnaire was piloted in 4 HEIs with 100 students and subsequently refined based on analysis of the findings and expert statistical advice.

The resulting questionnaire had 17 questions, a combination of open questions and questions which required a response on a 5-point Likert scale. There were three main sections: Section A determined the students’ backgrounds; Section B focused on users of MLS; and Section C focused on non-users of MLS. First year service mathematics classes have the largest percentage of at-risk students and are the main target of MLS in terms of student retention and progression, so it was decided to issue the questionnaire to these cohorts only. Evaluation sheets are usually distributed within MLSCs but this can lead to bias as users already rate the MLSC to some extent if they attend it (Lawson et al., 2003). With this in mind, it was decided that the questionnaire should be issued in appropriate lectures to get a blend of user and non-user feedback and to reduce bias. The questionnaires were anonymous and there were no identifying characteristics. The questionnaire, in the first large scale survey of its kind, was issued to members of staff involved in the provision of MLS in HEIs in Ireland and they were asked to distribute paper copies in the appropriate first year service mathematics lectures during the second semester of the 2010-11 academic year. Service mathematics refers to users of mathematics (e.g. engineering, science, business etc), rather than mathematics specialists (e.g. pure or applied mathematicians) (Burke et al., 2012).

The HEIs surveyed were Universities or Institutes of Technology (IoTs), and these have different and complementary roles and missions within HE in Ireland. At undergraduate level Universities focus on Level 8 (Honours Degree programmes), and IoTs emphasise career-focused HE offering Level 8 programmes but also programmes Level 7 (Ordinary Degrees) and Level 6 (Higher Certificates). IoTs also have a larger proportion of Adult Learners and students from disadvantaged areas and are stronger than the Universities in part-time and flexible provision (<http://www.heai.ie/en/node/981>). In the IoTs that participated in

the survey, the ratio of Level 8: 7: 6 students was 49:38:11% which is similar to the 53:37:9% proportion of Level 8: 7: 6 students in IoTs nationally in the 2011-12 academic year.

A total of 1633 completed questionnaires were returned from 9 HEIs (5 Universities and 4 IoTs) comprising enormous quantities of both qualitative and quantitative data. Two graduate students were hired to input the data into SPSS, and SPSS was also used to analyse the quantitative data. NVivo was used to analyse the qualitative data. A general inductive approach was used to analyse the data guided by the specific research questions (Thomas, 2003). Data was read and analysed by two researchers independently, one from this panel of researchers and an external person to identify emerging themes. Further details on the analysis to date for all respondents (traditional students and Adult Learners combined) can be found in (Mac an Bhaird et al., 2013; Ní Fhloinn et al., 2014; Ní Fhloinn et al., to appear).

Results

In Section A of the survey questions were asked which focused on students' backgrounds. Of the 1633 respondents, there were 221 (13.5%) Adult Learners, 73% of these were male and 91% were full-time students. In terms of students' mathematical background, they were given the 4 options outlined in Table 1. Generally, a minimum of OL mathematics would be needed for most service mathematics courses in HEIs and this is reflected among respondents with only 18 of the 1563 students who provided their LC results in the survey having studied mathematics at FL. If they had not taken the LC, then they could select the Other option.

A lower percentage of Adult Learners (than of the overall respondents) had taken HL, and higher percentages (compared with the overall) in the remaining three categories, with the majority studying mathematics at OL.

Table 1. Mathematical backgrounds of Adult Learners and of overall survey respondents.

Higher Level LC	Ordinary Level LC	Foundation Level LC	Other	Total
33.79% (541)	62.71% (1004)	1.12% (18)	2.37% (38)	Respondents (1601)
9.9% (20)	73.76% (149)	4.46 % (9)	11.88% (24)	Adult Learners (202)

When the breakdown of the disciplines that students were in was considered, we found, for most discipline areas, the proportion of Adult Learners was in line with the overall proportions of survey respondents, see Table 2.

Table 2. Degree Programmes of Adult Learners and of overall survey respondents.

Subject	No. of Adult Learners	%	No. of Respondents	%
Science	80	36.2	583	35.7
Engineering	50	22.6	236	14.45
Business	55	24.9	484	29.64
Arts	7	3.2	67	4.10
Education	6	2.7	90	5.51
Computing	23	10.4	171	10.47
Total	221	100.0	1631	100.0

Section B focused on MLS users. The majority of Adult Learners 136 (61.5%) availed of MLS, compared to only 32.2% of traditional learners. A Chi-Squared Test for independence indicated a statistically significant association exists ($p < 0.001$) between type of student (i.e. Adult or traditional learner) and whether a student uses MLS, Adult Learners were more likely to seek MLS than traditional learners. In terms of gender 68.3% of female Adult Learners compared to 43% of female traditional learners used MLS, and 59.4% of male Adult Learners in comparison to 23.3% of male traditional learners availed of MLS.

The mathematical backgrounds of both users and non-users of MLS among the Adult Learner sample were very similar, and the percentage breakdown was close to that of the Adult Learner population (see Table 1). When we considered subject discipline, the proportions of Adult Learners using MLS was very similar to the proportions of overall Adult Learners in each subject discipline (see Table 2).

Students who availed of MLS were asked, in an open-ended question, to comment on why they first decided to use MLS. There were 577 comments from attendees which were coded using GIA and the majority fell into 6 main categories as outlined in Table 3. This table contains comments from 122 of the 136 Adult Learners who responded.

Table 3. Frequency of Adult Learner reasons for using MLS

Categories of comments	Frequency of comments (n=122)	Sample comments
Extra help	38.52%	“Needed help with maths”, “I had gone to the tutorials and still had trouble with a particular area”, “I wanted help with a maths problem and to understand where I was going wrong”, “Because the pace of the main lectures were too fast and I wasn’t keeping up”, “I had to catch up on missed lectures”.
Background/Ability: Comment about being away from Maths for a while prior to entry (from mature students) or comment suggesting poor confidence in maths ability	19.67%	“Hadn’t done maths in ages so I needed extra help”, “Because I haven’t studied maths in ten years and really felt quite daunted by the thoughts of returning to study maths”, “Coming back to study after a long break, needed all the help at hand!”, “Because I am not great at maths”.
Assignments/Exams : Looking for help with specific aspect of coursework assessment during the semester (upcoming test, assignment) or attending for revision or prep for end of term exams	13.93%	“Struggling with maths assignments”, “I was stuck on understanding a part of an assignment and was spending a lot of time trying to figure it out”, “To help with revision”.

Struggling	9.02%	“I was struggling with the subject”, “Was lost with maths”.
Improve Understanding: Positive comments about attending to try to improve or gain better understanding	5.74%	“Because I thought it will be a great idea to use drop-in clinic if I want to get good grades.”
Mathematics Difficult	2.46%	“Because I find maths very difficult.”

A comparison of the frequency of responses in each category given by Adult Learners compared with the overall population of users provides some interesting differences. The frequency of responses from Adult Learners showed they are much more likely to make comments indicating that they:

- look for help as they have a long time away or suggesting poor confidence in their mathematical ability (19.67% as against 7.45% frequency of response),
- seek general extra help (38.52% as against 20.62% frequency of response)
- are struggling (9.02% as against 5.03% frequency of response).

In contrast, the frequency of responses from Adult Learners shows they are much less likely to make comments indicating that they:

- seek help specifically to get assistance with particular coursework assessment or revision for tests (13.93% as against 41.25% frequency of response)
- attend MLS to improve or gain better understanding (5.74% as against 15.94% frequency of response).
- state they find mathematics difficult (2.46% as against 9.71% frequency of response).

MLS users were asked to rate, on a 5-point Likert scale, the specific services available in their HEI and they were also given the opportunity to comment. The main support offered was a drop-in centre, so we will focus on that support in this paper. The distribution of ratings and responses from Adult Learners for the other services (e.g. ICT supports, workshops, support tutorials) are in line with that of the overall cohort.

All nine HEIs had a drop-in centre and 519 users rated them. 119 were Adult Learners and 89% of these rate it as worthwhile. There were 244 additional comments, 57 from Adult Learners and coding of responses placed them into the following three main categories:

20 (38.5%) relating to satisfaction levels with the service provided, 19 of which were positive, “Very helpful – I am even starting to enjoy maths now”, “Would not have a clue what I was doing if it was not for support”. 23 (40.4%) related to the physical resources, including staff and contact hours of the centres. Without exception, all comments stated that all of the above should be extended, “Class size was small for the amount of students”, “If

there were more opening hours and people available as it is very busy” and “Sometimes a long waiting time; too busy”. 9 (17.3%) related to the quality of tutors; 5 positive, 1 negative and 3 which were positive and negative, “Always as helpful as they can be with the exception of one of the tutors who tends to be very rude and arrogant”.

In Questions 11-15, MLS users were asked about their perception of the impact of MLS on various aspects of their education, the questions had a 5-point Likert scale and they could also comment on their answers. Students were asked to rate the impact of MLS had on their confidence. 539 users responded, 124 were Adult Learners and 67% of these rated the impact as helpful in comparison to 56% of all users. There were 106 additional comments, 21 from Adult Learners with 20 of these positive, “It has helped me a lot. I don’t need to struggle alone to figure out things that I don’t understand”, “Still find it difficult but have a better understanding of maths”. For all users, approximately 75% of comments were positive.

Students were also asked if MLS had impacted on their mathematics performance in tests or examinations to date. There were 526 responses, 115 from Adult Learners and 65% of these stated that it had an impact, in comparison to 56% of all users. There were 103 additional comments, 21 by Adult Learners, 16 of which were positive (90% of overall comments were positive), for example: “I would have failed if the extra help had not been there”.

Students were asked to rate how MLS had helped them cope with the mathematical demands of their courses. There were 530 responses, 119 from Adult Learners and 72% of these indicated that MLS had been helpful in comparison to 65% of all users. There were 55 additional comments, 14 from Adult Learners, 12 of which were positive, for example “It has been a huge help”, “Wouldn’t be able to do maths without all the extra services and wouldn’t have a hope of passing the year. One of the (two) negative comments stated “Some of the tutors in the centre might be good at understanding maths but not good at teaching it”.

In Question 11 students were asked if they had ever considered dropping out of their studies for mathematics-related reasons. 128 of the 136 Adult Learners answered this question with 25 (19.5%) stating that they did consider dropping out, this is a similar proportion to that of the overall student population. Question 12 asked (those who answered yes to Question 11) if MLS had been a factor in them not dropping out. 22 of the eligible 25 Adult Learners answered and 17 (77%) of these stated that MLS was an influencing factor in their decision not to drop out (compared to 62.7% of the overall population). Additional comments included: “Greatly. It has given me the confidence to turn maths as my worst subject into one of my best” and “Encouraged me to trust that my worries were normal and that practice would improve me”. 8 students left comments stating that they never considered dropping out because of the MLS that was available to them, “Never felt the need because of the support provided” and “No, but did worry about failing maths before using these facilities”.

Section C of the survey focused on students who had not availed of MLS. 85 (38.5% of) Adult Learners (compared with 67.8% of traditional learners) stated that they did not use the MLS facilities provided in their institution. In Question 16, non-attendees were asked to select from 7 fixed options, as to why they did not avail of MLS. For Adult Learners, the frequency of response in each category is interesting when compared with the overall 1041 students who did not use MLS, see Table 4 (note that students selected more than category).

Table 4. Frequency of reasons for not using MLS between Adult Learners and all students.

Category of response	% of Adult Learners who did not avail of MLS (n=85)	% of all students who did not avail of MLS (n=1041)
I do not need help with Maths	43.53%	48.13%
The times do not suit me	41.18%	28.34%
I did not know where it was	5.88%	17.87%
I hate Maths	3.53%	14.51%
Other	15.29%	12.78%
I was afraid or embarrassed to go	8.24%	11.43%
I never heard of the MLSC	11.8%	8.36%

In terms of individual respondents, it is worth noting that of the 85 Adult Learners who did not avail of MLS, 43.53% of these stated that they did not need help. In comparison, for the 941 (67.8%) traditional learners who did avail of MLS, 48.9% of these stated that they not need help. We can see in Table 4 that a larger percentage of responses from Adult Learners stated that the times did not suit and that they had not heard of the MLSC. The proportions of Adult Learners responding that they hated mathematics, did not know where MLS was or were afraid or embarrassed to go, were much lower than in the overall population.

There was also an opportunity to comment on answers to Question 16 and 34 Adult Learners did so. 20 comments stated that they did not need help or were able to work it out by themselves; 8 comments stated that the session timings did not suit them due to timetable or living circumstances; 2 stated that they never heard of the MLSC services; 2 comments related to a reluctance to attend: “Just felt a bit uncomfortable; felt the questions I had may seem a bit irrelevant”. These responses were consistent with overall student comments.

Table 5. Frequency of comments from Adult Learners who are non-users of MLS about what would encourage them to avail of MLS.

Category	% of Responses (n=41)
Go if needed	46.34%
Results/Exams	0%
Better times	19.51%
More Information	19.51%
Resources/Location	4.88%
Advised to go	2.44%
Student Feedback	2.44%
Miscellaneous	4.88%

In Question 17, non-users of MLS were asked to comment on what would encourage them to use the MLS facilities. The responses were coded into categories using GIA and Table 5 below gives the breakdown of responses from the 41 Adult Learners who answered. Compared with the overall responses, Adult Learners were more likely to comment that they would access MLS if they needed. They were less likely to comment on resources/location

or the need for student feedback or advice as reasons that would encourage them to avail of MLS. No Adult Learners mentioned exams or results as a prompt for them to access MLS.

Discussion and conclusion

In this paper we have considered the data concerning Adult Learners in our large-scale student evaluation of MLS. We also compared, where possible, these results with the overall cohort or with traditional learners. Our two main research questions were:

1. What are the motivational factors of Adult Learners who seek mathematics learning support (MLS)?
2. Why do some Adult Learners of mathematics not seek MLS?

When we considered the backgrounds of the respondents, we did not find a significant difference between Adult Learners and the overall cohort in terms of the disciplines that they were studying. This will be investigated further in the next stage of our analysis when we consider the breakdown of results in terms of the individual institutions that respondents attended. However, as one would expect, Adult Learners did present with a wider range of mathematical backgrounds than the overall cohort, with a smaller proportion taking HL and a higher percentage taking OL. This is consistent with research elsewhere, e.g. Gill (2010).

When students who engaged with MLS were considered, there was a statistically significant association (Chi-Squared Test, $p < 0.001$) between student type (i.e. Adult Learners or traditional) and whether a student uses MLS, demonstrating that Adult Learners are more likely to seek support than traditional learners. This supports other research, e.g. Ní Fhloinn (2007) who states that Adult Learners in DIT seek support much earlier than traditional learners, even as early as the first day of term. However, in our study, we found no significant difference in the mathematical backgrounds of Adult Learner users and non-users of MLS.

Partial answers to our first research question are provided when the reasons why students engaged with MLS were investigated. Analysis suggests that Adult Learners in our study were more likely than traditional students to mention the following reasons for engaging: having been a long time away from education; poor confidence in their mathematical ability; seeking general extra help; struggling with mathematics. In contrast, Adult Learners were much less likely than traditional students to mention the following reasons: to get help with specific coursework assessment or as revision for tests; to improve or gain better understanding; to state they find mathematics difficult. Being an Adult Learner, having not studied mathematics in any formal sense for a long time lends itself to having gaps in knowledge due to forgotten or perhaps never learned material. Lawson (2008) states that some students avoid support due to a fear of embarrassment or feeling that they just have too many mathematical problems to deal with. This gap in knowledge appears to act as an impetus rather than an obstacle for the Adult Learners in our study to engage with support “As I have been out of the education system for many years I felt I needed the extra support”. These Adult Learners were motivated to engage because of their worry about gaps in their mathematical knowledge and the length of time they had been away from studying mathematics “As a mature student I needed a refresher”. Wolfgang and Dowling (1981) may partially explain this finding as they maintain that traditional and Adult Learners have different motivations and approaches to study. Safford (1994, p50) supports this stating that

while Adult Learners may carry ‘intellectual baggage’, they are generally self-directed and making the decision to return to education implies a motivation for change and growth.

A significantly smaller proportion of Adult Learners did not avail of MLS when compared to the overall cohort. In terms of our second research question, we considered the reasons given by students for non-engagement with MLS. According to Ashcraft and Moore (2009) avoidance is often the consequence of mathematically anxious students. Bibby (2002) reports that math anxiety and shame of own mathematics ability are reasons that students fail to seek help with mathematics. In a study carried out by Grehan et al (2011, p79) at NUI Maynooth, the reasons divulged for lack of engagement with MLS included ‘fear; lack of personal motivation; the anonymity of large classes; and to a lesser extent the lack of awareness of support services’. Symonds et al. (2008) list a fear of embarrassment and a lack of information regarding the whereabouts of the mathematics support as reasons why students do not engage. Our findings largely contrast with those just mentioned. The largest proportion of responses from both Adult Learners and the overall cohort who did not engage with MLS indicated that they simply did not need to: “Good service for students – just didn’t need to avail of it”; “I would definitely find time to attend if I needed to”. It is reassuring that many of those who do not utilise the resources provided simply do not feel the need. Only 13% of Adult Learners stated that they did not know where it was and/or had not heard of the support and just 4 stated that they were afraid or embarrassed to go “Just felt a bit uncomfortable, felt the questions I had may seem a bit irrelevant”. As we discussed earlier, fear and embarrassment were more of a motivation to attend rather than not attend MLS.

Overall, respondents were very positive about the MLS experience they received in their institution, with Adult Learners especially so, e.g. users of MLS reported increased confidence in the mathematical abilities and finding it easier to cope with the mathematical demands of the courses “I’ve had a fear of maths all my life so with MLC help I’ve become more confident”. It is clear from the comments that MLS provides a mathematical lifeline, so to speak, for many Adult Learners: “I would be seriously lost without the MSC and the extra maths classes ran. Now I actually like maths”; “Excellent and I credit the help I receive here to me passing all my maths tests so far”.

Many of the comments highlighted the important role of MLS tutors. Lawson (2008) states that students attend MLSCs precisely because they offer emotional and MLS to students who suffer from mathematics anxiety. FitzSimons and Godden (2000), and Safford (1994) recommend the provision of this warm supportive environment in which individual needs are met and Adult Learners of mathematics can thrive. The quality of staff is crucial to the success of MLS (Lawson, et al., 2003) and in particular in relation to the education of Adult Learners (FitzSimons & Godden, 2000). Gill (2006) states that the one-to-one attention students receive in MLSCs is most highly favoured. Some of the responses in this study referred to how they preferred the teaching approach used in the MLSCs to those in their regular tutorials “People in the MLSC explain the questions or doubts you have the way the people in the tutorials should”.

However, Lawson et al. (2003) states that not everyone will make a good MLS tutor and this is reflected by the small number of negative comments about certain MLS tutors, e.g. “Possibly some training in social skills for some of the tutors”. Benn (1994) encourages teachers to tread carefully when dealing with Mature Students of mathematics as it will

influence how students perceive the subject. It is in the nature of MLS evaluation that both positive and negative comments can be used constructively. To this end, the IMLSN is in the process of developing and collating MLS tutor training materials which can be used to help ensure best practice in the recruitment and training of tutors. There were some other negative comments, e.g. in relation to the timing of the drop in centre or classes, the volume of students in attendance and hence the lack of one-to-one attention at busy times: “It’s sometimes very crowded and the instructors cannot get to you”, “Sometimes the wait for assistance is 30-45 minutes”. These findings resonate with those of Lawson et al. (2003) who state that MLSCs are inclined to be very busy at certain times, such as at examination time, and there will be waiting times as a result. Again, these comments were not standard across the survey and will be of more relevance to the individual institutions when further analysis is presented.

It is very difficult to claim that MLS is responsible for increases in retention or student success rates in mathematics (Lawson et al, 2003). Mac an Bhaird et al (2009) tell us that we cannot take full credit as a number of factors are in play when it comes to student progress such as motivation etc. However, the findings from this study indicate a high level of Adult Learner satisfaction with the services provided by the MLSCs throughout Ireland, and many Adult Learners indicated that MLSCs are responsible for their not dropping out of their studies. “It was a very valuable experience, whereby without it I would have certainly failed”.

References

- Ashcraft, M. H. (2002). Math anxiety: Personal, educational, and cognitive consequences. *Current directions in Psychological Science*, 11, 181-185.
- Ashcraft, M. H., Moore, A. M. (2009). Mathematics anxiety and the affective drop in performance. *Journal of Psychological Assessment*, 27, 197-205.
- Benn, R. (1994) Mathematics: Breaking down the barriers. In D. Coben, (Ed.) *Proceedings of the 1st Inaugural Conference of Adults Learning Mathematics* (pp. 24-29). Retrieved June 20th, 2014, from <http://www.alm-online.net/images/ALM/conferences/ALM01/proceedings/ALM01-proceedings-p24-29.pdf?7c979684e0c0237f91974aa8acb4dc29=1nq6kn4hq32cacibkun7m6neb6>.
- Benn, R., (2000). Mathematics: Certainty in an uncertain world? In D. Coben, J. O'Donoghue, G. E. FitzSimons (Eds.) *Perspectives on adults learning mathematics: Research and practice* (pp.109-118). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Bibby, T. (2002). Shame: An Emotional Response to Doing Mathematics as an Adult and a Teacher. *British Educational Research Journal*, 28(5), 705–721.
- Burke, G., Mac an Bhaird, C., O'Shea, A. (2012). The impact of a monitoring scheme on engagement in an online course. *Teaching Mathematics and its Applications*, 31(4), 191-198.
- Carmody, G., Wood, L. (2005) Bridging the gap for first year students with the use of peer tutoring. In M. Horne, B. Marr (Eds.) *Connecting voices in adult mathematics and numeracy: practitioners, researchers and learners* (Proceedings of the Adults Learning Mathematics (ALM) 12th annual international conference, pp. 76-81). Melbourne: ACU National.
- Coben, D. (2003). *Adult numeracy: review of research and related literature*. London: National Research and Development Centre (NRDC).
- Cronin, A., Cole, J., Clancy, M, Breen, C., O'Se, D. (2016) *An audit of Mathematics Learning Support provision on the island of Ireland in 2015*. National Forum for the Enhancement of Teaching and Learning in Higher Education, Dublin.
- Diez-Palomar, J., Gimenez Rodriguez, J., Garcia Wehrle, P. (2005) Adults' dialogic productive mathematical interactions in the classroom. In M. Horne, B. Marr (Eds). *Connecting voices in adult mathematics and numeracy: practitioners, researchers and learners* (Proceedings of the Adults Learning Mathematics (ALM) 12th annual international conference, pp. 97-103). Melbourne: ACU National.
- Elliot, S. & Johnson, S. (1994). Adult learners in Higher Education; Academic Maths Support. In D. Coben, (Ed.) *Proceedings of the 1st Inaugural Conference of Adults Learning Mathematics* (pp. 85-89).

- Retrieved June 20th, 2014, from <http://www.alm-online.net/images/ALM/conferences/ALM01/proceedings/ALM01-proceedings-p85-89.pdf>.
- Faulkner, F., Hannigan, A., Gill, O. (2010). Trends in the mathematical competency of university entrants in Ireland by Leaving Certificate mathematics grade. *Teaching Mathematics and its Applications*, 29, 76-93.
- Fitzsimons, G. E., & Godden, G. (2000). Review of research on adults learning mathematics. In D. Coben, J. O'Donoghue, G. E. FitzSimons (Eds.) *Perspectives on adults learning mathematics: Research and practice* (pp.13-45). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Galligan, L., Taylor, J. A. (2008) Adults returning to study mathematics. In H. Forgasz, A., Barkatsas, A. Bishop, B, Clarke, S. Keast, W. Seah, P. Sullivan (Eds.) *Research in Mathematics Education in Australasia 2004 – 2007* (pp. 99–118). Sense Publishers: Rotterdam.
- Gill, O. (2006). *What Counts as Service Mathematics? An Investigation into the 'Mathematics Problem' in Ireland*. (Unpublished doctoral dissertation). Department of Mathematics and Statistics, University of Limerick, Limerick.
- Gill, O. (2010) Evaluating the impact of a refresher course in mathematics on adult learners. In H. Christen, J. Diez-Palmar, J. Kantner, C.M. Klinger (Eds.). *Maths at Work – Mathematics in a changing world* (Proceedings of the Adults Learning Mathematics (ALM) 17th annual international conference, pp. 37-46). Oslo: Vox, ALM.
- Gill, O., Johnson, P., & O'Donoghue, J. (2008) *An audit of mathematics support provision in Irish third level institutions*. CEMTL: University of Limerick.
- Gill, O., Mac an Bhaird, C., Ní Fhloinn, E. (2010). The Origins, Development and Evaluation of Mathematics Support Services. *Irish Mathematical Society Bulletin*, 66, 51-64.
- Gill, O., O'Donoghue, J. (2006) Mathematics support for adult learners. In V. Seabright, I. Seabright (Eds.) *Crossing borders – research, reflection and practice* (Proceedings of the Adults Learning Mathematics (ALM) 13th annual international conference, pp. 43-53). Queens' University Belfast: ALM, CAMET.
- Golding, G., O'Donoghue, J. (2005) Using topic maps to support adults' mathematics learning. In M. Horne, B. Marr (Eds.) *Connecting voices in adult mathematics and numeracy: practitioners, researchers and learners* (Proceedings of the Adults Learning Mathematics (ALM) 12th annual international conference, pp. 120-128). Melbourne: ACU National.
- Grehan, M., Mac an Bhaird, C., O'Shea, A. (2011). Why do students not avail of mathematics support? *Research in Mathematics Education*, 13(1), 79-80.
- Higher Education Authority (2016) *Key facts and figures*. Retrieved 4th Sept, 2017, from <http://hea.ie/assets/uploads/2017/06/HEA-Key-Facts-and-Figures-201516.pdf>
- Lawson D., Croft, T., & Waller, D. (2012) *Mathematics support past, present and future. Innovation, Practice and Research in Engineering Education*. Retrieved 20th June, 2014, from http://www.academia.edu/2715773/Mathematics_support_past_present_and_future.
- Lawson, D. (2008). *Mathematics Support Centres: Who uses them and who doesn't? Why not?* Keynote presentation at the 3rd Irish workshop on Mathematics Learning and Support Centres, NUI Maynooth.
- Lawson, D., Croft, A.C., & Halpin, M. (2003). *Good Practice in the Provision of Mathematics Support Centres* (2nd edn.). Birmingham: LTSN Maths, Stats and OR Network. Retrieved 20th June, 2014, from www.mathcentre.ac.uk/resources/guides/goodpractice2E.pdf.
- Lee, S., Harrison, M., Pell, G., & Robinson, C. (2008). Predicting performance of first year engineering students and the importance of assessment tools therein. *Engineering Education*, 3(1), 44-51.
- Mac an Bhaird, C., Morgan, T., & O'Shea, A. (2009). The impact of the mathematics support centre on grades of first year students at the National University of Ireland Maynooth. *Teaching Mathematics and its Applications*, 28(3), 117-122.
- Mac an Bhaird, C., Ní Fhloinn, E., Fitzmaurice, O., & O'Sullivan, C. (2013). Student Non-Engagement with Mathematics Learning Supports. *Teaching Mathematics and its Applications*, 32, 191-205.
- Matthews, J., Croft, T., Lawson, D., & Waller, D. (2012) *Evaluation of mathematics support centres: a review of the literature*. Loughborough: sigma. Retrieved 20th June, 2014, from <http://www.mathcentre.ac.uk/resources/uploaded/52487-evaluation-of-msc-7.pdf>.
- Ní Fhloinn, E. (2007) Assisting adult learners within a Maths Learning Centre. In T. Maguire, N. Colleran, O. Gill & J. O' Donoghue (Eds.) *The Changing Face of Adults Mathematics Education: Learning from the Past, Planning for the Future* (Proceedings of the Adults Learning Mathematics (ALM) 14th annual international conference, pp. 233-240). Limerick: University of Limerick, ALM.
- Ní Fhloinn, E., Fitzmaurice, O., Mac an Bhaird, C., & O'Sullivan, C. (2014). *Student perception of the impact of mathematics support in higher education*. *International Journal of Mathematical Education In Science And Technology*, DOI: 10.1080/0020739X.2014.892161
- Ní Fhloinn, E., Fitzmaurice, O., Mac an Bhaird, C., & O'Sullivan, C. Gender differences in the level of engagement with mathematics support in higher education in Ireland, to appear.

- O'Donoghue, J. (2000). Perspectives in teaching adults mathematics. In D. Coben, J. O'Donoghue, & G. E. FitzSimons (Eds.) *Perspectives on adults learning mathematics: Research and practice* (pp.229-234). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- O'Donoghue, J. (2004). *An Irish perspective on the "Mathematics Problem"*. Plenary lecture at Irish Symposium for Undergraduate Mathematics Education 2 (ISUME2), University College Dublin.
- Pell, G. & Croft, T. (2008). Mathematics support—support for all? *Teaching Mathematics and its Applications*, 27(4), 167-173.
- Perkin, G., Lawson, D., & Croft, T. (2012) *Mathematics learning support in Higher Education: the extent of current provision in 2012*, Loughborough: sigma. Retrieved June 20th, 2014, from <http://www.mathcentre.ac.uk/resources/uploaded/52789-mls-in-uk.pdf>.
- Safford, K. (1994) Introduction to Algebra for Adult Students, In D. Coben, (Ed.) *Proceedings of the 1st Inaugural Conference of Adults Learning Mathematics* (pp. 40-50). Retrieved June 20th, 2014, from <http://www.alm-online.net/images/ALM/conferences/ALM01/proceedings/ALM01-proceedings-p40-50.pdf?7c979684e0c0237f91974aa8acb4dc29=1nq6kn4hq32cacibkun7m6neb6>.
- Singh, E. (1993). The political dimension of adult numeracy: Conclusions of a survey into attitudes to mathematics. In C. Julie, D. Angelis, & Z. Davis (Eds.) *Political Dimensions of Mathematics Education 2: Curriculum Reconstruction for Society in Transition* (pp. 335-341). Cape Town: Miller Maskew Longman (Pty) Ltd.
- Symonds, R., Lawson, D., & Robinson, C. (2007). The effectiveness of support for students with non-traditional backgrounds. *Teaching Mathematics and its Applications*, 26(3), 134-144.
- Symonds, R., Lawson, D., & Robinson, C. (2008). Promoting student engagement with mathematics support. *Teaching Mathematics and its Applications*, 27(3), 140-149.
- Thomas, D. R. (2003). *A general inductive approach for qualitative data analysis*. Retrieved June 20th, 2014, from <http://www.frankumstein.com/PDF/Psychology/Inductive%20Content%20Analysis.pdf>.
- Tusting, K., & Barton, D. (2003). *Models of Adult Learning: A Literature Review*. Leicester: National Research and Development Centre for Adult Literacy and Numeracy.
- Wolfgang, M.E., & Dowling, W.D. (1981). Differences in motivation of adult and younger undergraduates. *Journal of Higher Education*, 52(6), 640-648.

Incorporating Study Skills Training into an Elementary Algebra Course

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Abstract

Pass rates in developmental mathematics courses are very low across the United States and the attrition rates are high. The purpose of the intervention undertaken at an urban community college was to address the above problems. Five instructors incorporated study skills training into an elementary algebra course in sections randomly selected for treatment. Other sections taught by the same instructors were designated as control. All participating instructors attended a workshop on study skills training conducted by an expert in that area. The workshop included outlines of seven mini-lessons as well as suggestions on how to incorporate them into the course. A detailed description of the mini-lessons is presented in this paper. The results show that the attrition rate was significantly lower in the treatment section and that the pass rates were slightly higher, although not significantly so.

Key words: developmental mathematics, study skills, mathematics learning, student retention.

Background

This paper was inspired by a grant funded study, conducted during Fall 2010 semester at the Borough of Manhattan Community College (BMCC) of the City University of New York, USA. The purpose of the study was to test a specific approach for increasing pass rates and reducing attrition rates in a developmental mathematics course offered at BMCC.

In the past, the mathematics department at BMCC has undertaken a number of initiatives aimed at improving success in developmental courses. These included Supplemental Instruction in which tutors attended classroom sessions in order to enhance student learning. In addition, the tutors also helped students outside of the formal classroom setting on a weekly basis. Other initiatives included faculty workshops designed to improve teaching methods as well as student workshops which reviewed subject matter in preparation for exit examinations.

Notably, none of the aforementioned initiatives addressed such important student variables as study skills, time management, attitudes, and persistence. Research, however, has documented that only about 25% of the variation in students' performance is attributable to the quality of teaching of the subject matter. Another 25% is explained by students' affective variables such as attitudes, study habits and skills, dispositions, and math and test anxiety (Bloom, 1976; Nolting, 2008), with the remaining 50% attributable to cognitive entry skills (aptitude, prerequisite knowledge of the subject). According to Nolting, students' affective characteristics are the most neglected area in colleges today.

A number of researchers have explored the affects of mathematics study skills training on student learning and grades. Paul Nolting (1991) did research on students who repeated an algebra course one to four times. The experimental group in his study received math study skills training in the form of a one-hour math study skills course which met for two hours a week for the first eight weeks of the semester. The results indicated that the experimental group had a significantly higher passing rate than the control group.

Valencia Community College in Orlando, Florida, received a Title III grant to examine the effects of math study skills courses on their Elementary Algebra passing rates. Their model involved providing a one-hour math study skills course that ended at the midterm. Their findings indicated a significant improvement in students' grades.

West Virginia Wesleyan College in Buckhannon, examined the effects of teaching study skills to students with disabilities. The instructor taught mathematics study skills in her Elementary Algebra course during the first half of the semester. The pass rate in her class was significantly higher than the average course pass rate for the school (Houghton Mifflin, n. d.). Based on the above evidence, it was apparent that training in mathematics study skills may have a positive affect on student performance in Elementary Algebra. Consequently, the authors decided to incorporate study skills training into developmental mathematics instruction. Ideally, we would have liked to offer a separate mathematics study skills course, but because this was not a feasible alternative at our institution, we decided to include formal study skills training in selected sections of Elementary Algebra.

Intervention

Five instructors teaching two or more sections of an Elementary Algebra course participated. Each instructor incorporated study skills training into one section of the course. The instructor's other section was taught without any specific intervention.

The instructors were given a workshop conducted by a consultant (Dr Peskoff) who shared his expertise on incorporating study skills into the developmental mathematics classroom. The consultant developed a series of lesson plans based on a study skills workbook written by Alan Bass (2008). The lessons covered a variety of topics such as "navigating" the textbook, effective note taking, completion of homework assignments, preparing for exams, and coping with mathematics anxiety. The workshop was offered immediately preceding the beginning of the semester and was three hours in duration. Throughout the workshop, instructor participation was encouraged. The following handout, prepared by Dr Peskoff, was distributed at the workshop and served as a reference for instructors during the semester.

Integrating Study Skills and Time Management Strategies into the Classroom

The goal of this workshop is to enable instructors to motivate mathematics students to study effectively and to help them learn the appropriate study skills. Each student will receive a copy of the Bass workbook entitled "Math Study Skills". However, it is the instructor's job to see that his or her students are willing and able to use the information.

This workshop will begin by asking the audience to participate in a "hands on" exercise to enable them to appreciate the important role that study skills play in effectively learning mathematics.

Subsequently, we will then discuss the most important points contained in each chapter of the Bass workbook, emphasizing how you can implement them in your classroom. The material will be organized into seven different topics. The first topic discusses four chapters from Bass and the remaining six topics discuss one chapter each. It is recommended that you require students to read the appropriate chapters in Bass before you review them. You can assign the exercises as homework, or perhaps do one or two of them from each chapter in class as part of your presentation.

Suggested Timetable:

In order to enable students to benefit from the material as soon as possible, it is suggested that all of the information be initially presented (and then later reviewed) during the first twenty to thirty minutes of the first seven to eight class meetings (assuming a course meets twice per week for one hour and forty minutes). Accordingly, each of the seven topics can be presented in approximately one-half hour. Nonetheless, you may choose to spend proportionately more time on certain chapters (such as math anxiety) and less on others (such as your class notebook).

It should also be noted that throughout the course of the entire semester (even after the first eight meetings), students should always be reminded of appropriate study skills (such as completing homework) whenever necessary.

In other words, after the initial presentation over the first few weeks, the material should still be constantly reinforced, especially when students appear frustrated or apathetic.

Topic One: Introduction, What Makes Math Different, Learning Styles, and Retention

Note: Because this topic covers four chapters, it may take more than one-half hour to cover in class.

Chapter One: Introduction

You should ask students about their past experiences studying math. You will probably get a lot of feedback! Many students will say they felt frustrated or even angry in former math classes. Ask them why. Use their responses to focus on ways to improve their understanding of math and achieve success in your class. Motivate the students to read Bass precisely because by helping them study more effectively, it will decrease their anxiety and increase their success. Remind students that the skills learned here can be used in future math courses also.

Chapter Two: What Makes Math Different

Ask students how learning math is different from learning English, Psychology, or History. See what they have to say. Bass refers to math as a “skill-based subject.”. Similarly, I emphasize that there is a focus on understanding and problem solving rather than mere memorization. Remind students of the importance of doing homework.

You may wish to give students a simple in-class exercise such as “list three important reasons to do homework after every class.” Actually there are more than three reasons, and we will discuss them together during this workshop.

This may be an opportune time to remind students of the benefits of using the mathematics laboratory for help with homework.

Chapter Three: Learning Styles

You may briefly want to mention the three learning styles (visual, auditory, and kinesthetic) presented in Bass. The goal here should be to let students know that not everyone learns the same way but that you will try your best to accommodate each student's best style of learning (which is not an easy task).

If time permits, you can have students complete the “yes/no” exercises for each learning style in Bass.

Whatever learning style works best for a student, he or she should be encouraged to practice solving problems as much as possible.

Chapter Nine: Retention and General Study Strategies

This chapter contains a lot of information which may be difficult for you to present within a limited time frame. You might want to discuss the importance of note cards as an important retention technique. Present one of the examples from Bass in class and perhaps ask students to create one or two note cards of their own for the next class. Some students may like note cards and others not, but as Bass states to his audience “You owe it to yourself to try note cards.” We will discuss these in more detail during the workshop.

Some of the other important strategies that you should discuss (also listed on p. 72 Bass, 2008) include doing homework effectively (**and regularly**), reading your textbook, creating a vocabulary list, reworking and reviewing notes. Keep in mind that students often have limited time and get easily frustrated, so although these approaches can be presented, they may not be the best alternative for everyone.

Nonetheless, I would strongly recommend that you constantly emphasize the importance of reviewing class notes and doing homework after every class meeting. Of course, you can monitor this either electronically or by simply collecting or “spot checking” homework in class.

Topic Two: Math Anxiety

Many students feel very anxious and even “dread” studying math. Although there are many reasons why math anxiety develops, many students have had negative experiences in the past (perhaps stemming back to elementary school) when they attempted to study math.

Your focus should be on helping students feel more comfortable studying math. The best way to achieve this is to **practice** math (even if it's the last thing a student feels like doing) as much as possible.

You may wish to first discuss the “avoidance” behaviors Bass presents (2008, p. 27) which accompany math anxiety and then review the seven strategies Bass lists (2008, p. 28) as a “cure” for math anxiety. We will review these during the workshop.

For homework (or if you prefer, in class), you may wish to implement a simplified form of the Self-Talk exercise in Bass. Ask students to write down a negative thought they

have about studying math and then a positive thought that can replace it. Subsequently, ask them to write a negative behavior and then a positive behavior that can replace it. We will also review these during the workshop.

Topic Three: Managing Your Time

Most of this material is not unique to the study of mathematics. However, it is harder to be motivated to manage one's time when (a) you are extremely overwhelmed by job and family responsibilities and (b) you are trying to find time to study a subject you find difficult, if not frustrating. You should emphasize these constraints to your students so they know you understand the challenges they face.

In addition, you may wish to briefly review the list of strategies presented in Bass. In particular, I would emphasize maintaining a regular study schedule which includes homework after every class. Lack of study leads to lack of understanding which leads to "cramming" which often leads to failure and the perpetuation of a vicious cycle of math anxiety. Disciplined study, however, can lead to success!

Topic Four: Your Class Notebook

Bass recommends using a three ringed notebook but I'm not sure how many students will comply with this idea. A good approach to emphasize is that a student's notebook should be used to organize his or her work and should contain different sections. A notebook should be thought of as a portfolio which contains handouts, class notes, homework, exams/quizzes, and perhaps a glossary of vocabulary terms (although some students may prefer to have the glossary as part of their class notes).

The key here is **organization** so no material gets misplaced.

Topic Five: Your Textbook and Homework

One of the most important challenges for students is how to effectively use their textbook (and ancillary materials).

Bass recommends surveying the material, surveying the assigned homework, and then reading the section. You may wish to go through this process using one of your homework assignments as a model. Keep in mind though, that many students do not read the textbook. They simply use it (or photocopied pages from it) to do homework. Perhaps some students can achieve success in this manner, but you should try to emphasize the importance of using the textbook to accompany and enhance the material presented in class rather than merely for homework assignments.

Doing homework is perhaps one of the most important activities of all. Bass (2008, p. 51) discusses "How to do Homework."

The key point to emphasize to your students is that they should review their class notes and handouts before attempting a homework assignment to ensure that they remember the material they learned. Most assignments go from easier to harder problems so students may get stuck near the end of an assignment. They should be motivated not to give up. If they are stuck, they should mark the question, and ask their instructor (or tutor) to review it as soon as possible.

Another strategy that Bass presents to avoid frustration and anxiety is to always end on a “positive note” with an easier problem that you know how to do, perhaps from an earlier lesson.

You may wish to model the process of completing a short homework assignment (perhaps five problems) for your students, including what to do when you “get stuck.” This will be discussed in more detail during the workshop.

Topic Six: Class Time and Note taking

The key points here are arriving on time to class, being an active listener during class, and taking organized notes during the lesson. Each lesson should begin with a clean sheet of paper (and the date and topic written on top). Bass recommends that students should “space your notes out” so that details can be filled in later. Although it is important to take comprehensive notes, it is equally important to listen. The emphasis should be on “understanding, not dictation.”

Usually, during the course of a lesson, students will be given an opportunity to solve problems to practice what they have learned. You should try to implement this strategy as much as possible. Walk around the room looking at students’ work and encourage them to go to the blackboard.

Both Bass and I emphasize the importance of asking questions in class. In sum, students should be encouraged to ask questions, but too many questions from an underprepared student may impede the learning of others.

Topic Seven: Test Taking

It can be argued that if a student has been studying regularly, then preparing for an exam should require minimal effort. This may be slightly exaggerated, but nonetheless, if students have implemented the correct study skills, then they should essentially need to review their class notes, homework, and practice exams (or sample questions) to prepare for an examination. Bass emphasizes the importance of a practice test as a dress rehearsal and I agree. Accordingly, students should be encouraged to stay up to date and avoid cramming so that they can prepare confidently for an exam primarily by reviewing and practicing what they have already learned.

As an instructor, it is useful to provide students with frequent short quizzes and practice problems so that they not only have plentiful material to practice but can also feel confident.

You should emphasize that an exam does not have to be stressful if one feels adequately prepared.

Bass recommends that while taking an exam, students should first write down any necessary formulas before looking at the questions. Subsequently, they should “survey the entire test,” do the easiest problems first followed by the hard problems, always be “mindful of your time,” and finally review the entire exam with any time that remains.

(Peskoff, 2010, complete).

Discussion and Conclusion

The workshop was successful. Subsequent interviews with the participating instructors confirmed that all were completely satisfied with the workshop and found the information presented to be both useful and relevant. Moreover, they reported that the workshop thoroughly prepared them to teach study skills to their students. At the end of the semester, the instructors reported that it would be advantageous to have more time allotted to teach mathematics study skills (without sacrificing the time needed to teach mathematics).

The sections in which students received study skills training had a significantly lower attrition rate than the control sections. This difference may be attributed to the fact that participation in a formal study skills program (as part of a mathematics course) contributes to better time management skills, increased confidence, and a lower level of anxiety. All of these factors tend to lead to an increase in persistence and a decrease in learned helplessness. The pass rates were slightly higher in the “study skills” sections than in the control sections, but the difference was not statistically significant. One reason for this result might be that both the “study skills” and control sections had the same amount of total class time. In the “study skills” sections, some of this time had to be utilized to teach study skills in lieu of mathematics content while in the control sections, all of the class time was dedicated to teaching mathematics. This discrepancy can be regarded as a “double edged sword.” The authors hope to correct this imbalance in future studies by designating additional class time for teaching study skills so that the time used to teach mathematics does not need to be sacrificed. This extra time may be included in the course as a supplementary “laboratory” hour dedicated to the learning of study skills.

In conclusion, although our study clearly demonstrated the benefits of incorporating study skills into a developmental mathematics course, we believe that the amount of time spent teaching mathematics content should not be reduced. In other words, the time allotted for study skills training should supplement rather than replace the time allotted for mathematics instruction. In addition, the authors plan to investigate the effect of study skills training in college level mathematics courses.

References

- Bass, A. (2008). *Math Study Skills*. Pearson Education, Boston, MA.
- Bloom B. (1976). *Student characteristics and school learning*. McGraw-Hill, New York, NY.
- Houghton Mifflin. (n. d.). *Math Study Skills*. Retrieved from (http://college.hmco.com/instructors/ins_teachtech_foundations_module_mathstudy_skills.html)
- Nolting, P. (1991). *The effects of Counseling and Study Skills Training on Mathematics academic Achievement*. Academic Success Press, Bradenton, FL.
- Nolting, P. (2008). *Winning at Math: Your Guide to Learning Mathematics through Successful Study skills*, Fifth Edition, Academic Success Press, Bradenton, FL.

Workplace and Vocational Education

Adult Maths and Everyday Life: Building Bridges, Facilitating 'Transfer'

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Abstract

How should we teach mathematics so as to support adults' functioning satisfactorily in their work and everyday lives? We can draw on the range of activities that the typical adult is involved in, but this range will vary greatly across the group in a typical basic education or college pre-calculus course. Thus, the mathematics taught must be able to be 'transferred' or generalised to other contexts. My approach to this problem provides a critical alternative, both to traditional transfer theories and to situated cognition, in showing learners and teachers how to build bridges between different practices, particularly between school or college and work.

Introduction

The *transfer* (or generalisation) of learning means the use of ideas and learning from one context in another. This might involve:

- (i) the *harnessing* of out-of-school activities and thinking in the teaching of college subjects;
- (ii) the *application* of learning from college contexts to work or everyday activities;
or
- (iii) the use of a school subject like mathematics outside of its own domain, in nursing, engineering or business studies (cf. Maas, 1998).

This is clearly an especially important set of issues for mathematics since it is claimed to have wide applicability across the curriculum, and outside the school or college. Yet students often 'fail' to accomplish transfer. And views on the reasons are in conflict, leading to widespread controversy.

Views on the Transfer of Learning in Mathematics

The discussion has been vibrant not only in psychology, but also in education in the last 10 or 20 years. Here *traditional* approaches include views favouring the use of behavioural learning objectives, "basic skills" approaches, and "utilitarian" views (e.g. Cockcroft Report, 1982). They share several ideas. A problem or "task", and the mathematical thinking involved, can be described in abstract, e.g. as a 'percent', or a 'proportional reasoning' problem; hence it is claimed to be possible to talk about "the same mathematical task" occurring across several different contexts. For traditional views, that the "transfer of

learning", e.g. from school to everyday situations, should be relatively straightforward - at least for those who have 'understood' the maths properly.

However, there are problems. As indicated above, one cannot *depend* on transfer being accomplished, by a particular learner, in a particular situation. And recent research has shown striking differences between performance in school tasks, and that in work, or everyday situations (e.g. Lave, 1988; Nunes et al., 1993).

Thus critical positions have emerged - in particular, the *strong form* of situated cognition. Its proponents argue, citing Jean Lave (1988), that there is a *disjunction* between doing maths problems in school, and in everyday life, as these different contexts are characterised by different *structuring resources* (e.g. ongoing activities, social relationships). Subjects' thinking is *specific* to these settings, and to the different practices in play in them. Thus transfer of learning from school / academic contexts to outside ones is pretty hopeless.

However, there are problems with the situated cognition account, too. First, in its strong form, the view threatens a cul-de-sac: we are offered a proliferation of differently situated types of mathematical thinking, with high boundaries between them, and claims that the use of one type of thinking in another context is basically impossible (cf. Noss & Hoyles, 1996b, ch.2). Second, this approach seems to assume that practices and communities of practice can be seen as 'natural' - whereas I argue the need for description and *analysis* of the bases of different practices - in language or 'discourse'.

Meanwhile, Jean Lave's more recent work (Chaiklin and Lave, 1993; Lave, 1996) has moved on, acknowledging that no practice could ever be completely closed, or completely separated from other practices. Her approach consists of studying learning within communities of practice, and the social relations, and identities across them.

This brief discussion (see also Evans, 1999) suggests a need for a reformulation of the problem of transfer. Four crucial issues are discussed in the next section. Currently, a number of areas, besides mathematics education, are contributing to this reformulation, including developmental psychology (e.g. Nunes et al., 1993) and cognitive psychology (e.g. Anderson et al., 1996), and sociology of education (Muller and Taylor, 1995); see below. My approach further draws on discourse theory and poststructuralism; see e.g. Walkerdine (1988); Walkerdine and Girls & Maths Unit (1989); Evans and Tsatsaroni (1994, 1996); Evans (1999, 2000a).

Conceptualising Boundaries and Bridges

(1) how to characterise and differentiate the various contexts of thinking, activity and learning, and the related practices at play in them

In my approach, *practices* are activities such as school mathematics, research mathematics, nursing, banking, apprenticeship into tailoring, and shopping. Each practice is constituted by *discourses*.

Discourses are systems of ideas expressed in terms of *signifiers* and *signifieds*; signifiers are words, sounds, gestures, etc. and signifieds are conceptions, or what is meant. These discourses give *meaning* to the practice by expressing its *goals* and *values*, and *regulate* it in a systematic way, by setting down standards of performance. Within a community of practice, there is a set of social relations (power, difference) - with different members of the community taking up different *subject-positions*. For example, the basic positions available in

school mathematics are normally "teacher" and "pupil"; in shopping or street-selling, they would be "seller" and "buyer".

This approach, like situated cognition, recognises different practices as in principle distinct - e.g. school maths and everyday practices like street selling. But, my approach aims to avoid the *cul-de-sac* (see above), and to go further - by analysing the discourses involved through their relations of signification - relations of similarity and difference between signifiers and signifieds, and also devices such as metaphor and metonymy. So far this draws on de Saussure's structural linguistics. Going further, poststructuralist ideas about the inevitable tendency of the signifier to slip into other contexts, thereby making links with other discourses, and producing a play of multiple meanings, provide insight into meaning-making in mathematics; see the discussion of "shopping with mummy" below, and also e.g. Walkerdine (1988, Ch.2) on children's use of language to indicate relations of size, Evans and Tsatsaroni (1994, p.184).

Thus, rather than attempting to specify the context of a school maths problem by looking only at its wording - or by naming the context as if simply based in "natural" settings, as researchers we can describe it as socially constructed in discourse. This means:

(a) analysing the practices at play in the context, that would be involved in the 'positioning' of participants (Evans and Tsatsaroni, 1994); and

(b) attending to particular signifiers and their relations of similarity and difference, e.g. in reading interview transcripts.

(2) how to describe the relations between practices, and communities of practice, e.g. the boundaries or bridges between them

Contrary to the hopelessness of the strong form of situated cognition, I aim to build bridges between practices, by identifying areas where out-of-school practices might usefully "overlap" or inter-relate with school mathematics. This requires first of all that distinctions are made between those relations of signification in the learner's everyday practices that provide *fruitful* 'points of inter-relation' with school maths, and those that may be *misleading*. An example of a misleading inter-relation would be the attempt to harness the use of "more" in the home - where its opposite is *no more* (as in "no more ice cream for you") - to help teach "more" vs. *less* as an oppositional couple at school. The pupils are likely to be confused because what appears to be 'the same' signifier has a different meaning (signified) in the home and the school discourses (Walkerdine and Girls & Mathematics Unit, 1989, pp.52-53).

Thus, Walkerdine argues that activity within one discourse - say, playing a particular card game - will help with (i.e. can be "harnessed" for) school maths in those, and only those, aspects of the game which are both contained in school maths and which enter into *similar* relations of signification (Walkerdine, 1988, pp. 115 ff.). This would suggest that knowing the order of precedence among the 13 cards of a traditional deck (2,3,4, ... 9,10, Jack, Queen, King, Ace) would help a child to learn to count (1,2,3,, ...) - but only up to a point. There are *similarities* in the identical orderings of cards and natural numbers between 2 and 10 - but, also, especially, the *difference* between the Ace and '1' must be made explicit. So we can broaden Walkerdine's stipulation of 'similar relations of signification' to mean 'similar or specifiably different'.

(3) acknowledging the importance of affect, motivation, etc.

Most accounts of mathematical thinking, including situated cognition, largely ignore the area of emotion. But "meanings are not just intellectual" (Walkerdine & Girls and Maths Unit,

1989, p.52). Thus, whenever a teacher reaches outside of mathematics for an example as illustration, the mathematics is "at risk"; e.g. when illustrating maths in the context of shopping with "Mummy", if the mother "has financial difficulties, ... is sick, far away or deceased" (Adda, 1986, p.59). This is because of the fundamental character of language, its ability to produce "multiple meanings", as argued by poststructuralists (see above). Thus another reason that a particular set of relations of signification may not succeed in attempts to *harness* everyday life for school purposes, is that these relations may be *distracting* or *distressing* - and not only misleading.

Affect can be seen as the energy that powers reason (Buxton, 1981). Here, affect is understood as an emotional charge attached to particular words, gestures, and so on. This charge can flow from one signifier to another, along *chains of meaning*, by *displacement* (Evans and Tsatsaroni, 1996, p.355; Evans, 2000a). (NOTE 1)

The quality and intensity of affective charges may often be a major influence in the success or failure of many attempts at transfer - an influence that has so far been largely ignored in the mathematics education literature.

Implications for Teaching

(4) designing pedagogic practices that will facilitate harnessing and transfer

Besides seeking out fruitful (non-misleading, non-distracting) points of inter-relation, we must structure the pedagogic discourse so as to work systematically through a process of 'translation'. This involves a series of steps, where the signifiers and signifieds linked in one set of signs are transformed into a new set of signs, thereby creating new meanings. The several steps are held together by *chains of meaning* (Walkerdine, 1988, p.128ff.).

A simple example is that of a mother who, in discussing with her child the number of drinks needed for a party of the child's friends, manages to teach the child to count - by the following transformations from one step to another (Walkerdine, 1988, p.129):

Step

- | | |
|---|---|
| 1 | Child (<i>signified</i>) |
| | Name (<i>signifier</i>) |
| 2 | Name (<i>signified</i>) |
| | Finger (<i>iconic signifier</i>) |
| 3 | Finger (<i>signified</i>) |
| | Spoken Numeral (<i>symbolic signifier</i>) |
| 4 | Spoken Numeral (<i>signified</i>) |
| | Written Numeral (<i>symbolic signifier</i>) |

At the first step, the mother-teacher, encourages the child to form a sign linking the name of each child (*signifier*) with the "idea" of that child (*signified*). At each subsequent step, a new signifier (gesture, numeral) is linked to a new signified, which had been the signifier at the previous stage; each step thereby creates a new set of signs. The chain of meaning moves as follows: actual child (more precisely, *the idea of the child*) - name of child - iconic signifier - spoken symbolic signifier - written symbolic signifier.

Here, the different steps do not really represent different discourses, but they nevertheless show how a series of carefully constructed links between signifiers and signifieds could

provide the bridges for crossing boundaries between discourses - here, between home practices and school maths.

Schliemann (1995), in a paper concerned with the viability of harnessing maths from everyday settings to help with learning school maths, reaches a conclusion similar to Walkerdine's (above) about the necessary conditions for transfer or generalisation:

... mathematical knowledge developed in everyday contexts is flexible and general. Strategies developed to solve problems in a specific context can be applied to other contexts, *provided that the relations between the quantities in the target context are known by the subject as being related in the same manner as the quantities in the initial context are.*

(p.49, my emphasis; NOTE 2)

A set of useful guidelines for teaching / learning for transfer, can be based on the analysis above, and augmented from Anderson et al. (1996). An important principle is clearly to show learners how to perform a detailed analysis of the shared or similar components - *and the different aspects* - of the initial and target tasks. For a fuller discussion, see Evans (1999c).

Implications for Research

In order to specify the context of a maths problem attempted in a particular setting, some of our research effort must be aimed at producing and analysing interview transcripts. To illustrate, I include a brief reference to one of my own research interviews: the responses of "Donald" to one of the problems presented to a sample of social science undergraduates - which concerned a graph showing how the price of gold varied over one day's trading in London (Evans, 1998, 2000a).

For these interviews, two practices were judged to be 'at play' - on the basis of the setting, the language used in the letter of invitation, the interviewer's scripted talk, and so on. These two discursive practices were 'college maths' (CM) and 'research interviewing' (RI). In addition, I judged, mostly from the particular subject's talk, what was the 'predominant positioning' of each during each crucial episode of their interview. In general, RI was considered to open up the possibility of the subject's positioning being in a practice from their non-college or previous 'lives'; for the graph question, this non-college practice was often a business practice of some kind.

The interview analysis shows several things:

- (i) Donald is apparently able to focus on *discursive similarities and differences* between college maths and business maths (BM). He seems able to read the diagram as a "chart" (BM) or as a "graph" (CM), and to recognise the connections between a "trend" and a "gradient" (respectively).
- (ii) He is also aware of the different objectives in using the graph. In *business*, the objectives are competitive, to make comparisons across personnel or groups, or over time; in *college maths*, the objectives are more analytical, focused on the qualities of the curve, including the rate of change. He is aware of *different values and standards of regulation*, in particular of *precision*, required in the two discourses.

(iii) He is also open about the *different feelings* evoked by the two practices. For example, his awareness of the different goals of the two practices (see above) is sometimes painful (Evans, 1998, 1999b).

(iv) He is able and willing to use both college maths and money-market maths. Further he seems able to choose which practice to use to address the problem in the interview, to decide whether or not to apply his (more precise) college maths methods of calculating gradients to the problem of saying during which period of the day the price of interest was rising faster. Though not certain, it also appears that Donald is able to bridge the two practices, i.e. to transfer his college maths methods to deal with a problem involving charts, assuming he was convinced of the need.

Conclusions

1. Continuity between practices (e.g. school and out-of-school activities) is not as straightforward as traditional views assume. Hence scepticism is in order about claims that transfer is in principle straightforward.
2. Like situated cognition researchers, we can acknowledge at least that transfer is not dependable and often difficult. But it is not impossible, and hence we can be more optimistic than these other approaches suggest.
3. In teaching and learning, bridges between practices can be built, by analysing the *similarities and differences* between discourses (e.g. school vs. everyday maths), so as to identify fruitful "points of inter-relation" between school maths and outside ("target") activities.
4. The inter-relationships of thought and feeling have received insufficient emphasis in most discussions of transfer. They are important because they contribute to the inevitable tendency of language to flow in unexpected ways and generally to assume multiple meanings within different practices - which constitutes a severe limitation on the possibilities of any intended transfer.
5. Yet this ability of a signifier to form different signs also provides the basis for any transfer possibilities. Thus, though the successful crossing of bridges cannot be guaranteed "risk-free", this paper has sketched some steps it is *necessary* to follow. Thus, for anything like transfer to occur, a "translation" across discourses would have to be accomplished, as summarised in (3). This translation is not straightforward, but it often will be possible.
6. We need further study of transfer from school to work, including a focus on sign systems, and more widespread workplace studies in the styles of Recife (e.g. Nunes et al., 1993) and the London Institute (e.g. Noss and Hoyles, 1996).
7. Given the links between the notion of transfer and the traditional views criticised above, as well as widespread dissatisfaction with the notion (e.g. Lave, 1988, 1996), I propose that the term should be replaced either by 'translation' or by 'generalisation'.

Notes

1. Indeed, insights from psychoanalysis can allow us a fuller consideration of the affective (Walkerdine, 1988; Evans and Tsatsaroni, 1996; Evans, 2000a).
2. As with Walkerdine's position, I would want to broaden Schliemann's stipulation of 'the same relations of quantities' to mean 'similar or specifiably different'.

Bibliography

- Adda, J. (1986). Fight against academic failure in mathematics. In P. Damerow et. al. (Eds.), *Mathematics for all* (pp. 58-61). Paris: UNESCO.
- Anderson, J. R., Reder, L.M. and Simon, H.A. (1996). Situated Learning and Education, *Educational Researcher*, 25, 4, 5-11.
- Carraher, D. and Schliemann, A. (1998), The Transfer Dilemma. Paper presented at symposium "If Not Transfer, Then What?", Annual Meeting of the American Educational Research Association, San Diego; submitted for publication in *Journal of the Learning Sciences*.
- Chaiklin, S. and Lave, J. (Eds.) (1993). *Understanding Practice: Perspectives on activity and context*. Cambridge: CUP.
- Evans, J. (1998). Problems of transfer of classroom mathematical knowledge to practical situations. In F. Seeger, J. Voigt, and U. Waschesio (Eds.), *The Culture of the Mathematics Classroom*. Cambridge: CUP.
- Evans, J. (1999). Building Bridges: Reflections on the problem of transfer of learning in mathematics, *Educational Studies in Mathematics: Special Issue on the Contexts of Teaching and Learning Mathematics*. (in press)
- Evans, J. (2000a). *Adults, Practices and Numeracy: Mathematical Thinking and Emotions in Context*. London: Falmer Press.
- Evans, J. (2000b). Adult Maths and Everyday Practices: Building Bridges, Facilitating 'Transfer'. In Diana Coben, John O'Donoghue, Gail FitzSimons (Eds.), *Perspectives on Adults Learning Mathematics: Research and Practice*, Kluwer.
- Evans, J. & Tsatsaroni, A. (1994). Language and subjectivity in the mathematics classroom. In S. Lerman (Ed.), *The culture of the mathematics classroom*. Dordrecht, NL: Kluwer.
- Evans, J. & Tsatsaroni, A. (1996). Linking the cognitive and the affective in educational research: Cognitivist, psychoanalytic and poststructuralist models. *British Educational Research Journal: Special Issue on Poststructuralism and Postmodernism*, 21(3), 347-358.
- Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life*. Cambridge: Cambridge University Press.
- Lave, J. (1996). Teaching as Learning, in Practice. *Mind, Culture, Activity*, 3, 3, 149-164.
- Maas, J. (1998), Technology transfer - a useful metaphor for university level maths courses for engineers and scientists; pp.58-62 in D.Coben and J.O'Donoghue (Eds.), *Adults Learning Mathematics - 4: Proceedings of ALM-4, the Fourth International Conference of Adults Learning Maths - A Research Forum*, University of Limerick, Limerick, Ireland, 4-6 July 1997.
- Muller, J. and Taylor N. (1995). Schooling and everyday life: knowledges sacred and profane. *Social Epistemology*, 9, 3, 257-275.
- Noss R. and Hoyles C. (1996). *Windows on Mathematical Meanings*. Dordrecht: Kluwer.
- Nunes, T., Schliemann, A. and Carraher, D. (1993), *Street Mathematics and School Mathematics*. Cambridge: Cambridge University Press.
- Walkerdine, V. (1988). *The mastery of reason: Cognitive development and the production of rationality*. London: Routledge & Kegan Paul.
- Walkerdine, V. & Girls and Mathematics Unit (1989). *Counting girls out*. London: Virago.

Post-script

In looking back on this conference contribution from some 20 years ago, I acknowledge that, within the mathematics education and the adult mathematics education communities, the debates on learning transfer have moved on. Nonetheless, the problem of transfer continues to be considered as central, as it always will be – in any society which supports a formal education system as one of its institutions.

Key ideas in the approach I take remain broadly the same. These start from the basic idea of a *practice*, as a set of activities that human beings, generally working together, create and support as a recurrent part of their lived experiences. These practices are defined and regulated by *discourses*, which are sets of ideas, expressed as relations between *signifiers* and *signifieds*. These discourses have effects in terms of power, since they *position* people unequally within the discourse – and affect the way they act, speak, think and feel. Examples are given in the paper and in the references, including those added below.

In later work, often with colleagues, I have expanded on and developed these ideas in several ways. The overall conceptual framework and a more developed set of examples is presented in Evans (2000a). It is extended in Evans, Morgan and Tsatsaroni (2006), where we consider a classroom example from Portugal, using ideas from Bernstein's sociology (2000) and Critical Discourse Analysis (Fairclough, 2003). In this latter article, we introduced the terms (a) *structural analysis*, for what I called 'analysing the practices at play' in the original conference paper (p.4); and (b) *textual analysis*, for analysing the dynamics of interaction, as represented in interview transcripts. In the later article (2006), we foreground *affect* and *emotion*, as energisers of what happens in the classroom, including in processes of learning and learning transfer. This is an example of the idea that transfer is an emotional, as well as a cognitive, process.

In fact, the idea of 'transfer' is very general, so it is worth exploring other sorts of transfer that are widely discussed nowadays. For example, 'knowledge transfer', is sometimes considered as the third sector of higher education, in addition to research and teaching (e.g. Ozga & Jones, 2006). In the original article, I also referred to 'technology transfer', the re-application of techniques designed for one setting in another (e.g. Maasz, 1998).

However, especially in conditions of globalisation, transfer as a movement of ideas or techniques from one setting to another is too simple. One should be careful even of the idea of transfer as a 'generalisation'. It is more helpful to consider transfer as a 'translation' between *contexts*, or even a 'transformation'. This leads us naturally to discuss the sorts of contexts that could be at issue; see Evans, Alatorre, van der Kooij, Noyes, and Potari (2010) for a review.

Significant theoretical work on learning transfer has been done in the last 20 years. For example, the various chapters in Tuomi-Gröhn and Engeström (2003) consider transfer using the metaphor of boundary-crossing. One chapter in that collection, Tuomi-Gröhn, Engeström and Young (2003), compares the approaches to transfer of 'cognitive-constructivist', situated and 'activity theory' approaches.

Nevertheless, at this stage, I think that what the literature needs at least as much is the description of situations where transfer has worked in practice. A very simple example with very young learners learning to count is given in the conference paper above (pp.6-7). A more challenging example is described in Evans (2000b); this paper discusses how adult students' learnings about how to use a derivative function to calculate the maximum height of an object thrown up into the air might be applied to calculate the point at which surplus or profit might be maximised, depending on the quantity of a commodity sold. In another example, Carreira, Evans, Lerman and Morgan (2002) show how a higher education teacher's attempt to harness an example from choices of drinks in everyday life to help students learn about indifference curves in academic economics discourse is disrupted by the students' calling up of a third, geographical discourse from their earlier studies.

At this stage, in adult mathematics education, my view is that we need more studies like those above, where case studies from a teacher's practice can be analysed using theoretical resources. These same resources will often be further developed themselves by the encounter between theory and practice.

Additional References

- Bernstein, B. (2000). *Pedagogy, Symbolic Control and Identity: Theory, Research and Critique* (revised ed.). Lanham: Rowman and Littlefield.
- Carreira, S., Evans, J., Lerman, S., and Morgan, C. (2002), Mathematical Thinking: Studying the Notion of 'Transfer'; in A. Cockburn and E. Nardi (eds.), *Proceedings of 26th International Conference of*

- Psychology of Mathematics Education Group (PME-26)*, Vol. 2, pp185-192, University of East Anglia, Norwich, UK, 21-26 Aug., 2002.
- Evans, J., Alatorre, S., van der Kooij, H., Noyes, A., and Potari, D. (2010), Plenary Panel: Mathematics in Many Settings; in M. M. F. Pinto and T. F. Kawasaki (eds.), *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp109-139. Belo Horizonte, Brazil: PME.
- Evans, J., Morgan, C., and Tsatsaroni, A. (2006). Discursive positioning and emotion in school mathematics practices. *Educational Studies in Mathematics*, 63(2), 193-207.
- Fairclough, N. (2003). *Analysing Discourse: Textual Analysis for Social Research*. London: Routledge.
- Ozga, J. and Jones, R. (2006). Travelling and Embedded Policy: the case of knowledge transfer. *Journal of Education Policy*, 21, 1 (January), 1-17.
- Tuomi-Gröhn, T. and Engeström, Y. (eds.) (2003). *Between work and school: new perspectives on transfer and boundary-crossing*. London Pergamon.
- Tuomi-Gröhn, T., Engeström, Y. and Young, M. (2003). From transfer to boundary-crossing between school and work as a tool for developing vocational education: an introduction, pp.1-15 in T. Tuomi-Gröhn and Y. Engeström (eds) *Between work and school: new perspectives on transfer and boundary-crossing*. London: Pergamon.

KEYNOTE ADDRESS

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Mathematics in Use: insights from a study of the workplace

Report by Mieke van Gorenestijn

Professor Celia Hoyles' Keynote Address focused on aspects of a research study¹ of the mathematics used in three professions: investment banking, nursing and commercial aviation. Professor Hoyles presented examples of the routine practices used, but also the surprises – where conflict among practitioners provoked more articulated discussion of the models in use.

Nursing

Drug calculations involve nurses working with proportions to obtain a dose, given a doctor's prescription of the amount to be administered and the stock concentration of the drug. Nurses often recall the rule they learned in their training:

$$\frac{\text{amount you want}}{\text{amount you have got}} \times \text{volume it is in}$$

However, observations on the wards showed that nurses rarely followed this rule. Instead they used a variety of finely-tuned strategies, which was cued by specific kinds of drugs. In practice it appears that it is the drug itself and its clinical treatment which has meaning for the nurse. It is the drug itself which gives nurses directions about how to deal with the relations needed to calculate the volume of the dose. The numbers are not simply quantitative measures of a drug. They are part of the drug and acquire their meanings in relation to the drug, its properties and action and the recommended doses for different weights and conditions of patients. A general formula is not applicable in these situations. The nurses studied used their professional expertise to develop appropriate mathematical models for drug dosage calculations in very specific situations, for example, one nurse said: "with Odansetron you only need to halve it".

Commercial Aviation

Navigation by pilots in part involves relating the speed and heading of a plane to the wind velocity in order to fly to a specified destination. During actual flights it appears that navigation is not only a matter of applying basic calculations using a triangle of velocities, as described in instruction manuals. Pilots did their calculations more by 'feeling' where the wind was coming from, and co-ordinating pieces of their knowledge in terms of specific

characteristics, not of planes in general, but of the behavior of the type of plane, a Fokker 100, they were used to flying.

Expertise is characterized by diversity. The pilots and nurses showed fragments of individual knowledge applied in different ways in very specific situations. General mathematical relationships (the triangle of velocities and the drug calculation formula) are not sufficient to actually manage particular situations. It is also the pilots' and nurses' own feelings and practical knowledge about the materials they use that lead them to make the right decisions in specific situations.

Some of the typical mathematics questions arising in the workplaces studied are illustrated as follows.

Mathematics Questions from the Workplace

Investment Baking

1. Suppose you want £100 in one year. You have the chance of buying a simple interest instrument (say a CD) paying 8% or an instrument (like a Treasury Bill) which offers a discount over the year again of 8%. Which would you choose and why?

2. Suppose you have a nominal certificate of deposit, an NCD (a type of simple interest instrument), face value £100 issued on March 1st, with 365 days to maturity carrying a 4% coupon payable on maturity. Put another way, you can calculate the value at maturity by assuming an interest rate of 4%. What would this value be?

Now suppose the prevailing rates change on March 1st and the NCD trades at 3%. If you want to sell the NCD, will its price go up or down?

Calculate the price at which you would sell, assuming a day basis of 365.

Suppose the number of days to maturity is 180, at what price should you sell?

Nursing

1. Belinda needs to give 120mg of an antibiotic prepared in 100mg per 2ml vials. What volume does Belinda need to give?
2. Many drugs are given at regular intervals, but occasionally a drug is given first at one frequency and then at another, the 24hr dose running constant. A patient was prescribed an antibiotic, vancomycin – at 600mg every 6 hours for 24 hours, then at 1200mg every 12 hours thereafter. As with many drugs, the dosage level in the blood needs to be kept high enough to be effective but if the level is too high the drug becomes toxic. At midday two nurses, Wanda and Betty, check when to give the first 1-hourly dose, the last 6-hourly dose having been given at 6 o'clock that morning. When do they give the 12 hourly dose?

Commercial Aviation

1. Imagine you're flying over the beacon at 'Spy' (or Spijkerboon), and you have been requested to fly directly to Eelde. You find the track for Eelde at 0-6-3, ground speed 200, but you still need to readjust your heading to take account of the wind, which is 50 knots from a bearing of 030. Could you find both your heading and true air speed?
2. Imagine you are now nearing Hamburg and you need to consider the wind conditions at the airport. It is snowing, although both runways have been cleared of snow and gritted. The latest surface wind report for both runways is 0-8-0 at 16kts (knots).

The breaking actions at each runway are: point-2-9, point-4-2, point-4-6 for runway 0-5, and point-4-2, point-4-0, point-3-3 for runway 1-5. Which runway would you choose to land on – if any? Explain your reasoning.

In another situation, the cross-wind limit is the standard 33kts, and wind speeds are around 50kts. What is the range of safe direction 'off the nose'?

Note

1. This ESRC-funded project was entitled 'Towards a Mathematical Orientation Through Computation Modelling' (1.1.1996 – 31.3.1998) Project Directors were Richard Noss and Celia Hoyles; Research Officer: Stefano Pozzi.

Reference

Noss, R. & Hoyles, C. (1996) *Windows on Mathematical Meanings: Learning Cultures and Computers*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Mathematics at Work

Adults and Artefacts

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1 An explanation and a dilemma

As most of my arguments in this text come from a workplace context, I start with a description, if not definition of what I take as ‘work’: The ‘Advanced Learner’s Dictionary of Current English’ (Hornby et al., 1960: 1492) offers seven definitions for ‘work’. I cite the first two of them: ‘bodily or mental effort directed towards doing or making something; the expenditure of energy (by man, machinery, forces such as steam, electricity, etc. or by forces of nature)’ and ‘occupation; employment what a person does in order to earn money’. In this text, I will concentrate on work with an identifiable purpose / within a certain social system, which is done by human beings to earn their living. Nevertheless, I also look into the mathematics used by ‘ordinary’ citizens in everyday situations.

In research on mathematics used by adults, especially in workplace contexts and vocational training and education, we soon have to face a specific dilemma: Politicians, managers and business administration people often speak of a growing societal use of mathematics. We hear statements like ‘The application of mathematics is extended to more and more important fields of human activities’. If we look into the history of science, we additionally find that mathematics is used in more and more disciplines. On the other hand, research on mathematics in vocational, workplace contexts hears the ‘normal’ worker say that s/he is not using any mathematics (maybe apart from elementary arithmetic), or even worse: Mathematics gradually disappears from the workplace and/or the workers’ attention. Survey studies with adults came to a similar result: On a broad basis, the persons interviewed stated that they seldom, if ever use mathematics apart from arithmetic in their everyday life. How can we understand this contradiction?

2 A Case in Industrial and Social History: Mathematics of weighing

In order not to start with too global an approach, the paper first offers a historical case study of the disappearance of mathematics from societal perception (most of this section is a slightly adapted reprint from Strässer 2002). The case study analyses the standard procedures of weighing and pricing in small and medium businesses showing the growing implementation of mathematics into various workplace tools (either material or organisational). It looks into a ‘standard’ situation such as weighing some three kilos of potatoes and hence telling the price of this merchandise.



Figure1: Traditional beam scales



Figure 2: Analogous weighing & pricing

Weighing 1: traditionally

In the past and even nowadays in marketplaces and old-fashioned shops, weighing was done with a pair of beam scales (see Figure 1 next page; all illustrations in this paper by courtesy of BIZERBA, one of the important producers of balances in Germany) and normed weights. The goods were placed in one scale and normed weights into the other (in complicated cases also into the one containing goods) to have the beam completely balanced. The weight of the goods can be read off the balance by adding and/or subtracting the weight in the respective scales. The price was then calculated separately (either mentally or in writing on a sheet of paper) by multiplying the unit price of the merchandise with the weight read from the beam scales. As for the mathematics involved, we see the partitioning of the weight according to the weights available, addition (and subtraction) to calculate the overall weight and a proportional model for the pricing.

Weighing 2: analogously

In Germany, since 1924, this way of weighing & pricing was slowly replaced by the introduction of a different analogous type of balance, in Germany called ‘Neigungsschaltwaage’ (also: ‘Fächerkopfwaaage’, a name related to the form of the balance; see Figure 2 above). This type of balance still uses a simple proportional model. The partitioning and adding of weights was done ‘automatically’ by the balance because the hand of the balance would move to the right proportionally to the weight of the merchandise – indicating the total weight on top of the scale. Using the handle on the left hand side of the balance (in figure 2), the seller could set an interval of weight the balance showed, the interval was to be read off from two small fields in the upper left and right corner of the balance (see figure 3 next page). In addition, the pricing by multiplication was taken over by this new artefact: the price

could be read off the scale at the correct place of the hand indicated by the unit price. The correct reading of prices and eventually adding the price of whole units of weight were therefore the only essential competences the seller should have. Mathematical ‘interpolation’ was necessary in case of very large or very small or odd prices, which could not be found on the hand and scale of the balance. Following information from BIZERBA, around 70% of the prices read off these balances were incorrect. Nevertheless, this type of artefact was widely used in Germany, until the 1980s at least.

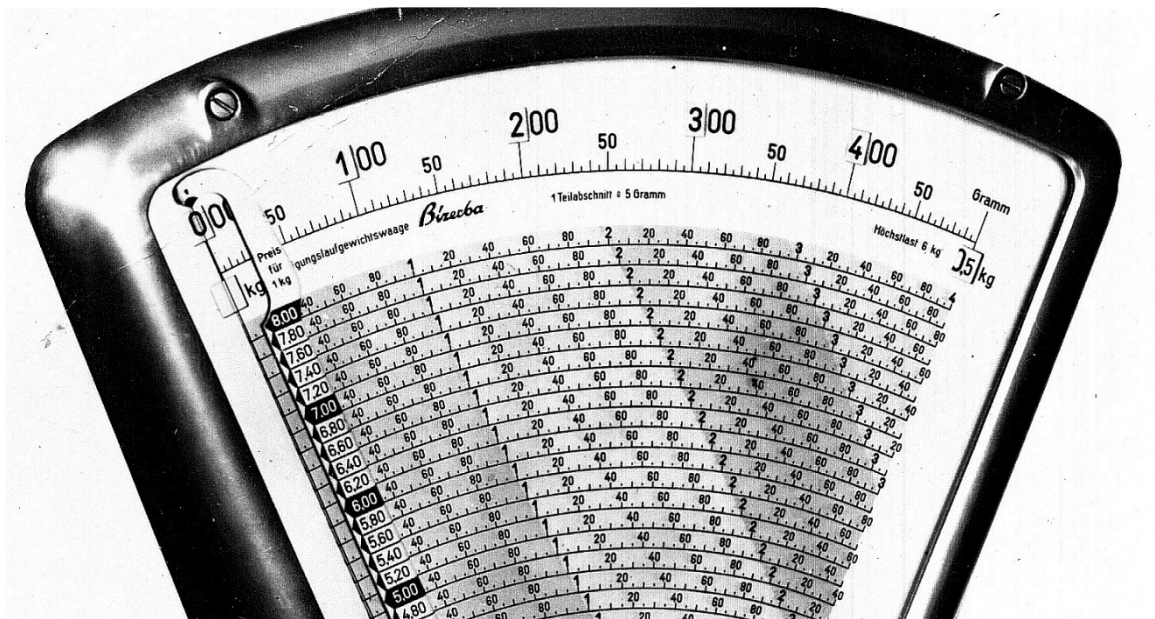


Figure 3: details of analogue weighing

Weighing 3: digitally

Nowadays, especially in larger shops and supermarkets, you would find digital balances (see Figure 4), which directly offer prices for the merchandise put onto them with printouts of prices to be pasted to the goods – if the shop ever sells goods which are not pre-packed. The proportional model is still in use but hidden from the perception of the buyer. Reading weights and prices has become easy, while interpolation of large and small prices is unnecessary. ‘Odd’ prices (like the ‘famous’ 3.99 \$) have only come into use with these balances or pre-packed merchandise. What is left to be done by the seller or the buyer is keying in either the unit price or an identification number or symbol for the goods to be purchased. Normally, addition of several goods and identification of the individual seller is done automatically.

Mathematics travels up the professional hierarchy to managers who decide on (quantity) discounts and special offers. Programmers of computerised systems for checking the flow of goods in a company are responsible for a constant and realistic flow of information on the amount of goods offered in the shops, stored in ‘just-in-time’ delivery or to be ordered. Even the cash flow of the money in the enterprise is linked to these information systems to help securing the economic success of the company.



Figure 4: weighing & pricing digitally

3 Mathematics integrated in artefacts

3.1 Lessons to be learnt from the case study

Looking back to the case described above, the disappearance of mathematics from everyday perception – at least from the perception of the actual buyers and sellers – can indeed be illustrated. In fact, mathematics (addition/subtraction of weights, calculation and addition of prices) is progressively turned into algorithms, automated and integrated into machines (into ‘artefacts’, see below) and thus hidden from the notice of those involved in the activity. If the job runs smoothly and routinely without unfamiliar and unforeseen events (the worst case would be the breakdown of the electricity supply), practitioners tend to rely on well-known routines for repetitive problems. These routines are often implemented in tools (like machines for calculating, scales to read, charts to fill and the like). Difficulties when using mathematics tend to be simplified, if not totally avoided, by algorithms and routine activity flows. Book-keeping with its longstanding formalised set of concepts and practices (like discount and increase, recording of transactions by means of accounts, double entry book-keeping etc.) can serve as an additional illustration of how complicated workplace practices are made routine by ‘simple’ algorithms which do not call for mathematical competences. As long as the workplace does not present unexpected situations, these tools go unrecognised and ‘hide’ the mathematics they incorporate. Nevertheless it would be wrong to state that mathematics disappears altogether or becomes less important socially. On the contrary, the third phase of the weighing clearly shows the growing social importance of mathematics. The dilemma described in the introduction to this paper exactly describes the situation: At the very same time as mathematics disappears from societal perception it becomes more and more widely used and more and more important.

Is there a chance of ‘rediscovering’ mathematics in vocational and everyday situations? Recent research on mathematics in vocational contexts offers a somewhat deceiving answer

to this question. It is only in non-routine and non-standard situations, when usual practices fail or do not cover the situation to be faced at the workplace (the ‘breakdowns’ or unfamiliar situations), that (even qualified) practitioners go back to unfamiliar, maybe innovative procedures. ‘They apply a fragment of professional knowledge, a half-remembered rule from school mathematics or a novel, though generally unsuccessful, use of a familiar tool’ (Noss et al. 1996, p. 14; see also Magajna 1998 and for the non-understanding of workplace mathematics Hogan 1996, p. 288 or Hogan&Morony 2000). Here again the tool shows up as one way to somehow manage non-routine problems. Computers and finely tuned software can even be used to offer a micro-world for exploring non-routine, unusual situations (cf. Noss et al. 1996; for a more detailed discussion of computer technology as a special type of tool see below).

These ideas in mind, we can easily describe the trend of hiding mathematics in algorithms and routines. By integrating mathematical concepts, relations and procedures into various types of tools (be it rules to follow, charts to fill in, computer technology to handle or other machines to use), mathematics tends to disappear gradually from the sight of the worker. Even if mathematics is increasingly used on a global level, the individual professional does not notice this development, as they tend to describe the ongoing process as a gradual disappearance of mathematics from their workplace. It is only in non-routine and ‘breakdown’ situations that mathematics shows up again and may be (re)invented to cope with the ‘new’, non-tool-governed situation.

3.2 From ‘artefacts’ to ‘instruments’

For a more detailed description of this process, concepts like ‘artefact’ and ‘instrument’ are most helpful. Wartofsky defined ‘artefacts’ as ‘anything, which human beings create by the transformation of nature and of themselves: thus also language, forms of social organisation and interaction, techniques of production, skills’ are artefacts’ (Wartofsky 1979: xiii; for details see loc. cit. pp. 201ff). Obviously, the concept ‘artefact’ (sensu Wartofsky) is wider than the word ‘technology’, which is normally only used for the hardware in information and communication technology (ICT). ‘Artefacts’ – embracing also mental constructions and organisational settings - is a reminder that (social) organisation and (individual) competencies are important aspects of work and social life.

In order to better structure this vast range of artefacts embraced by Wartofsky’s definition, he himself offers a helpful list of levels of artefacts: ‘Primary’ artefacts are those ‘directly used in the production’, while ‘secondary’ artefacts are ‘used in the preservation and transmission of the acquired skills or modes of action or praxis by which this production is carried out’. ‘Tertiary’ artefacts ‘constitute a domain in which there is a free construction in the imagination of rules and operations different from those adopted for ordinary ‘this-worldly’ praxis’ (Wartofsky 1969: 202/209). Using this classification of artefacts, didactics often seems to be mainly concerned with the analysis of secondary artefacts – thus losing a chance to better understand the basic use of primary artefacts and the mathematics integrated into these tools. On the other hand, disciplinary, especially ‘pure’ mathematics sometimes seems to produce ‘tertiary’ artefacts, which only later turn out to be effective tools to be integrated into primary artefacts (see the history of permutations in abstract algebra which – some 50 years later - turned out to be very efficient in coding theory and the development of security systems). Information and communication technology (ICT) present the whole range

of orders of artefacts. For the drawing sector, CAD-programs may be presented as primary artefacts, while Dynamical Geometry Software (DGS) was developed as a learning tool, i.e. a secondary artefact (for an overview see the issue of ZDM edited by Strässer 2002). Some computer games can definitely be taken as tertiary artefacts.

The row of artefacts by Wartofsky (primary – secondary - tertiary) classifies according to different purposes of the tools. From an ergonomic (and didactical) approach, French colleagues studied how workers learned to use complicated, high technology tools, they looked into ‘How do human beings learn to use machines?’ These researchers, more specifically Rabardel (1995: 117f), came up with a helpful distinction for the analysis of artefacts: For an analysis of (learning with) artefacts, it is appropriate to distinguish the artefact (the tool itself) from an ‘instrument’, ‘a mixed entity made up of both artefact-type components and schematic components that we call utilization schemes (in the original French: ‘schèmes d’utilisation’). This mixed entity is born of both the subject and the object’ (Rabardel&Samurcay 2001). Figure 5 below is a schematic representation of an ‘instrument’:

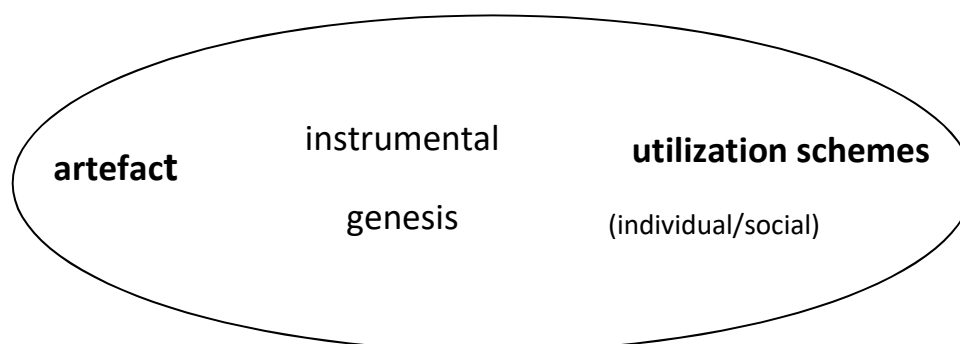


Figure 5: ‘Instrument’ – a unity of artefact and utilisation schemes

The construction of utilization schemes for a given artefact is called ‘instrumental genesis’ (see Rabardel&Samurcay 2001). When learning to use a new artefact, the user/learner first has to find out about the use of the artefact – ‘instrumentalisation’ of the artefact sensu Rabardel. An experienced user of an instrument may develop (new) utilization schemes – a development called ‘instrumentation’ by Rabardel (for a discussion of these evolving utilization schemes in the case of computer algebra systems ‘CAS’ see Artigue 2002).

3.3 On information and communication technology (ICT)

Today, a comment on information and communication technology (ICT) is in place, as it is usually taken as the key technology, the most important artefact of present day economy and society.

The ‘banking study’ (Noss et al. 1996) clearly shows that ICT can (and usually does) play an ambivalent role: ICT is a most effective artefact to hide mathematics. Its (schemata of) use can be reduced to keying in information to get a decision produced by a built-in, hidden

algorithm. This is a prototypic example of the hiding of mathematics by its integration into artefacts. Local examples from banking may be bookkeeping, calculation of interests, loans and other financial artefacts. Technical drawing with CAD and the material production using CNC-machines may be good illustrations for this role of ICT from other areas of production and distribution.

On the other hand, ICT can be a most effective opener of black boxes, a most effective way to rediscover the mathematics hidden in all sorts of instruments. Noss et al. (1996) report in detail on the 'eye-opening' potential of simple 'what-if'-programs (offering the possibility of varying the interest rate or the like). As illustrated by this, ICT can be used

- to de-grey the black boxes,
- to show the mathematical relations and
- to explore the inherent, implemented relations.

Normally, this opening of black boxes will be done by means of simulation, that is exploration off the workplace, studying the inherent relations by varying the parameters available for change. In some workplace situations (like power stations and the production of chemicals), simulation may even be the only way to learn about the reality of the workplace. Because of the risk of material, financial and/or time losses possibly incurred, an un-experienced worker, a 'novice' would never be allowed to use these instruments in 'real time'. Exploration by ICT-based simulation may be the only way to learn about the situation and become a worker competent to control these processes by her/himself.

4 Conclusion on research methodology and democracy

In order to better understand the use of mathematics not only in the workplace, an investigative research methodology is needed because of the integration of mathematics into sometimes very complicated instruments. Traditional survey and interview studies produce only limited information on the actual use of mathematics – as can be seen with some traditional projects on mathematics at the workplace and in vocational education. According to my experience, research into the use of mathematics in the workplace has two basic options: The first and most widely used nowadays is participant observation in an ethnographic style (and this is often used especially in Anglo-Saxon countries like Australia). Historical case studies - like the one I presented in the second section – are a second and additional way to find out about mathematics not only in the workplace. Both approaches are more or less 'passive' methodologies, which try not to interfere and change the situation they are researching. There may be also more 'active' approaches to create opportunities for the qualified worker to show her/his workplace practice and the mathematics they use therein. Both approaches, 'active' and 'passive' research methods need a thorough and intensive analysis of data to (re-)discover the mathematics hidden in the workplace. It may come as a surprise that I want to end this text on 'mathematics – adults and artefacts' with a remark on 'democracy' and 'maths for all'. I strongly believe that a competent worker should

- be in control of her/his workplace,
- understand, not only handle the artefacts at his/her place,

- know about the mathematical models in use.

Together with understanding social organisation (including the mathematics used), these competences may be a description of a critical citizen in a democracy. To make it more explicit: to me, mathematics is part of the knowledge and competences needed by a fully functional citizen in a real democracy.

References

- Artigue, M. (2002). Learning Mathematics in a CAS Environment: The Genesis of a Reflection about Instrumentation and the Dialectics between Technical and Conceptual Work. *International Journal for Computers in Mathematical Learning* 7(3): 245-274.
- Hogan, J. (1996). A methodology for using classroom teachers to gather information about using mathematics from the work place. In P. Clarkson (ed.), *Technology in Education: Proceedings of the 19th annual conference of the Mathematics Education Research Group of Australasia (MERGA)* (pp. 283–289). Melbourne: MERGA.
- Hogan, J. & W. Morony (2000). Classroom Teachers Doing Research in the Workplace. In A. Bessot et al. (eds.) *Education for Mathematics in the Workplace* (pp. 101–113). Dordrecht: Kluwer.
- Hornby, A. S., E. V. Gatenby, et al. (1960; 11th ed.). *The Advanced Learner's Dictionary of Current English*. London: Oxford University Press.
- Magajna, Z. (1998). Formal and informal mathematical methods in work settings. In A. Watson (ed.): *Situated Cognition and the Learning of Mathematics* (pp. 59–70). Oxford: Dept. of Educational Studies, Univ. of Oxford.
- Noss, R. & C. Hoyles (1996). The Visibility of Meanings: Modelling the Mathematics of Banking. *International Journal of Computers for Mathematical Learning* 1(1): 3-31.
- Noss, R., Hoyles, C. & Pozzi, S. (1998). *Towards a Mathematical Orientation through Computational Modelling Project*. ESRC End of Award Report, Mathematical Sciences Group, Inst. of Education, Univ. of London.
- Noss, R., C. Hoyles, et al. (2000). Working Knowledge: Mathematics in Use. In A. Bessot et al. (Eds.), *Education for Mathematics in the Workplace* (pp. 17–35). Dordrecht: Kluwer.
- Rabardel, P. (1995). *Les hommes et la technologie. Approche cognitive des instruments contemporains*. Paris: Armand Colin.
- Rabardel, P. & R. Samurçay (2001). *From Artifact to Instrument-Mediated Learning. New challenges to research on learning*. International symposium organized by the Center for Activity Theory and Developmental Work Research, Center for Activity Theory and Developmental Work Research, University of Helsinki (also available on internet).
- Strässer, R. & R. Zevenbergen (1996). Further Mathematics Education. In A. Bishop et al. (eds.), *International Handbook on Mathematics Education* (pp. 647-67). Dordrecht: Kluwer.
- Strässer, R. (2000). Conclusion. In A. Bessot et al. (Eds.), *Education for Mathematics in the Workplace* (pp. 241 – 246). Dordrecht: Kluwer.
- Strässer, R. (2002). On the disappearance of Mathematics from society's perception. In: *Developments in Mathematics Education in German-speaking Countries. Selected Papers from the Annual Conference on Didactics of Mathematics, Bern, 1999* (pp. 124-133). H.-G. Weigand et al. (eds.). Hildesheim - Berlin, Franzbecker.
- Strässer, R. (2002). Research on Dynamic Geometry Software (DGS) - an introduction. *Zentralblatt für Didaktik der Mathematik* 34 (3): 65.
- Wartofsky, M. W. (1979). *Models. Representation and the Scientific Understanding*. Dordrecht - Boston – London: Reidel.

Sociomathematics:

Researching Adults' Mathematics in Work

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Abstract

Research is always an answer to questions. Within mathematics education research, the problems concern the relationships between humans and mathematics in different contexts. Concepts like ethnomathematics, numeracy, mathematics-containing qualifications and competences have expanded the problem field and paved the way for studies of adults' mathematics in working life. A critical perspective is opened up when studies concern the functions of mathematics education in society and in people's lives. On the basis of previous studies of adults, mathematics and work, I present my preliminary construction of the analytical concept sociomathematics (the problem field concerning the relationships between people, mathematics and society).

“Le sens du problème est le nerf du progrès scientifique.”

(Gaston Bachelard, 1927)

What does it mean “researching adults' mathematics in work”? Research is always an answer to questions - whether these are explicit or implicit. As the French philosopher Gaston Bachelard puts it: the sense of the problem is the nerve – motor, activator, energiser – of scientific progress. Within the field of research in mathematics education, the questions concern the relationships between humans and mathematics: Why should adults learn mathematics in our world? What kind of mathematics do adults learn in our world? How do adults learn mathematics in our world? These are some of the possible questions hidden in the title of the conference ALM10, “Adults learning mathematics to live and work in our world”, and in each question it is possible to focus on adults' cognitive, affective or social relationships with mathematics. The subject field is constituted by the problem field of mathematics education “in all its complexity”, i.e., the subject area is ‘always-already’ structured and delimited by the concrete forms of practice and knowledge that are currently regarded as mathematics (Wedge, 2001) and by “our world”. Thus, Gail FitzSimons's formulation and study of the question “What counts as mathematics in adult and vocational education?” is central (FitzSimons, 2002).

In the three questions concerning adults and mathematics that I formulated above, the relationships between humans and mathematics are extended with a context called “our world”. But what do we actually mean by “mathematics” and by “mathematics education in all its complexity”? Are society and culture as contexts for adults learning mathematics seen as unimportant boundary conditions in our studies of teaching, learning and knowing or as a central part of the study? At the ALM conference, I presented a preliminary definition of sociomathematics as a subject field and a problem field of mathematics education research.

The borderland between adult education and mathematics education

Research problems concerning the subject area “adults’ mathematics in and for the workplace” are formulated in a dialectic relationship with practice and the subject area is growing into a subject field. Apparently simple questions like “Who is an adult?” are investigated and the answer in form of a definition affects our practice (See Safford, 1998). In the international research forum “Adults Learning Mathematics” (ALM), we have debated the identity of our research domain and the specificity of our research questions at the annual conferences since 1997. In this debate, a central question has been: Where is the research domain situated? (Wedegé, Benn, Maasz, 1998; Coben 2000). This isn’t just an academic question without any consequences for practice and research. When we know where we are – or want to be – in the scientific landscape, we know something about scientific legitimacy and about criteria of quality and relevance.

During this process, metaphors and illustrations of the scientific landscape have been created. Roseanne Benn made an illustration with concentric circles to show that we have a series of neighbouring disciplines (Benn in Wedegé et al., 1998). I claim (Wedegé, 2001) that “adults’ mathematics in and for the workplace” is situated in the borderland between research in mathematics education and in adult education from where we import and reconstruct concepts, theories, methods and findings (see figure 1). The construction or reconstruction of conceptual frameworks is an important task in research.

The development of adult education research to an independent academic field is closely associated with the institutionalisation of adult learning. But, although the development of the field of practice is an important *criterion for relevance*, the criterion of relevance is not just the ability of research to answer the problems in the field of practice, but also to criticise and reformulate these problems (Olesen & Rasmussen, 1996). Within the field of mathematics education research, relevance to the practice of teaching or learning mathematics is also a criterion of quality. As mentioned above, the subject area is ‘always-already’ structured and delimited by the concrete forms of practice and knowledge that are currently regarded as mathematics teaching, mathematics learning and mathematics knowing. However, a critical perspective might be opened up when studies concern the functions of mathematics education in society and in people’s lives. (Wedegé, 2000). ‘Critical mathematics education’ is one of the forms this critique has found within mathematics education research (Skovsmose, 1994). ‘Folk mathematics’ and ‘ethnomathematics’ are other forms (Mellin-Olsen, 1987, D’Ambrosio, 1985, Knijknik, 1997).

It is an important part of the self-conception in the research field of adult education that it cannot be subordinated in a disciplinary context (such as a sub-discipline in pedagogics, psychology or sociology), but that inter-disciplinarity is a significant feature (Olesen & Rasmussen, 1996). The field of mathematics education research also makes use of concepts, methods and results from other disciplines (psychology, sociology, linguistics, anthropology, philosophy). In the beginning, the studies were *multi-disciplinary*. According to Brousseau, the original approach in research was to deduce the consequences for mathematics education from findings in the other disciplines. To create *inter-disciplinarity* the imported conceptual frameworks have to be reconstructed (Brousseau, 1986). In adult vocational and further

education, the reasons for teaching and learning mathematics are to be found outside mathematics. That is another reason why inter-disciplinarity is essential, both in education and research, and reconstruction of conceptual and theoretical frameworks from other disciplines is a central task (Wedeg, 2001).

In addition, mathematics education research has a specific relationship to *mathematics* as a scientific discipline, as a social phenomenon, and as a school subject. What is recognized as mathematics, and what is not, is important to research, and it is also a political question; a question about mathematics and power (Mellin-Olsen, 1987; FitzSimons, 2002). The question – what do we mean by mathematics – is crucial to the construction of our subject field.

Three approaches to adult's mathematics in work

In the scientific landscape where the research area “adults’ mathematics in work” is situated, I have investigated three (partly overlapping) approaches to the subject area “adults learning and knowing mathematics”, see figure 1.

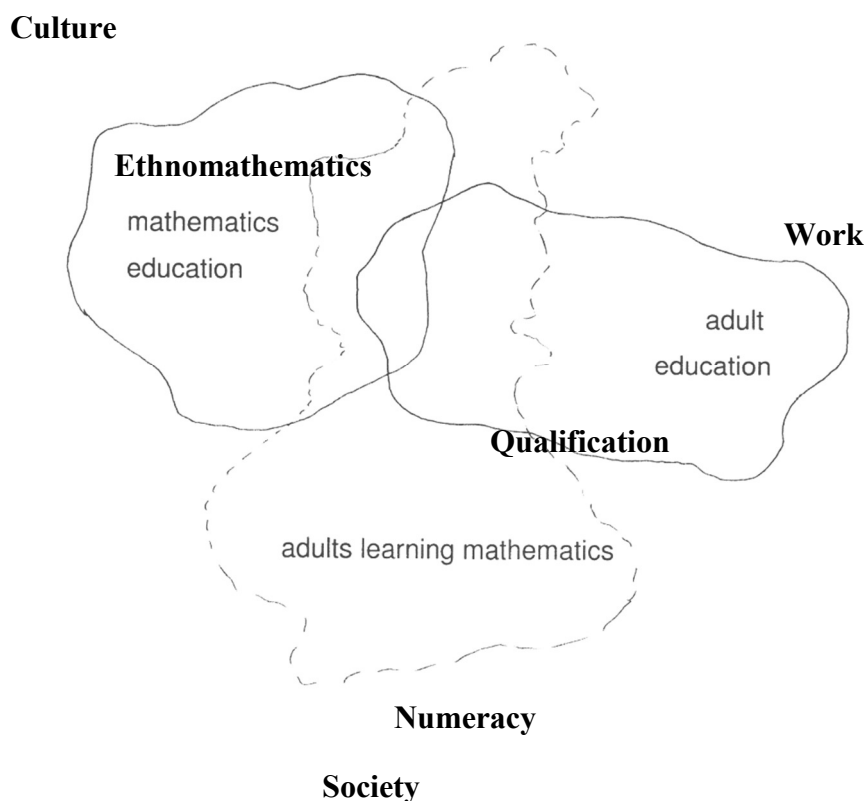


Figure 1. ‘ALM’ situated in a borderland: three contexts (culture, society and work) and three correlated concepts.

I claim (Wedeg, 2001) that the conceptual constructions of ethnomathematics and folk mathematics, as well as the concepts of numeracy and of qualification, have expanded the problem field of mathematics education research and, thus, paved the way for studies of adults’ mathematics in working life. (Evans, 2000; Gerdes, 1996; Groenestijn, 2002; Mellin-Olsen, 1987). A socio-cultural approach is common in the studies of ethnomathematics, adult numeracy and mathematics in the workplace (not *for* the workplace). Today it is scientifically

legitimate to ask questions concerning people's everyday mathematics and about the power relations involved in mathematics education.

Ethnomathematics

Ethnomathematicians argue that the techniques and truths of mathematics are a cultural product and stress that all people – every culture and subculture – develop their own particular forms of mathematics. D'Ambrosio contrasted *academic mathematics* (the mathematics taught and learned in schools) with *ethnomathematics*, which he describes as the mathematics “which is practised among identifiable cultural groups such as national-tribal societies, labour groups, children of a certain age bracket, professional classes, and so on” (D'Ambrosio, 1985:45). Ethnomathematicians adopt a broad concept of mathematics, including counting, locating, measuring, designing, playing and explaining (Bishop, 1988), and they emphasise and analyse the influences of socio-cultural factors on the teaching, learning and development of mathematics (Gerdes, 1996).

Numeracy

In adult mathematics education, the term ‘numeracy’ is used but it is unclear where the dividing line between the two terms, numeracy and mathematics, should be drawn. At ALM9 in 2002, John O'Donoghue discussed mathematics versus numeracy – asking the question “Mathematics or Numeracy: Does it really matter?” His answer was clear and affirmative: “Yes, it really does matter in a number of important ways.” (O'Donoghue, 2002). If mathematics is defined to include numeracy we have moved out of the mathematics classroom and into society and adults' everyday lives. I find that a significant difference between adult numeracy and mathematics is that the idea of society and the need for mathematics in adult life are incorporated in numeracy but not necessarily in mathematics. Johnston and Yasukawa defined numeracy as a critical awareness, which builds bridges between mathematics and the real world. In their teaching of numeracy it is the relationship, the negotiation, between mathematics and the world that has become the core concern (Johnston & Yasakawa, 2001). For my purpose, I have selected three of the most recent working definitions of numeracy:

Numeracy is the ability to process, interpret and communicate numerical, quantitative, spatial, statistical, even mathematical, information, in ways that are appropriate for a variety of contexts, and that will enable a *typical member of the culture or subculture* to participate effectively in activities that they value (Evans, 2000:236, my italics).

Numeracy consists of functional mathematical skills and understanding that in principle all people need to have. Numeracy changes in time and space along *with social change and technological development*. (Lindenskov and Wedege, 2001:5, my italics)

Numeracy encompasses the knowledge and skills required to effectively manage mathematical *demands in personal, societal and work situations*, in combination with the ability to accommodate and adjust flexibly to new *demands in a continuously rapidly changing society* that is highly dominated by quantitative information and technology. (Groenestijn, 2002: 37, my italics)

Although it doesn't appear explicitly in the definitions, these three numeracy studies are concerned with the relationships between people, mathematics and society. When talking

about numeracy as a competence it is competent people in society, not only mathematical competence in society. By numeracy is understood “adult numeracy”.

Qualification

In society, there are complex relationships between technology, mathematics and people. The conception of *technology* on the labour market as consisting of three mathematics-containing elements - technique/machinery, human qualifications/ competences, and work organization - and their dynamic interrelation is crucial in my studies of adults’ mathematics in work. In the concept of qualification a dualism is incorporated: qualification is both seen as a characteristic of the requirements to skills and abilities of the job function and as a characteristic of the skills and abilities of the (potentially) working person. Human *qualifications* are defined as human knowledge, skills, characteristics and attitudes relevant in the interaction with technique and work organisation in a work or job function on the labour market.

In this manner, a conceptual framework for reflection on the relation between mathematics education, work and technology is offered (Wedeg, 2000).

In the Danish adult education research, the theoretical construction of the concept of qualification has been a central task because adult education is closely connected with work as an individual and a social phenomenon (Olesen, 1994).

Sociomathematics – a preliminary definition

On the basis of previous studies of people and mathematics in society – others’ and my own – I give a preliminary definition of an analytical concept, which encompasses the studies of for example numeracy, ethnomathematics and workplace mathematics in a single term. By *sociomathematics* I mean

- a problem field concerning the relationships between people, mathematics and society, and
- a subject field combining mathematics, people and society – as we may find it for example in ethnomathematics, folk mathematics, adult numeracy and mathematics-containing qualifications.

As a problem field, sociomathematics is defined by a socio-cultural perspective on mathematics education. As a subject field, sociomathematics is defined by a specific approach to the subject area of people, mathematics and society, see figure 2.

Sociomathematical problems concern:

(1) people’s relationships with mathematics (education) *in* society and vice versa.

People’s relationship might be seen as cognitive, affective or social according to the given perspective of a specific study. This relationship is the key issue, but to investigate this problem you have to study two other problems:

(2) the functions of mathematics (education) in society and vice versa, and

(3) people learning, knowing and teaching *in* society.

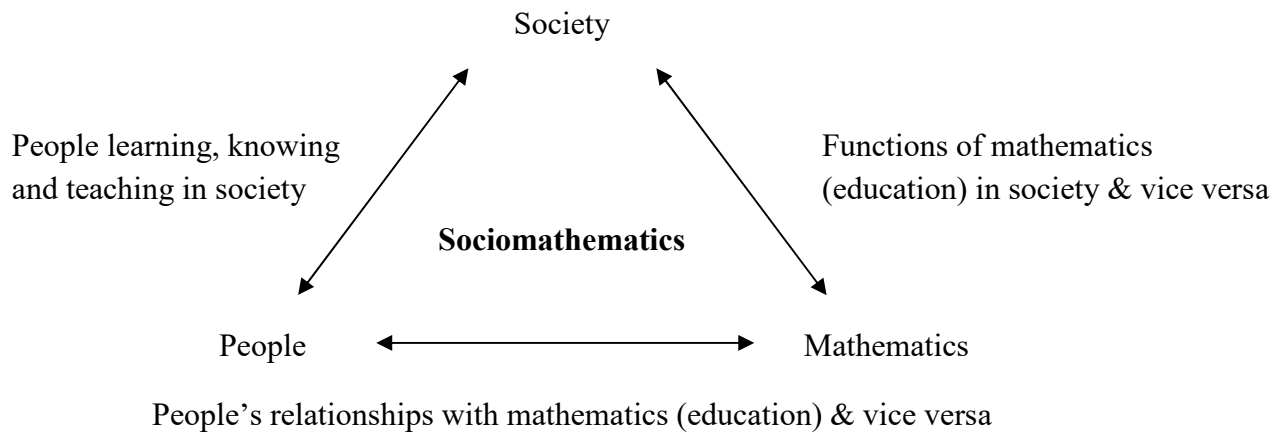


Figure 2. Sociomathematics as a subject field

A *sociomathematical concept* (an example): When Ole Skovsmose studied students' learning obstacles in mathematics he doesn't stop with their cultural background but also involves the students' *foreground* i.e. the opportunities provided by the social, political and cultural situation:

When a society has stolen away the future of some group of children, then it has also stolen the incitements of learning. (Skovsmose, 2002:9)

A *sociomathematical study* (an example): When I studied adults' motivation or resistance to learning mathematics I don't stop with the situation given by Lave's socio-psychological concept of community of practice but also involve Bourdieu's sociological concept about the adult's *habitus*:

(...) the habitus of a girl born 1922 in a provincial town as a saddler's daughter, of a pupil in a school where arithmetic and mathematics were two different subjects at a time where it was "OK for a girl not to know mathematics", and the habitus of a wife and mother staying home with her two daughters is a basis of actions (and non-actions) and perceptions. Habitus undergoes transformations but durability is the main characteristics (Wedeg, 1999:222)

In my terminology, sociomathematics is the name of a subject field and a specific problem field just like ethnomathematics (see Gerdes, 1996).

Sociomathematics, ethnomathematics and socio-mathematical norms

The concept of ethnomathematics has been a very important construction in my studies of unskilled and semi-skilled workers' mathematics in the workplace (Wedeg, 2000). But I never use the word "ethnomathematics" in the Danish vocational context where we talk about "workplace mathematics" or "everyday mathematics" instead. In many languages and situations, the prefix 'ethno' has connotations with reference to biological characteristics, colour of skin etc. At the Second International Congress on Ethnomathematics, in Brazil, Ole Skovsmose presented a strong reservation about the use of the word "ethnomathematics". However, his reservation is not to do with the meaning of "ethno" in the literature of ethnomathematics where according to D'Ambrosio, it simply refers to "environment" – e.g., culture and society: mathematics is acted out in many different ways in different cultures and

by different groups. What is emphasised in ethnomathematics are the connections between culture and mathematics. Mathematics is always socially embedded. Thus “engineering mathematics” and “mathematics in semi-skilled job functions” also represent different branches of ethnomathematics (Skovsmose, 2002).

I find that “sociomathematics” could be an answer to this terminological problem. However, sociomathematics is not just a translation of the word ethnomathematics into a more “clean” word. I found my inspiration in *sociolinguistics* i.e. relationships between language and society constituted as a scientific field within linguistics. But there is an important difference: sociomathematics is a field within mathematics education research (studying people’s relationship with mathematics in society), not a subdiscipline of mathematics.

During the ALM conference, Mercedes de Agüero told me that mathematics shared by a cultural subgroup of only two persons could be regarded as “ethnomathematics”. I wouldn’t call a phenomenon like this “sociomathematics”. The critical approach to ethnomathematics defined by Gelsa Knijnik (1997) is clearly an example of what I would name “sociomathematics”. Her study isn’t just about people’s competence in a small cultural context but about a larger political context making power relations visible. In a sociomathematical study the demands from society (labour market, educational system, democracy) are explicit.

As far as I know the term “sociomathematics” hasn’t been used before in this meaning. However, at the level of the social context of the classroom, Paul Cobb and his colleagues developed the term “sociomathematical norms” in an interpretive framework for analyzing mathematical activity with a social dimension (classroom social norms, sociomathematical norms and classroom mathematical practices) and a psychological dimension (beliefs about roles and mathematical activity in school, mathematical beliefs and values, and mathematical conceptions) (Cobb, 1996). In this framework the social category of socio-mathematical norms is correlated with the psychological category of mathematics beliefs and values. In my terminology, studies of socio-mathematical norms in a classroom would be called “sociomathematical” if the students’ relationships with mathematics *in* society are explicit on the agenda. I.e. the function of mathematics education in society or society’s function in mathematics through for example the students’ gender, race or class.

Studying adults, mathematics and work in society

Within the community of practice and research “Adults Learning Mathematics”, questions about the relationships between adults, mathematics and society is on the agenda: numeracy in all its complexity is a keyword. If we consider the societal justification of adult educational programmes, qualification for the labour market and the technological development is a key argument in the political debate concerning adult education and lifelong learning. The same is true if you look at mathematics education. However, if you look through the proceedings from the first nine conferences (ALM1 – ALM9), you will find relatively few papers combining adults, work and mathematics and even fewer with a sociomathematical perspective (see figure 3).

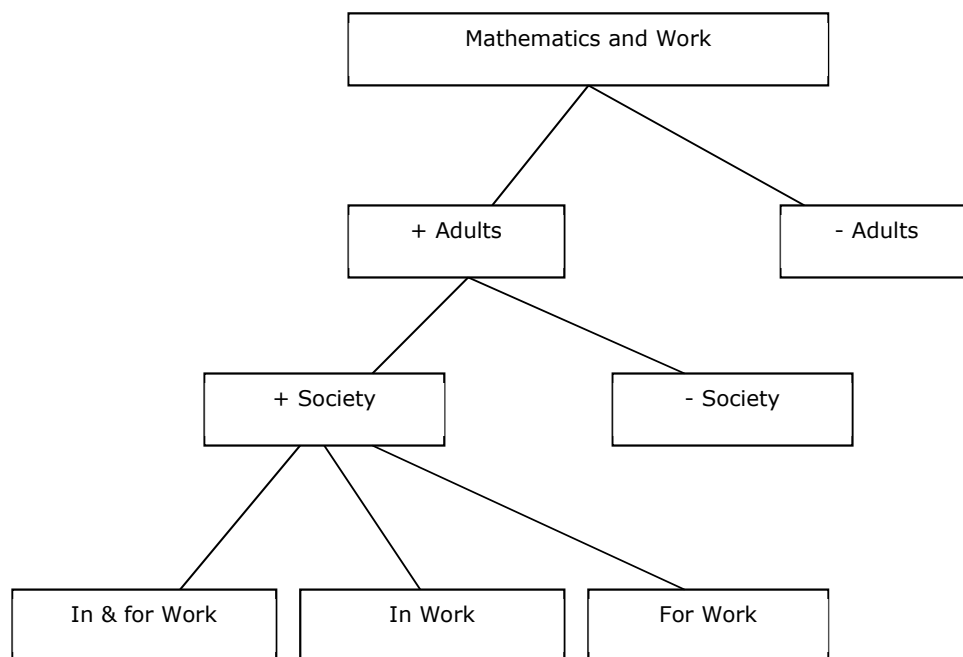


Figure 3. Adults – Mathematics - Work – Society: number of papers in the proceedings from ALM1 to ALM9.

Only 14 papers combine adults, mathematics, work and society. Here are some examples of problems being studied: developing technological competence at the workplace, consequential transition between school and work as an alternative to transfer, numeracy at the workplace (in & for work); recognition of the mathematics in female work (in work); a forklift driver’s course (for work).

Why do we have so few studies combining adults, mathematics, work and society?

It’s a bit of a paradox when working life is a top scorer as an argument for mathematics education. The working place is however a strange place for most of the mathematics teachers who have always lived and worked in school settings. In addition, getting access to a workplace as a researcher is not easy. Before you start the study, negotiating access to the workplace can be a complicated and complex process. According to Zevenbergen, there is always a “gatekeeper” who must give his or her consent before you start, as in any ethnographic project. This can be the secretary who allows you access to the employer, it may be the manager, the union delegates and so on. It is also important to have an idea of what might be the potential problem areas in the process. Within the workplace setting, there are a number of identified areas of concern that are unique to this context and research. Many of these problems are union issues associated with salary, piecework contracts and job security (Zevenbergen, 2000).

In order to analyse and describe numeracy in the labour market, I have investigated selected firms within four lines of industry: building and construction, the commercial and clerical area, the metal industry, and transport. My object was to identify and describe mathematics in semi-skilled job functions and to analyse how mathematics knowledge at work is interwoven with the workers’ qualifications (Wedeg, 2002a). In this project, the employers’ and

employees' associations helped me gain access to the workplaces and to find a relevant contact person, who was also representative of the firm. In relation to the difficulties mentioned above, it was important to have a contract signed by the firm, the worker and the researcher (Wedge, 2003).

Why sociomathematical research in the workplace?

Within the area of workplace mathematics (Bessot & Ridgway, 2000) it is in particular the following two problems that generate research: Mathematics is integrated in the workplace activities and often hidden in technology. The so-called 'transfer' of mathematics between school and workplace – and vice versa – is not a straightforward affair.

Mathematics is hidden from the perception of the worker by artefacts (material tools, workplace procedures or organisational features) (Strässer, 2002) or hidden from the consciousness of the adults due to their self perception and/or conception of mathematics (Evans, 2000, Wedge, 1999, 2002a & b). Research combining adults, mathematics and society in workplace studies opens the possibility to:

- Make the adults' mathematics visible in their competences at the workplace
- Make mathematics visible in the qualifications demanded from the labour market

When we as sociomathematicians are researching the human element of mathematics-containing technology in society the two concepts – competence and qualification – and the dialectic relationship between them should be taken seriously: on the one hand society's requirement to adults' qualifications and on the other hand adults' mathematics-containing competences in the workplace context. When lifelong mathematics education becomes a structuring principle in the educational system, this will be even more important.

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References

- Bessot, A. & Ridgway, J. (eds.) (2000). *Education for mathematics in the workplace*. Dordrecht: Kluwer Academic Publisher.
- Bishop, A. J. (1988). *Mathematical enculturation*. Dordrecht: Kluwer Academic Publishers.
- Brousseau, G. (1986). Fondements et méthodes de la didactique des mathématiques. *Recherches en Didactique des Mathématiques*, Vol. 7, No. 2, 33-115.
- Cobb, P. (1996). Accounting for mathematical learning in the social context of the classroom. In C. Alsina et al. (eds.) 8th International Congress on Mathematical Education: Selected lectures (pp.85-99). Sevilla: S.A.E.M. 'THALES'. (Published in 1998.)
- Coben, D. (2000). Perspectives on research on adults learning mathematics. In Coben; O'Donoghue, J.; FitzSimons, G. E. (eds.) *Perspectives on Adults Learning Mathematics* (pp.47-52). Dordrecht: Kluwer Academic Publishers.
- D'Ambrosio, U. (1985). Ethnomathematics and its Place in the History and Pedagogy of Mathematics. *For the Learning of Mathematics*, 5(1), 44-48.
- Evans, J. (2000). *Adults' mathematical thinking and emotions: A study of numerate practices*. London: Routledge Falmer.
- FitzSimons, G. E. (2002). *What Counts as Mathematics? Technologies of Power in Adult and Vocational Education*. Dordrecht: Kluwer Academic Publishers.

- Gerdes, P. (1996). 'Ethnomathematics and Mathematics Education.' In A.J. Bishop et al. (eds.), *International Handbook of Mathematics Education* (pp. 909-943). Utrecht: Kluwer Academic Publishers.
- Groenestijn, M. van (2002). *A Gateway to Numeracy. A study of Numeracy in Adult Basic Education*. Utrecht: Utrecht CD-β Press.
- Johnston, B.; Yasakawa, K. (2001). Numeracy: negotiating the world through mathematics. In Atweh, B.; Forgasz, H.; Nebres, B. (eds.), *Sociocultural Research on Mathematics Education. An International Perspective* (pp. 279-294). Mahwah: Lawrence Erlbaum Associates, Inc., Publishers.
- Knijnik, G. (1997). 'Mathematics Education and the Struggle for Land in Brazil.' In FitzSimons, G. (ed.), *Adults returning to Study Mathematics. Papers from Working Group 18. ICME8* (pp. 87-91). Adelaide: The Australian Association of Mathematics Teachers.
- Lindenskov, L. and Wedege, T. (2001). 'Numeracy as an Analytical Tool in Adult Education and Research.' *Centre for Research in Learning Mathematics, Publication no.31*, Roskilde University.
- Mellin-Olsen, Stieg (1987). *The Politics of Mathematics Education*. Dordrecht: Kluwer Academic Publisher.
- O'Donoghue, J. (2002). 'Mathematics or Numeracy: Does it really matter?' To be published in *Policies & practices for adults learning mathematics: opportunities and risks. Proceedings of the 9th conference of ALM*. London: King's College London.
- Olesen, H. Salling (1994). Qualification Research. Basic Concepts and Danish Research. In Tøsse, S. et al. (eds.), *Social Change and Adult Education Research. Adult Education Research in Nordic Countries 1992/93*. Trondheim: Tapir.
- Olesen, H. Salling & Rasmussen, P. (eds.) (1996). *Theoretical Issues in Adult Education – Danish Research and Experiences*. Frederiksberg: Roskilde University Press.
- Safford, K. (1998). Who is an adult? How does our definition affect our practice? In Groenestijn, M. and O'Donoghue, J. (eds.), *Mathematics as part of Lifelong Learning. Adults Learning Maths - A Research Forum. ALM-5. Proceedings of the Fifth Conference of ALM, 1-3 July 98, Utrecht* (pp. //?). London: Goldsmiths University of London. (pp. 96-102)
- Skovsmose, O. (1994). *Towards a Philosophy of Critical Mathematics Education*. Dordrecht: Kluwer Academic Publishers.
- Skovsmose, O. (2002). 'Students' foreground and the politics of learning obstacles'. *Centre for Research in Learning Mathematics, Publication no. 35*, Roskilde University.
- Strässer, R. (2002). 'On the disappearance of Mathematics from society's perception.' In H. G. Weigand, N. Neill, A. Peter-Koop et al. Hildesheim (Eds.) *Developments in Mathematics Education in German-speaking Countries. Selected Papers from the Annual Conference on Didactics of Mathematics, Bern, 1999*. (pp. 124-133) Berlin, Franzbecker.
- Wedge, T. (1999). 'To know or not to know – mathematics, that is a question of context.' *Educational Studies in Mathematics*, **39**, 205-227.
- Wedge, T. (2000). 'Technology, competences and mathematics.' In D. Coben, J. O'Donoghue, & G. E. FitzSimons (Eds.), *Perspectives on adults learning mathematics: Research and practice* (pp. 191-207). Dordrecht: Kluwer Academic Publishers.
- Wedge, T. (2001). 'Epistemological questions about research and practice in ALM.' In Safford, K., Schmitt, M.J. (eds.) *Conversation Between Researchers and Practitioners, The 7th International Conference on Adults Learning Mathematics (ALM7)* (pp. 47-56). Medford, Massachusetts: Tufts University.
- Wedge, T. (2002a). 'Numeracy as a basic qualification in semi-skilled jobs.' *For the Learning of Mathematics*, Vol.22/3, 23-28.
- Wedge, T. (2002b). "'Mathematics - that's what I can't do" – People's affective and social relationship with mathematics.' *Literacy and Numeracy Studies: An International Journal of Education and Training of Adults*, 11(2), 63-78.
- Wedge, T. (2003). Mathematics at work: researching adults' mathematics-containing competences. To be published in *Nordic Studies in Mathematics Education*, May 2004.
- Wedge, T.; Benn, R.; Maazs, J. (1998). 'Adults Learning Mathematics as community of practice and research.' In Groenestijn, M. and O'Donoghue, J. (eds.), *Mathematics as part of Lifelong Learning. Adults Learning Maths - A Research Forum. ALM-5. Proceedings of the Fifth Conference of ALM, 1-3 July 98, Utrecht* (pp. 54-63). London: Goldsmiths University of London.
- Zevenbergen, R. (2000). 'Ethnography and the situatedness of workplace numeracy.' In Bessot, A. & Ridgway, J. (eds.) *Education for Mathematics in the Workplace* (pp. 209-224). Dordrecht: Kluwer Academic Publishers.

Adult Numeracy/Mathematics in Australian Workplaces

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Abstract

Numeracy in the workplace is much more complex than the simple application of mathematical knowledges and skills learned in school or vocational education. Building on a burgeoning corpus of research into how mathematics/numeracy is used in workplaces around the world, I visited several Australian workplaces to gain further insights. Implications for the vocational and workplace mathematics education will be drawn.

Introduction

Numeracy (or doing mathematics) in the workplace is much more complex than the simple application of mathematical skills learned in school or vocational education. Building on a burgeoning corpus of research into how mathematics/numeracy is used in workplaces around the world, over decade ago I visited nine Australian workplaces: a fund-raising ‘trivial challenge’ production office, a modular shed construction company, a local post-office, a short-term home rental company, a graphic design company using CNC machinery, a local playgroup, a small hairdressing salon, a wholesale power tool warehouse, and an aged-care hostel. A second, smaller project involved investigating how mathematics/numeracy is learned on the job in the case of chemical spraying and handling. The activities of chemical preparation, application, transport, handling and storage undertaken by operative workers are high risk activities in terms of occupational health and safety of workers, their clients and in relation to environmental damage. They place high demands on workers’ numeracy and literacy skills. Sites visited included parks, vineyards, orchards, plant nurseries, golf courses, and chemical warehouses.

Literature Review

Workplace studies show that mathematical elements in workplace settings are subsumed into routines, structured by mediating artefacts, and are highly context-dependent. The mathematics used is intertwined with professional expertise at all occupational levels, and judgements are based on qualitative as well as quantitative aspects. The constraints and affordances of most workplaces mean that the conditions for doing mathematics at work are very different from the school setting (FitzSimons, 2012), and the visibility of mathematics at work often becomes problematic. Mathematics at work often bears little resemblance to school mathematics in that the actual problems (as distinct from routine procedures) need to be recognised and defined by workers (very often in collaboration with others), the medium of communication is rarely, if at all, pen-and-paper alone, and workers rarely recognise what they are doing as mathematics per se, but merely regard it as commonsense.

In FitzSimons (2005) I reviewed the extant literature considering the differences between mathematics and numeracy, the uses of mathematics and numeracy in the workplace, and offered curricular and pedagogical implications. At the time of original publication of this article, there had been little attention given to how numeracy is learned in the workplace. More recently, as co-editor of a special issue on characterising vocational mathematics education, I prepared a commentary which identified three main areas where school and vocational mathematics curricula may be distinguished: (a) the purpose, (b) the considerable range of stakeholders, and (c) the fact that the students are undergoing a process of transition between school and work (FitzSimons, 2014).

Drawing on Bernstein (2000), FitzSimons and Boistrup (2017) argued that doing mathematics at work requires people to skilfully contextualise their theoretical knowledge of the *vertical discourse* of mathematics and reconcile it with the vertical discourse/s of their particular vocation, together with their specific knowledge of the social and cultural *horizontal discourse* of their actual workplace practices. Reconciling these sometimes competing discourses means that the best mathematical solution may not be not necessarily the best in practical situations, and may not be workable at all. The importance of clear and meaningful communication through all available modalities cannot be over-emphasised, especially in safety critical situations as workers need to interact with others whose relevant mathematical (& other) knowledges and skills may vary widely from their own. Vocational and workplace mathematics (or numeracy) education implies a fundamentally different curriculum due to its different goals (or intended outcomes), yet encompassing underpinning mathematical knowledges and skills in ways that enable the generation of locally ‘new’ knowledge in order to solve problems which cannot always be known in advance. Future workers also deserve the mathematics education to sustain them in meeting unforeseen changes, especially due to technological innovation, throughout their work and life trajectories.

Methodology

This research was framed by an activity theory approach which offered a unit of analysis for enabling a theoretical account of an object-oriented, collective, and culturally mediated activity system in all its complex interactions and relationships. The minimum elements of this system include the object, subject, mediating artefacts (signs and tools), rules, community, and division of labour (see Engeström, 1987). Detailed observation of participants took place wherever possible, and semi-structured interviews were undertaken. Where permission was granted, the collection of artefacts took place, including actual samples of materials used (e.g., procedures, manuals, charts) and/or photographs. Interviews were audiotaped for further analysis. The interview data and observations were analysed to identify common themes, issues and potential strategies that could inform teaching practice.

Findings

Most, if not all, of the following competencies were observed in all workplaces: (a) collecting, analysing and organising information, (b) communicating ideas and information, (c) planning and organising activities, (d) working with others and in teams, (e) using mathematical ideas and techniques, (f) solving problems, and (g) using technology. Most if

not all of Bishop's (1988) pan-cultural activities were also observed: counting, designing, explaining, locating, measuring, and playing (or thinking hypothetically). Underlying mathematical knowledges and skills included algebraic thinking (e.g., spreadsheets), calculations (with & without a calculator) and associated relevant estimation skills, geometric thinking, logic, measurement, and the accurate storage, retrieval, display, and interpretation of data. Clear communications with other stakeholders and creative problem solving were essential. Other mathematics-related competences for those in positions of responsibility included high level skills in forward planning and organisation, as well as the ability to keep the operation financially viable and to meet other legal requirements (e.g. accurate and timely record-keeping) for accountability purposes. I now summarise the individual findings.

1. Fund-Raising: Trivia Challenge.

This worker organised a large-scale fund-raising competition for around 20 000 primary and secondary students in Victoria, from grades 3-10 divided into four levels; she researched the questions herself. The major tasks included mail-outs to schools, organising the question papers for individual schools at the relevant levels, arranging the finals at a major venue, purchasing prizes (token and large), and collecting the money raised. The underpinning knowledge required in order to meet the contingencies of the job, underlining the worker's creativity in problem solving, could be covered by six years of secondary mathematics. However, these skills were applied in the context of the workplace where criteria for success are very different. On a day-to-day basis the worker was largely responsible for her own work so it was imperative that she was well organised: There were always tensions between competing tasks and meeting deadlines.

2. The Local Post Office.

The major tasks observed were mail sorting, sales of postal goods and services, other financial services (e.g., bill paying for utilities), sales of stationery, calendars, tourist items, and so forth. The characteristics of creativity and problem solving were integral to the work of the local post office manager. One of the key issues was staying in business — that is, making enough money to cover costs (including his own salary). This needed scrupulous attention to monetary details (e.g., accurate entries on the cash register and safe, cash flows, trustworthy staff), measured creativity in seeking new items for profitable sale in a relatively small floor space, and giving customers a reliable, high level of friendly service. Calculations were of the essence in the varied planning roles. Design and storage (location) and retrieval of stock and records were also an essential ingredient. This business depended on the reliable use of statistics as a means of stock control.

3. Modular Shed Construction.

Here, the onsite construction of a modular shed behind the client's home was observed. Although the two workers both had less than six years secondary schooling, their skills were adequate to meet the level of numeracy required. However, what is not emphasised in school mathematics is the necessity of appropriately accurate and timely readings of measurements. Checking and re-checking was essential to the whole project. Problem solving — dealing with the unexpected — was always a real possibility. Sometimes it was needed in order to

recover from human or machine error, such as the faulty construction of the modular components delivered to the site. But a workable solution had to be found, even if it appeared costly in terms of time and/or money. Much of the communication seemed to be tacit — well known procedures and routines had already been established between these two workers. There was limited dialogue exchanged in the confirmation of measurements or the realignment of the construction, but gestures and other non-verbal forms were frequent. However, these exchanges are crucial and in the case of a breakdown would need to be clearly articulated and discussed or debated. Another critical feature, already established by the workers was the logical organisation of the project – from picking and loading the required modules onto the truck in sequence at the factory, to the final check of the completed shed.

4. The Short-Term Home Rental Business.

This work involved dealing with a range of activity groups (e.g., potential and actual renters, trades people, cleaners) also required mathematical understandings in order that the sequence of events for any one rental arrangement might take place as planned. Careful attention was paid to optimising the tenancy rates while offering flexibility to clients. Keeping track of money — cash flow was critical — was fundamental, and required systematic record-keeping as well as highly developed planning, teamworking, communication, and problem solving skills. This was in addition to the competent use of IT with spreadsheets and other templates, including those of the Australian Taxation Office. These required analytical, abstract, algebraic as well as numerical thinking — especially at times of technological breakdown. Efficient and effective use of the pocket calculator — especially in relation to percentages — was also essential.

5. Graphic Design & the CNC Machine.

The task observed was the cutting of clear panels for public telephone boxes. The importance of accuracy in measurement and communication was not underestimated as mistakes were costly in terms of expensive raw materials wasted and delays in meeting customer deadlines — expensive, not only for this job, but for the company's reputation with this and other customers. Because of the creativity inevitably associated with customised work, problem solving played a critical role in setting up the process and in maintaining the correct measurements throughout the job at hand. Attending to computer-numeric-controlled (CNC) machines, which operate in *three dimensions*, required the ability to think analytically and abstractly (i.e., algebraically & geometrically), while relating the machine settings to the concrete numerical/measurement demands of the task. Clearly, operating the onboard controller console needed specialised training from the supplier (or more experienced workers), so familiarity with technological-mathematical equipment was required. But, unlike the school situation, every calculation or keystroke had to be as accurate as possible — careless mistakes were not tolerated. Accuracy of measurement and knowing the limits of measuring tools were essential, requiring that the operator had a well-developed sense of the reasonableness (or otherwise) of readings from inputs or outputs.

6. The Local Playgroup. [observations still current]

There is still a widespread perception that teaching is an easy job, and for pre-schoolers in particular, very easy. The opposite remains the case. There is a significant requirement for forward planning of activities and availability of the materials required to support the children's activities. Constant monitoring of children and communication between staff is essential for the smooth flow of the programme, as well as for safety reasons. The educational side of early childhood education is also becoming increasingly important — not least in mathematics education. The preparation of new workers in this industry in Australia now requires a greater depth of knowledge and understanding of the mathematical principles and their associated pedagogies. There are also serious accountability requirements as regulation by government departments continues to tighten. Financial viability of the programme is always a pressure, so that the teacher/co-ordinator must operate strictly within a given budget.

7. The Local Hair Salon. [observations still current]

Knowledge of client needs, and preferences in style and colour need to be kept up to date on computerised records, with new requests accommodated at each visit. This requires the ability to understand the logic of the database system in order to enter and/or interpret data. Computer use requires a knowledge of prices to enable estimation of reasonableness; also ability to problem solve. Given the importance of stock control and accurate record keeping for economic viability and financial accountability for taxation purposes, there are serious requirements for mathematical skills and knowledges in the daily practice of a salon such as this. This is in addition to the 'invisible' skills of design and location which encompass not only the styling and colouring of hair, but also the layout of the salon (front and back of house). Measurement skills are most obvious in the preparation for hair colouring — although the level of accuracy is mostly determined by past experience. In an industry such as this, the ability to communicate with clients and suppliers is of the essence, incorporating mathematical understandings even though they may be largely invisible.

8. The Wholesale Warehouse.

This company sold a range of power tools and cutting machines (large and small) to hardware retailers. Record-keeping and stock control needed to be accurate and up-to-date in order to ensure the smooth flow of dispatches. Technologisation of both packaging and organisation was occurring rapidly, and workers needed to keep abreast of the developments, adapting to new technologies of management, equipment, and record keeping. There was a logical flow of tasks to be executed in an optimal way, keeping in mind the constraints of carriers' pick-up times and capacities. Computer use required a knowledge of spreadsheets, specialised data processing, and labelling. Choice of packaging materials and composition of consignment was context dependent — related to the nature of the sales items (e.g., shape, dimensions, value). Clear and accurate communication was of the essence, and generally overtly mathematical in nature: identification codes, locations, measurements, costs and times, for example.

9. The Aged-Care Hostel.

Although the advent of pre-packaged drugs appears to have removed preparing medicinal dosages, the most obvious indicator of mathematics used on the job for personal care workers, there were other less visible numeracies at work. For example, new ways of working included:

1. Quality auditing which has changed the way practices are undertaken; there are flow charts for every action.
2. Changes in legislation which have introduced the need for formalised training in areas such as privacy and dignity; also that photographs be attached to residents' records and medication lists for checking when medication is administered.
3. Emergency colour codes which need to be learned and responded to automatically.
4. In this hostel, a new fire detection system had been installed, with a complex diagram which needed to be interpreted in order to locate the source of any fire as a matter of urgency.

The small size of this hostel also required that some personal care workers were responsible for budgeting for food orders while maintaining good quality meals. The daily routines included medication rounds before and after meals, showering and dressing residents who required assistance, organising the totality of activities around mealtimes (e.g., setting tables, organising seating, cleaning up), as well as general cleaning and laundry. This worker also completed the bookwork for the Quality Audit. Mathematics-containing communication was of the essence: (a) with residents about their immediate needs; (b) with residents' families about their needs, monthly or as required; (c) with visiting doctors about residents' healthcare needs; (d) with the local pharmacy about supply (and sometimes clarification) of necessary drugs etc.; (e) with other personal care workers about residents and daily routines; and (f) with management about changes to routines, rosters, critical incidents, and completed time-sheets.

10. Chemical Spraying and Handling.

These tasks were undertaken by workers as just one aspect of a range of duties, and ranged from small-scale weed spraying from a back-pack to broad acre spraying drawn by a tractor. Safety was the ultimate consideration, both for people and for the health of the spray site and the local environment. For more detail, see FitzSimons, Mlcek, Hull, and Wright (2005).

Mathematics Skills and Concepts at Work

The following were identified as the underlying mathematics concepts in chemical spraying and handling: addition, subtraction, multiplication & division of whole numbers and decimals; ratio & proportion; measurement: length, area, volume, capacity, mass [usually metric]; estimation; and approximation. The following were identified as processes used by workers to undertake these calculations: estimation, pencil & paper methods, use of basic 4-function calculator; verbal or written communication with other workers; consultation with prescriptive calculations sheets and with historical records; completion of up-to-date records of chemicals used and their amounts; and consideration of other contextual factors, e.g., date/time; block area; crop; crop stage; weed/pest/disease targeted; chemical group; rate/hectare; litre spray applied; method of application; temperature; wind speed; wind direction; rainfall; and humidity.

Underlying mathematical concepts in the other nine workplaces included: algebraic thinking (for spreadsheets), calculations (with & without a calculator) and associated relevant estimation skills, geometric thinking, logic, measurement, and the accurate storage, retrieval,

display, and interpretation of data. Clear communications with other stakeholders and creative problem solving were essential. Other mathematics-related competences for those in positions of responsibility included: high level skills in forward planning and organisation, as well as the ability to keep the operation financially viable and to meet other legal requirements (e.g., accurate and timely record-keeping) for accountability purposes.

How did workers learn to do these calculations? Most of these basic calculations were taught initially in school prior to the post-compulsory years. In the case of chemical spraying, most, if not all, of the workers had the Farm Chemical Users Certificate, or equivalent, and the relevant calculations were revised and practised here, in (semi-) contextualised settings. That is, the students observed and experienced the actual measurement skills in practice sessions, but they lacked access to the authentic ongoing records of any one particular site which could provide a deep sense of meaningfulness to their calculations. For the other nine workplace observations, it seemed that most relevant learning was done in the contextualised workplace, through observation, reflection, and creative adaptation to the artefacts, problems, or objectives at hand. However, these needed to be supported by a firm foundation in school mathematics — beyond the minimal grades for certification — together with a determination to make sense of available data (current and historic), and a positive, creative approach to problem solving, especially in workplaces where timely and cost-efficient resolutions were imperative.

How did the workplace setting impact on how the calculations were done and how the processes were learnt by workers? In the chemical spraying workplace, calculations were always checked in some form by another person, whether the supervisor or the tractor driver, for example. Previous experience and historical data played a considerable role in determining reasonableness of answers. It also determined whether and how to approximate answers. Most importantly, the learning in the workplace varied from school mathematics education in that workers were always reminded to check their calculations for reasonableness, to ask repeatedly if they are not sure, and, above all, to consider personal safety while avoiding environmental or crop damage.

Implications

The task of preparing and applying chemicals requires that a complex set of variables must be taken into account by the person responsible. Numeracy in chemical spraying and handling is always a social-historical and cultural practice, involving the transformation of school-based mathematics. Estimation is always absolutely necessary, based on prior experience of the particular kind of spraying needed, or just sensible estimation of calculations by the novice. Common sense is of the essence. Judgements are needed as to when it is appropriate to approximate the chemical mixture and when it is not, and how this approximation may be usefully made. It is never acceptable to make a mistake in the actual process as it may threaten public and environmental safety, also the livelihoods of the operators and their managers. All calculations must be double-checked, and asking questions where any doubt exists is strongly and repeatedly encouraged. Confidence in undertaking the numeracy tasks comes from several sources, including the support of expert knowledge from the managers or co-workers within the workplaces as well as the internet. Team and group work is fostered as part of workplace practice. Artefacts are used as structuring resources to aid in formal calculations, or in other situations requiring assessment and evaluation.

Mathematics teaching *off-the-job*, as a complementary educational context, is also critical in supporting workplace numeracy. It is commonplace that pedagogy should attempt contextualise the skills and knowledges required, but this could be supported by integrating

work related tasks with relevant mathematical skills and knowledges extending beyond counting and measurement. Many designers work in a three-dimensional, concrete world: Somehow a bridge needs to be made so that vocational mathematics/numeracy education does not simply produce more text-based applications. Simulated work experience, through small self-contained episodes, would be a great advantage in preparation for work situations; also mini-projects with inbuilt uncertainties typical of the workplace (see, e.g., FitzSimons & Boistrup, 2017).

Apart from specialised computer training, most learning seems to be *on-the-job*. However, this ability to learn requires a strong general mathematics education, in order that the workers can quickly adapt to the idiosyncracies of their workplace contexts. Logical thinking and problem solving are examples of mathematical competences that can develop over the compulsory years and be refined by further formal education in numeracy as well as on-the-job. As noted above, the ability to communicate is of the essence, and this incorporates mathematical understandings even though they may be largely invisible most of the time. The development of authentic communication skills should form an integral part of any vocational mathematics/numeracy education.

Conclusion

This paper has given an account of workplace visits intended to illuminate the mathematical practices utilised by workers in their everyday practices. Any workplace activity is necessarily socially, culturally, and historically located. Mathematics or numeracy education for the workplace must also recognise this fact. The challenge is to adequately prepare students for whatever life or work circumstances they may confront.

References

- Bernstein, B. (2000). *Pedagogy, symbolic control and identity: Theory, research, critique* (Rev. ed.). Lanham, MD: Rowman & Littlefield.
- Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education*. Dordrecht: Kluwer Academic Publishers.
- Engeström, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research*. Helsinki: Orienta-Konsultit.
- FitzSimons, G. E. (2002). *What counts as mathematics? Technologies of power in adult and vocational education*. Dordrecht: Kluwer.
- FitzSimons, G. E. (2005). *Literature review. Learning numeracy on the job: A case study of chemical handling and spraying—Support document* (pp. 3-27). Adelaide: National Centre for Vocational Education Research. Retrieved August 2, 2017, from https://www.ncver.edu.au/_data/assets/file/0026/4895/nr3106s.pdf
- FitzSimons, G. E. (2013). Doing mathematics in the workplace: A brief review of selected recent literature. *Adults Learning Mathematics: An International Journal*, 8(1), 7-19.
- FitzSimons, G. E. (2014). Commentary on vocational mathematics education: Where mathematics education confronts the realities of people's work. *Educational Studies in Mathematics*, 86(2), 291-305.
- FitzSimons, G. E., & Boistrup, L. B. (2017). In the workplace mathematics does not announce itself: Towards overcoming the hiatus between mathematics education and work. *Educational Studies in Mathematics*, 95(3), 329-349.
- FitzSimons, G. E., Mlcek, S., Hull, O., & Wright, C. (2005). *Learning numeracy on the job: A case study of chemical handling and spraying*. Final Report. Adelaide: National Centre for Vocational Education Research. Retrieved August 2, 2017, from https://www.ncver.edu.au/_data/assets/file/0013/5152/nr3106.pdf

Understanding Workplace Mathematics from a Curriculum Development Perspective

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Introduction

Complementing concerns expressed by the Organisation for Economic Cooperation and Development (OECD) and Programme for International Student Assessment (PISA) a number of studies and surveys in the UK point to the inadequacies of the workforce in terms of their literacy and numeracy (see for example, Roberts, 2002). It is deemed particularly important that deficits in these areas are remedied as the rapidly changing nature of workplaces both now and potentially in the future, is considered. In the UK questions are currently not only being asked about how the curriculum might provide a more appropriate mathematical preparation for young people, but also how qualifications structures should be developed to make study of the subject more prominent. The recent report by Smith (2004) acknowledges that, “mathematics is fundamentally important in an all-pervasive way, both for the workplace and for the individual citizen.” Such considerations do not only exercise the minds of policy makers: young people themselves recognise the potential utility of mathematics in the workplace and often question their teachers about how, where and when they might use the topics that they are required to learn in school and college.

Studies of what, and how, mathematics is used in different workplaces might therefore be seen as potentially very useful sources to inform the debate about how mathematics curricula and associated pedagogy and assessment might usefully develop. However, as potential informative sources there are relatively few studies that appear to have proved particularly illuminating in this respect. Early studies, perhaps simplistically, tended to focus on the mathematics that employers considered important for their workers to be able to do, or understand, using existing or newly developed structures that were framed in the ‘formal’ or ‘academic’ discourse of mathematicians and mathematics educators. More recent studies have been greatly influenced by the realisation that the both the application of knowledge and perhaps more importantly its construction, is often heavily situated within a community of practice (see for example, Lave, 1988; Wenger, 1998). There are, therefore, an increasing number of workplace studies that are in the ethnographic tradition and have been analysed and described from a Cultural Historical Activity Theory (CHAT) point of view. Alongside such accounts a range of new constructs (for example, situated abstraction, general mathematical competences and techno-mathematical literacies) have been developed that attempt to describe the complex activity observed in workplaces in ways more familiar to those in the mathematics education community. In addition to these newly developed constructs mathematical modelling is perhaps potentially useful in allowing us to make sense of mathematical activity in workplaces in a way that might assist curriculum developers and policy makers bridge the college - workplace divide.

Theoretical frameworks

Cultural Historical Activity Theory considers how the activity of a community is mediated by a range of different influences. It builds on the fundamental thinking of Vygotsky about how the action of a subject is mediated by artefacts, tools and ‘instruments’ (Figure 1). These artefacts and instruments are often most easily identified by researchers as they seek to unpack the mathematical activity of workers. As Strässer (2003), amongst others, have illustrated instruments and artefacts developed over time in the workplace allow mathematics to become “hidden” or transparent to the worker. These are in many instances becoming increasingly enmeshed with new technologies and consequently “black-boxed”.

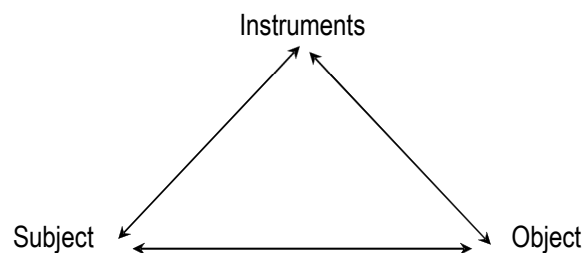


Figure 1. The mediation of action by “instruments”

The original idea triangle of mediation has been developed and extended by Vygotsky, Luria, Leont’ev and followers (for a summary of their work see Engestrom & Cole, 1997) allowing the unit of analysis to be extended to the ‘community of practice’ (Figure 2). The additional mediating influences of “community” and its “division of labour” and “rules” and “norms” may now be taken into account. Again these can be powerful in directing how mathematical activity is developed in workplace settings. For example, the totality of workplace activity is often distributed in such a way that mathematics is the preserve of the specialist, minimising the mathematics required of the majority of workers, thus reducing the potential for errors.

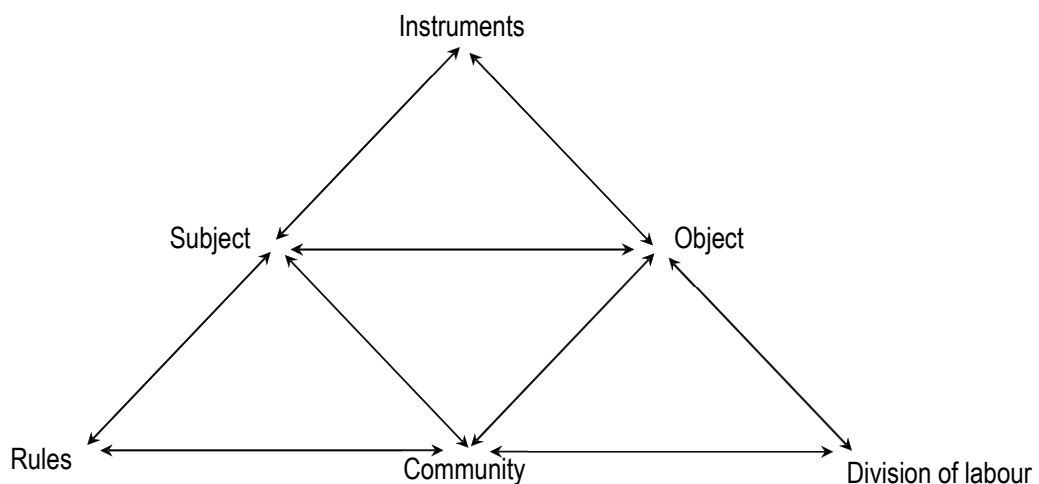


Figure 2. The expansion of the basic mediational triangle

I now turn to introduce briefly some of the constructs that have been developed by researchers from the mathematics education community as they struggle to describe the mathematics they observe to colleagues and educational policy makers in terms that might be well understood.

The idea of *situated abstraction* has been developed by Noss, Hoyles and colleagues (see for example Pozzi et al, 1998). This allows one to understand how workers may develop a generalised mathematical understanding, albeit perhaps one that holds only within the situational context of their work. This may well be within a discourse other than that of “standard” / “formal” mathematics but which may be mapped to this; indeed, this is perhaps necessary by the researchers when talking within their own community.

Within my own work, I and colleagues, have proposed the idea of *general mathematical competences* (see for example, Williams et al, 1999, Wake & Williams, 2000) which were developed from a mathematics education standpoint to take account of common ways in which students and workers might bring together and synthesise coherent bodies of mathematical knowledge, skills and understanding. Crucially this construct moves us away from thinking of the mathematics curriculum as being a collection of atomised mathematical skills towards consideration of common ways in which we bring together and use mathematics to analyse situations and solve problems.

Most recently Hoyles, Noss and colleagues (see for example, Kent et al., 2004) have proposed the construct of *techno-mathematical literacies* which they are developing to assist understanding of how mathematics in workplaces is not only very much grounded in day-to-day workplace activity but is often also highly integrated and dependent on the use of modern technologies. Indeed *techno-mathematical literacies* take account of how the mathematical activity of workers is often focused on interpretation of the outputs of complex mathematical models.

These different constructs, whilst not always referring explicitly to mathematical modelling, do, often by implication, suggest that mathematical modelling and mathematical models are often central to the activity of workers. A large number of mathematics educators have long been convinced of the importance of mathematical modelling as an activity with which students should engage as part of their mathematics curriculum. See the proceedings of the bi-annual conference of the ICTMA group for a comprehensive discussion of many aspects of this (for example, Lamon et al., 2003).

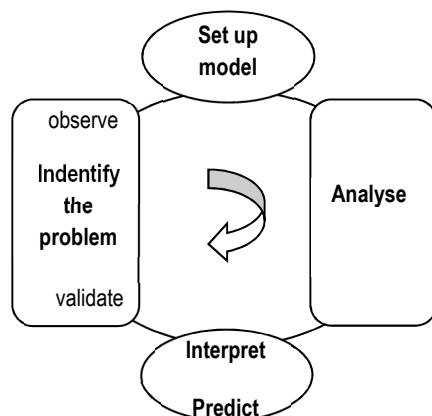


Figure 3. Schema representing the process of mathematical modelling

Here, I introduce the schema of Figure 3 as a framework around which to later develop my discussion of mathematical modelling in both workplaces and colleges. The left hand side of this can be considered to represent the real world – often the workplace in our considerations here – whilst the right-hand side we might think of as “mathematics”. At the centre of many studies is discussion of how similar or different this “mathematics” is in workplace and educational settings. The process of mathematical modelling is, of course, much more complex than the simplified schema of Figure 3 might imply. To gain some understanding of the likely complexity of the process see for example Niss, (1989). The mathematisation so that a real world problem might be described and analysed mathematically (referred to as “setting up the model” in Figure 3) is in effect mathematical modelling. However, this term has become used to describe the entire loop that also crucially involves the interpretation of the mathematical analysis. It will be important in later discussion to clearly distinguish differences between the process of setting up the model (translation from real world to mathematics) and the complete cycle, which may indeed be undertaken more than once in an attempt to develop a more sophisticated model. It is also important to recognise that “standard” mathematical models often exist and can be used in a variety of situations; in such cases the process of moving from “mathematics” to the “real world” may be emphasised. This is often described as *application* of mathematics rather than mathematical modelling as such.

Case Study

I turn now to describe in some detail just part of the activity of Alan, a railway signal engineer/designer. This case study was developed as part of a project “Maths into Work”¹⁷ in which we explored, with college students, workplace activity from a mathematics

¹⁷ This project was generously funded by a grant from Astra Zeneca to the University of Manchester

perspective. This is an extension of an earlier project in which a range of similar case studies were developed and which are reported and analysed in Wake and Williams, 2001. The methodology employed involved me as researcher working over a period with a group of three college students on a pre-vocational engineering course and their teacher: together we explored the activity of a number of different workers in a range of workplaces in which the students might in the future be expected to be employed. I report here data collected on two separate occasions: the first involved a site visit by myself, students and teacher to Alan's office where he described some aspects of his work, followed by a later discussion where I explored with the students their understanding of Alan's mathematical activity.

Alan, at the start of our visit, gave a comprehensive description of his current work and his background. He emphasised how he had initially followed a similar at college similar to the visiting students, became qualified at technician level in the field of electronics and then,

...jumped ship from electronics into the railway. I progressed through the drawing office as a junior designer, through being a designer, lead designer, and then design checker.

He went on to focus on one particular set of calculations he checks regularly: associated with the spacing and positioning of signals restricting train speeds and warning of signals ahead that require drivers to stop. As Alan explained,

we have speed-boards everywhere on the railway to limit the line speed for trains ...the driver sees a warning board: it's telling him in advance that there's going to be a speed restriction ahead ...it just looks like the same speed restriction that you see on the road ...the difference on the railway is ... we have to put a warning board in advance of that, because he [the train driver] needs to start applying his brakes, because he's got many tons of train behind him, he can't just apply his brakes and slow down to 40 ...we have to calculate the distances that he has to start braking at ... that's a similar calculation to what we do when we're doing signal spacing ...and the main thing there is literally working out average gradients over a stretch of track.

Alan then introduced us to a plan of a stretch of railway, part of which is shown in Figure 4 which I have annotated to show the "stop signal" at the end of the platform at Millom Station, a "distance signal" along the track before the station and a marker which shows the average gradient over a section of the track.

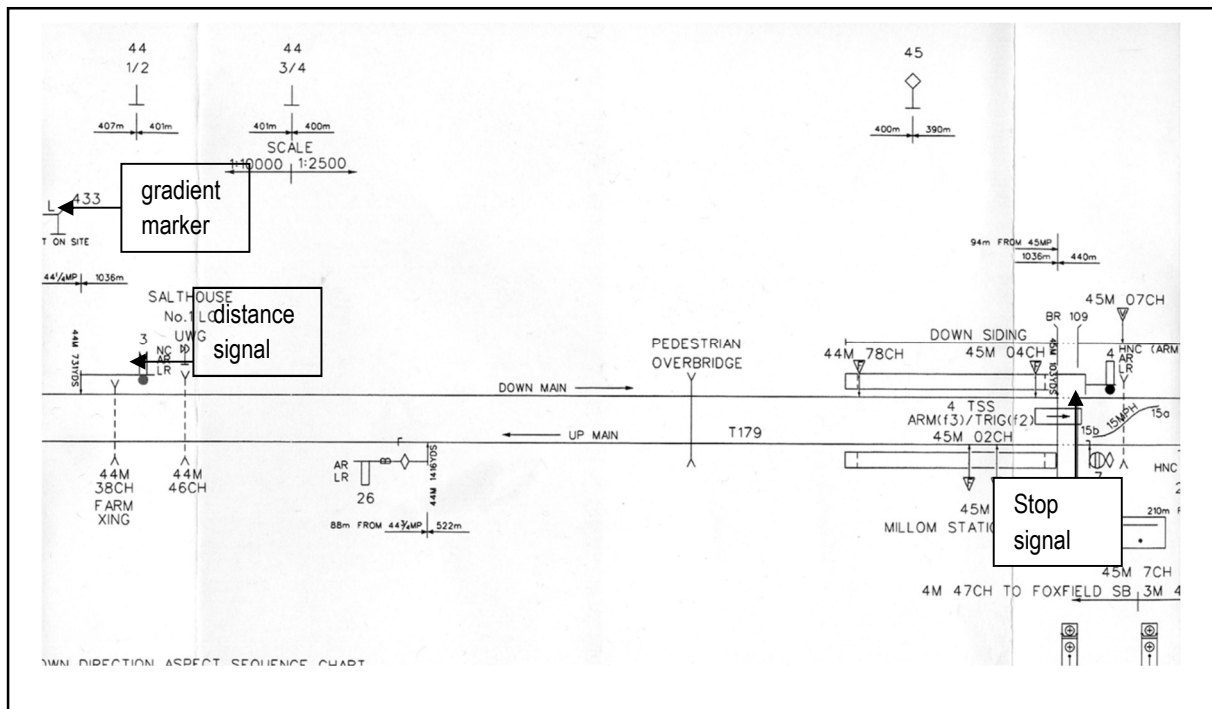


Figure 4. Part of plan representing railway track with gradient marker and “stop” and “distance” signals indicated.

For this particular stretch of track the average gradient between the distance and stop signals is constant (see Figure 4), but as Alan went on to explain,

we have to work out the average gradient over the whole section of line that we’re looking at for spacing these signals apart. And there might be six or seven different gradient changes over that entire distance, so we have to take the average, which is what the training notes explain.

Alan circulated the training notes (see Figure 6) before briefly describing the calculation procedure. These calculations formed the focus of a follow-up discussion between myself and the three students which I will describe in a little detail in due course. However, before doing so, I should complete Alan’s description of the entire process that he performs by introducing the final step in which he uses a Table (see Figure 5) to look up the required distance between signals. He uses the value of average gradient between the two signals indicated in Figure 4 (that is, 1 in 433 rising). In Alan’s words:

roughly there’s a gradient of in 1 in 500 rising, then we’d go to the chart for an approaching speed of 60 miles per hour. We’d read off the 60 miles per hour row and – for a rising 1 in 500 rise in gradient - would read off the required minimum signal spacing distance. Then we’d compare that to where we’ve actually placed the signal.

Alan explained that he always has to err on the side of safety and as a gradient of 1 in 500 rising lies between 1 in 200 rising and the level, it is safer to take the value associated with a level gradient as a rising track requires less braking distance due to gravity acting to slow the train down. Alan doesn’t need to think about this he knows to always use the value from the table “to the right”.

INITIAL SPEED (Mph)	GRADIENT								
	Rising					Falling			
	1 in 50 2.0%	1 in 67 1.5%	1 in 100 1.0%	1 in 200 0.5%	Level Level	1 in 200 0.5%	1 in 100 1.0%	1 in 67 1.5%	1 in 50 2.0%
20	175	180	195	215	240	275	320	395	520
25	240	255	280	315	355	410	485	625	840
30	320	340	380	425	485	575	700	895	1425
35	405	440	485	550	635	780	1010	1380	2237
40	495	550	620	720	865	1080	1420	1903	2237
45	630	710	805	935	1130	1435	1660	1903	2237
50	688	748	816	935	1130	1435	1660	1903	2237
55	770	831	901	984	1130	1435	1660	1903	2237
60	849	911	980	1061	1165	1435	1660	1903	2237

Figure 5. Table used to determine distance between signals

Now I turn to give something of the flavour of the follow-up discussions I had with the three students as they struggled back in college to make sense of Alan's workplace mathematics.

We first of all investigated how to find the distance between stop and warning signals using the gradient indicated in the Table of Figure 5.

Student 1: Yeah, or 1 in 433 - and it is *rising* - it couldn't go that high could it?

Student 2: I don't know how they use it.

Researcher: well look, ok... have you got any idea (to S2). Here you've got *level*, here you've got 1 in 200, and here you've got 1 in 100. But we want 1 in 400 so what do you think we might do?

S1: double it [indicating value in 1 in 200 column].

S2: yeah double it...

R:double the distance would make it *huge* wouldn't it? Yes, If you doubled ... like 60 miles [per hour]... we're working along this line here... So 1 in 200 is 1061 if you *doubled* it, that would be 2100 – well.... but on the level is only 1100 ?
...if we could slot in another column there between 1 in 200 and? Yes?

S1: I would just take the higher gradient ... or erm...

R: Or you could...oh you see, you could...erm. Ah, he [Alan] mentioned something about always using the one to the *right*, didn't he?...

S1: right yeah

R: so, where are we then? Which... I can't read that. What value have you got?

S1: 1165

R: that's 1165... what are they ... they're yards. Right, so what we've got to find out *now*, is...whether it is actually 1165 ...

S2: times 2

The students had great difficulty in understanding what had become crystalised in Alan's practice, that is, finding the value of distance associated with an average gradient, which if not indicated in the Table, requires using a safer distance by taking the value "to the right". As Student 2's final comment indicates his initial belief that a gradient of 1 in 400 somehow requires doubling values associated with a gradient of 1 in 200, is deeply entrenched.

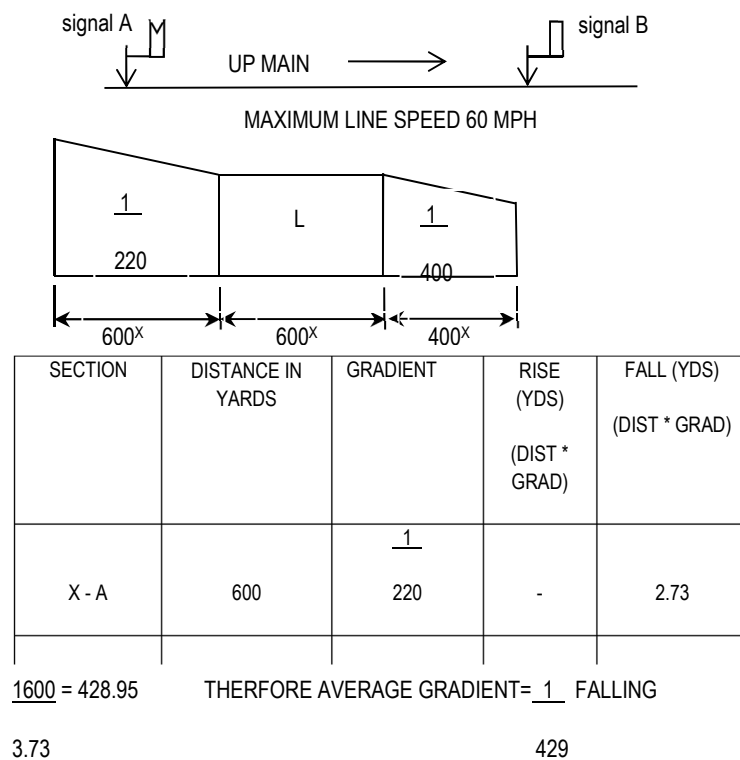


Figure 6. Training example calculation for average gradient

Although this example shows considerable and well organised detail of how to calculate an average gradient for a length of track, which includes three different gradients, the students struggled to make sense of this.

Researcher: Yes... So can you just explain what's going on in there [indicating the Table in Figure 6]?

Student 2: ... used different gradients for each slope and he's averaged it out...

R: yes can you sort of explain the detail ...

S2: you started adding them together --- adding the gradients together and divide by two.

R: Perhaps if we describe what each column is doing

Student 2 appears to associate finding an average with the school mathematics procedure of “adding the values together and dividing by the number of values” – presumably in this case discarding the level gradient. The ensuing discussion was lengthy even requiring explanation by myself of the basic concept of gradient, by drawing diagrams that illustrated, “for every 220 it goes along, it comes down 1, so when you’ve gone along 220 it’s come down 1”. Of course the worker’s familiarity with the procedure causes him no problems with his calculations, but coming to understand this for the first time is clearly demanding for the students.

Analysis and discussion

The Case Study illustrated here allows us to identify some general aspects of workplace mathematical activity and the difficulties that outsiders have in making sense of this. A CHAT analysis of this particular workplace allows us to consider the mediating influences of community, division of labour (note the hierarchy of workers and their activity as described by Alan) and rules and norms (importantly associated with safety considerations). However, with our focus on the activity of Alan as an individual, perhaps what is most prominent here are the ‘instruments’ that Alan uses and which to some extent mask the mathematical understanding upon which his activity is based. Consider, for example, the Table which he uses without thinking to find the appropriate distance between signals: this hides an understanding of the physical principle (and experience!) that a steeper gradient will result in gravity slowing down the train. To Alan this requires little, if any, thought with mathematical understanding having become transparent to him; however, as we have seen this proves a major obstacle to the students in their attempts to understand or re-create the mathematics.

It is clear that as outsiders we needed to have a flexible attitude to the way mathematics is used, and perhaps specially constructed, by workers such as Alan and his colleagues. The students appeared to be not well prepared to do this; for example, Student 2, on a number of occasions, was attempting to bridge to the college mathematics with which he is familiar but was only able to grasp for procedures which he was unable to adapt successfully.

I would like to turn now to consider Alan’s activity from a mathematical modelling perspective and how this might inform us about the type of activity we should include in our college mathematics curriculum that would support such workers, and perhaps more importantly how a worker coming to do such a job for the first time might be best prepared.

Alan works with a mathematical model of reality, that of average gradient: however, he does not build this from scratch, rather it is “industry standard” with important meaning in the workplace. Although, as he acknowledged, his calculations could be automated by the programming of a spreadsheet, Alan chooses each time to carry out the calculation from scratch using the agreed procedure. To reach the final answer to his problem he uses the results of his calculation of average gradient to look up the spacing between signals in a table and if values for the gradient he has calculated are not given here he uses the closest value given that is a safer case (that is, he chooses the value “to the right” of where the calculated gradient lies). To Alan this has become procedural; however, he does have a deep

understanding of the context which the mathematics models. Indeed, large tracts of the transcript of our workplace visit are of Alan explaining in great detail how what we saw in his office relates to actual track on the ground. Alan's activity might therefore be considered as *application* of mathematics in the terms discussed earlier.

However, as the later episodes, in which I explored the workplace mathematics with students perhaps indicates, it is important when attempting to make sense of such "industry standard" models that one has sufficient workplace contextual knowledge available as well as a flexibility in the adaptation of mathematics to new situations. This flexibility can be enhanced by having available a range of strategies on which to draw when attempting to make sense of established mathematics; for example we have found the ability to consider simple cases or extreme values useful (for further discussion of this see Wake & Williams, 2003).

This example suggests that workers do not always have freedom in how they set up their mathematical models, rather that their day-to-day activity centres on:

- organising numerical data in such a way that they can arrive at, or use, a model that is familiar or "standard" in their industry;
- interpreting the results of their *application* of such models in terms of the rich and complex context of the workplace.

It would therefore seem that the process of mathematical modelling has the potential of substantively informing how we might better organise and describe appropriate mathematical process within school and college curricula. At this stage I suggest that there is the need for further research and analysis of workplace activity from this mathematical modelling standpoint.

References

- Engestrom, Y. and Cole, M.: 1997, *Situated cognition in search of an agenda*, in Whitson, J.A. and Kirshner, D., (Eds.), *Situated cognition: social, semiotic, and psychological perspectives*, Lawrence Erlbaum Associates, Hillsdale, N.J.
- Kent, P., Hoyle C., Noss R. & Guile, D. (2004), *Techno-mathematical Literacies in Workplace Activity*, Paper presented to International Seminar on Learning and Technology at Work, Institute of Education, London (www.lonklab.ac.uk/kscope/ltw/seminar.htm)
- Lamon, S.J., Parker, W. A., and Houston, S.K. (2003). *Mathematical modelling: a way of life*. Chichester, U.K: Horwood Publishing.
- Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life*. Cambridge, UK: Cambridge University Press.
- Niss, M. (1989) Aims and Scope of Applications and Modelling in Mathematics Curricula, in Blum, W., Berry J. S., Biehler, R., Huntley, I. D., Kaiser-Messmer, G. & Profke, L. (Eds), *Applications and Modelling in Learning and Teaching Mathematics* (pp.22-31). Chichester, U.K.: Ellis-Horwood Publishing.
- Pozzi, S., Noss, R. and Hoyle, C. (1998), Tools in Practice, Mathematics in Use, *Educational Studies in Mathematics*, 36, 105-122.
- Roberts, G. (2002). *SET for success The supply of people with science, technology, engineering and mathematics skills*. HM Stationery Office, London.
- Smith, A (2004). *Making Mathematics Count*, HM Stationery Office, London.
- Strässer, R. (2003). Mathematics at Work: Adults and Artefacts. In J. Maasz & W. Schloeglmann, *Learning mathematics to live and work in our world, proceedings of the 10th International Conference on Adults Learning Mathematics* (pp.30-37) Austria, Linz.
- Wake, G. D. & Williams, J. S. (2000). Mathematics in pre-vocational courses. In A. Bessot & J. Ridgeway (Eds.), *Education for Mathematics in the workplace*(pp. 167-180). Dordrecht: Kluwer.

- Wake, G.D. & Williams J.S. (2001). *Using College Mathematics in Understanding Workplace Practice* Manchester: University of Manchester.
- Wake, G.D. and Williams, J.S., (2003) Using Workplace Practice to Inform Curriculum Design, in S.J. Lamon, W.A. Parker & S.K.Houston (Eds.), *Mathematical modelling a way of life* (pp.189-199). Chichester, U.K.: Horwood Publishing.
- Wenger, E.: 1998, *Communities of practice: learning, meaning, and identity*, Cambridge University Press, Cambridge.
- Williams, J.S., Wake G.D. & Jervis, A. (1999). General mathematical competence: A new way of describing and assessing a mathematics curriculum, in C. Hoyles, C. Morgan & G. Woodhouse (Eds.) *Rethinking the Mathematics Curriculum* (pp.90-103). London: Falmer Press.

Numeracy for Nursing: The case for a benchmark

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Abstract

In this paper we outline and share examples of our work in progress on an inter-disciplinary project to develop a benchmark for a key aspect of numeracy for nursing, 'Medication Dosage Calculation: Benchmark Assessment for Nursing', funded by NHS Education Scotland (NES). The project is exploring the key issues associated with determining the achievement of competence in nursing numeracy. It provides a real opportunity to establish a UK (and potentially also international) benchmark for nursing competence in numeracy at point of registration.

Numeracy is a key skill for professional practice in nursing (Hutton, 1997). From September 2008 the body regulating the profession in the UK (the Nursing and Midwifery Council, NMC) will require nursing students to achieve 100% in a test of numeracy in practice (NMC, 2007a) before they will be allowed to register as nurses, yet there are currently no national standards for teaching and/or assessment of numeracy during pre-registration nurse training. Without such a standard, the measure of numerical competence is:

... in the eye of the recipient of evidence of that competence, be it Higher Education Institutions, Regulators, Employers or Service Users. (Hutton, 2004)

Medication errors are an aspect of clinical governance which has been highlighted recently by the National Patient Safety Agency (NPSA) and this issue is currently targeted for remedial action (NPSA, 2006a, 2006b). The number of injuries and deaths that can be attributed to medication errors in the NHS is unknown but 9% of incidents reported to the NPSA in its pilot data audit involved medicines (NPSA, 2003); this is consistent with historical data but since calculation error is not separately identified we do not know how many of these were calculation errors. The Department of Health report on *Improving Medication Safety* (Smith, 2004) put some of the blame for medication errors on inadequacies in the education and training of both doctors and nurses. In Scotland, two recent reports, *Learning from Experience* (SEHD, 2003) and *Safe Today, Safer Tomorrow* (NHS/QIS,

2006), have focused on patient safety and risk management, including medication error.¹⁸

The NPSA, in partnership with the *British Medical Journal* (BMJ) Publishing Group, has provided a standardised on-line educational package for junior doctors (NPSA, 2006a) but as yet nothing has been instigated for nurses.

While medication errors are multi-factorial, lack of competence in numerical calculation is often cited as a key area of concern, especially with respect to medication dosage calculation (Weeks, Lyne, & Torrance, 2000). Numeracy in the healthcare context is much broader than medication calculation, but this is its most visible and commonly cited example, and greater consistency in assessment would strengthen support for enhancement of learning and teaching approaches.

Employers are charged with reducing error rates year on year (Department of Health, 2001) and new nurses can expect employers to include mathematics assessment as part of the interview process and within their employment. Anecdotal evidence suggests that many NHS employers are not confident that newly qualified nurses have the numeracy skills required for safe practice and some employers impose their own tests of numerical competency when selecting for Staff Nurse posts. These locally constructed tests are in all probability neither reliable nor valid and *ad hoc* testing of this nature is unlikely to provide assurance of competence. Service Managers and Practice Development Leads have also identified numeracy development needs amongst registered staff, but, in the absence of a benchmark, or diagnostic assessment, it is often difficult to determine which skills require development, or when competence has been achieved.

A recent Commission for Healthcare Audit and Inspection report, *The Best Medicine. The management of medicines in acute and specialist trusts*¹⁹ made the following recommendations, which would go some way to addressing this situation:

We recommend that the National Prescribing Centre lead a national exercise to identify existing best practice and then develop tools to test the medicines-related competency of staff in areas of high risk. These tools should be suitable for assessing all professional groups involved in prescribing and handling medicines. Trusts should also identify areas of medicine that are not adequately covered on training courses for each professional group of staff and implement actions to address deficiencies. (Commission for Healthcare Audit and Inspection, 2007, p.20)

¹⁸ The NPSA has adopted the terminology of the US National Co-ordinating Council for Medication Error Reporting and Prevention: "A medication error is any preventable event that may cause or lead to inappropriate medication use or patient harm while the medication is in the control of health professional, patient or consumer." (Smith, 2004, p.20)

¹⁹ Authorities and trusts are the different types of organizations that run the NHS at a local level.

The Benchmark project

Against this background, following on from a review of relevant literature (Sabin, 2001) the recommendations in the consultation on Healthcare Numeracy (NES Numeracy Working Group, 2006),²⁰ NHS Education for Scotland (NES) has brought together an interdisciplinary group of subject experts from across the UK to explore the key issues associated with determining achievement of competence in nursing numeracy. The project is entitled ‘Medication Dosage Calculation: Benchmark Assessment for Nursing’. We are focusing on medication-related calculations because this is the most common exemplar for nursing numeracy. Furthermore, incorrect calculation of medication dosage can result in harm to patients and to the reputation of the profession.

The NMC requires universities to judge nursing students’ mathematics ability at entry and at registration (NMC, 2004) and more recently, also at entry to branch (Adult, Child, Mental Health, and Learning Disability) (NMC, 2007c). We are focusing on proposing the establishment of a benchmark at entry to register because this is the point at which students become registered nurses, responsible for their professional practice within the NMC’s regulatory framework.

Regulation: Nursing numeracy as an element of fitness to practise, fitness for purpose and fitness for award

The NMC has a clear and specific remit of public protection through professional regulation and uses its statutory control over the approval and monitoring of nursing education to ensure that appropriate professional standards are fostered amongst new practitioners and that achievement is determined through robust assessment. To that end, the NMC has established key competencies to be achieved at the end of the first ‘common foundation year’ at ‘entry to branch’ and at the end of the programme, on ‘entry to the register’. Further, in identifying that entrants to pre-registration programmes require a foundation of literacy and numeracy skills from which to develop, for example, proficiency in communication and drug calculation skills relevant to professional requirements, the NMC has also set minimum entry requirements which must be met before applicants can begin their professional education. These requirements are outlined in the following sections.

Numeracy requirements on entry to pre-registration nursing programmes

²⁰ NES recommended:

1. A standardised suite of nationally recognised and validated Healthcare Numeracy provision that demonstrates sound academic progression and vocational relevance.
 2. Development of national guidance and practice placement standards regarding numeracy learning.
 3. Closer collaboration between Higher Education, Further Education and the service.
 4. National on-line resources for learning, teaching and assessment of healthcare numeracy.
 5. Development of a national multi-disciplinary benchmark assessment designed to determine competence achievement at the point of registration.
- Once developed, the standard could be used as a benchmark for employment purposes.

The NMC identifies standards for both literacy and numeracy as specific components within their general entry requirements (NMC, 2004). In providing guidance to support this, the NMC suggest that evidence of literacy and numeracy may be deduced from academic or vocational qualifications, through evidence of key/core skills abilities, or through the approved educational institutions' own processes, which may include portfolios or tests for those without formal qualifications. NMC also states that educational institutions are additionally entitled to set their own specific educational entry requirements, which may be at a higher level than those required by the NMC. It is acknowledged that it would be regarded as best practice if all key stakeholders (educators, students, regulators, employers and the public) could agree on educational entry requirements for nursing. In the absence of such an agreement, the NMC regulatory standards place the onus for determining suitability for entry to programmes on programme providers (i.e., the Higher Education Institutions which provide pre- registration nurse education programmes) (NES Numeracy Working Group, 2006).

Numeracy requirements within pre-registration nursing programmes: the assessment of health related numerical competence in the Essential Skills Clusters

Since the publication of the NMC *Standards of Proficiency for Pre-Registration Nursing Education Programmes* (NMC, 2004) and in response to concerns raised by registrants and within the media, the NMC has sought to strengthen the process by which the achievement of specific skills is determined at the point of registration. Recognising that skills are often not demonstrated in isolation but as composite activities, the NMC has characterised these as 'Essential Skills Clusters' (ESCs) and has consulted with UK stakeholders regarding their development (NMC, 2006). This consultation process culminated in the publication of NMC Circular 07/2007 (NMC, 2007c) which specifies the UK-wide generic skills (in 'care and compassion', 'communication', 'organisational aspects of care', 'infection prevention and control', 'nutrition and fluid maintenance', and 'medicines management') required to support the achievement of the existing NMC outcomes for entry to branch and the proficiencies for entry to the register. The ESCs are mandatory within the curriculum with effect from September 2008, but their interpretation is left to individual Higher Education Institutions (HEIs), as the NMC states:

- Essential Skills Clusters must be integrated into existing approved pre- registration programmes in a way that they remain visible and identifiable. It will be for programme providers to determine how these are used, incorporated and assessed.
- Providers approving new programmes (or re-approving existing programmes) must demonstrate the integration of ESCs into the curriculum and how they are to be used, incorporated and assessed. The application of the ESCs will be evaluated through ongoing NMC quality monitoring. (NMC, 2007b, p.3)

The requirements for pre-registration nursing programmes specify that at entry to branch, under the domain of 'care management', nursing students are required to demonstrate achievement of all 'common foundation' outcomes. These include demonstration of the "literacy, numeracy and computer skills needed to record, enter, store, retrieve and organise data essential for care delivery" (NMC, 2004, p.3).

Whilst there would appear to be an axiomatic relationship between numeracy achievement at the earlier stages (pre-programme and mid-programme) and achievement at point of entry to the register (award), there is presumably still the opportunity to improve skills from a low

base during the programme of preparation. Therefore, the NMC's focus on public protection through regulation places the determination of numerical competence at point of entry to the register as the key measurement point for achievement of competencies, including numeracy, since lack of achievement at this stage would undermine the assurance which registration provides. At entry to register all nursing staff are required to demonstrate the achievement of all professional competencies in order to show their fitness to practise and fitness for award.

'Health related numerical assessments' are specified in the ESCs set out in NMC Circular 07/2007 (NMC, 2007c) and mapped against the Standards (NMC, 2007d). Annexe 1 to the Circular sets out the requirements for health related numerical assessments as follows:

Summative health related numerical assessments are required to test skills identified (*) within the ESCs that encompass baseline assessment and calculations associated with medicines, nutrition, fluids and other areas requiring the use of numbers relevant to the field of practice:

- For entry to the branch, programme providers will use the ESCs to inform the nature and content of the assessment, including whether to assess through simulation. They will determine their own pass mark and number of attempts.
- For entry to the register, programme providers will use the ESCs to inform the nature and content of numerical assessment in the branch programme where a 100% pass mark is required and all assessment must take place in the practice setting. The number of attempts is to be determined by the education provider. (NMC, 2007b, pp.2-3)

The competencies required are set out in a series of statements in Annexe 2 to the Circular (NMC, 2007a). For example, under the ESC 'Medicines Management', it is stated that patients or clients "can trust a newly registered nurse to correctly and safely undertake medicines calculations" as follows: at entry to branch, s/he "Is competent in basic medicines calculations"; at entry to the register, s/he "Accurately calculates medicines frequently encountered within Branch". The associated "Indicative Content" is set out as: "Numeracy skills, drug calculations required to administer medicines safely via appropriate routes in Branch including specific requirements for children and other groups" (NMC, 2007a, p.25).

Strengths and limitations of the present regulatory framework with respect to numeracy

The increased level of specification of health related numerical assessments embodied in the new Essential Skills Clusters is welcome, as is the inclusion of 'nutrition, fluids and other areas requiring the use of numbers relevant to the field of practice' as well as medicines calculation. The focus upon medicines calculation within the ESCs reflects both its visibility as a form of embedded 'nursing numeracy' and its prominent media profile in relation to risk and public safety. In characterising the assessment of numeracy in terms of practice-based medication calculation, the NMC appears to be seeking to address the concerns of both those who view numeracy as entirely contextualised, and those who view it as a core skill *per se*, albeit in a nursing guise. The focus upon 'drugs frequently encountered' and 'via appropriate routes in the field of practice' is important, since this competence will apply across nursing branches as diverse as Mental Health and Children's Nursing. However, despite these improvements, the present dispensation has some fundamental limitations.

Firstly, health related numerical competence is not defined (although it is exemplified) by the NMC, an unfortunate omission given that numeracy is known to be a slippery and contested concept (Coben et al., 2003, p.9).

Secondly, the NMC acknowledges that: “The ESC skills statements have not been pre- tested for reliability or validity and may not be suitable for directly assessing competence”.

Programme providers are accordingly required “to determine and demonstrate how these are integrated, applied and assessed within local assessment schemes” (NMC, 2007b, p.3). This is problematic with respect to numeracy since we know that:

the assessment of calculation ability has either been based upon an assumption that the results of medication-calculation tests will correlate neatly with actual ability in clinical practice, or that observation and assessment of episodes of clinical performance will be able to infer the required level of underpinning knowledge. Fixed-point competence assessment is unlikely, in itself, to be any more useful than an individual test score at encouraging or determining the creative application of calculation skills in practice. The literature suggests that neither approach has, in isolation, been able to identify or support students and practitioners who struggle with this area of practice. (Sabin, 2001, pp.7-8)

Thirdly, the NMC entry requirements allow for a determination of suitable numerical competence to be made from a number of forms of evidence, and individual Higher Education Institutions may apply different criteria in satisfying themselves about a student’s numerical aptitude on entry. This has already led to wide variations in the interpretation and application of the NMC Standards, potentially based more upon student supply and demand than outcome analysis.

Crucially, the NMC requirement that nursing students must achieve a 100% pass in a test of drug calculations in practice in order to register as nurses is meaningless unless we know what is being assessed and to what standard. Uncontrolled testing will lead to each Higher Education Institution or practice area developing its own test, with no measures of reliability or validity; standards will be variable in the extreme. Furthermore, practice settings simply cannot guarantee that the student is exposed to the full range of medication dosage problems, for example, involving calculations of sub-, multiple- and unit-dose. We have to ask: 100% of what?

A benchmark assessment solution

Providing an authentic, but safe environment in which students may both practise and be tested on the skills required, and in which support for the development of these skills is incorporated, would seem to be a better way forward, with these skills specified in a benchmark assessment. Confidence that the nurses’ educational programmes will equip them with the necessary competence for safe medications dosage calculation should be the key factor.

The benefits of establishing a suitable national benchmark are considerable. A robust statement of the competence in numeracy required of nurses would give programme providers a clear standard at which to aim. A benchmark standard of numeracy incorporated into the final assessment would ensure that nurse education was fit for purpose. Such a

benchmark assessment would allow providers and nursing students to measure their progress towards the standard required for safe and effective practice as they progress from entry to branch to entry to register. A benchmark would also assure employers of registered nurses' competence with respect to numeracy and obviate the need for repeated testing of applicants for nursing jobs and nurses in service. If a specific numeracy competence standard were to be established at 'point of entry to the register', and robust tools developed to measure its achievement, then the relationship between that standard and levels of achievement, which might precede it at 'entry to branch' or 'entry to programme' could be determined and other benchmark measures subsequently established. Thus 'likelihood of success at entry to the register' could be calculated relative to 'level of achievement at point of entry', and this coefficient used to guide supportive intervention.

This further suggests that a test of ability to calculate drug dosages competently by the end of 'training' should be the culmination of a programme of education and formative assessment with suitable feedback to aid development, which begins at entry to the programme and is continuous throughout the three years of training.

How students are assessed against the standard and prepared for an assessment are also essential aspects of the benchmarking process and establishing a benchmark assessment would allow education providers to test out different approaches to supporting numeracy learning and teaching.

In the next section, we consider what such a benchmark assessment might look like and the principles that should inform its development.

Principles of adult learning and teaching

The teaching and assessment of numeracy for nursing should be consistent with research-based principles of adult learning and teaching in which we draw on constructivist and socio-cultural approaches (Coben, 2000; Tusting & Barton, 2006). These principles we characterise as follows:

- Learning is a purposeful, goal-directed activity building on prior knowledge and experience to shape and construct new knowledge and a social activity embedded in a particular culture and context.
- Effective learning requires that the learner understand not only the facts but the underlying principles, patterns and relationships acquired through the application of knowledge^{9,21}.

²¹ The assumption of the 'application' of knowledge is problematic. Engeström (Engeström, 2001) and others (FitzSimons & Coben, 2007; Kanis, 2003) have argued that knowledge is not simply 'applied', instead, locally 'new' knowledge is developed in ever-evolving workplace situations, drawing upon what has previously been learned but in creative ways appropriate to the context and situation at hand (Gail FitzSimons, private communication, 19 Sept., 2007).

- Knowing when and how to apply what has been learned (procedural knowledge) is central to expertise, and can be acquired only through practice in an authentic environment.
- Teaching involves informed interpretations of, and responses to, learners' approaches to learning.
- Metacognitive strategies can be taught.
- Scaffolding instruction helps learners to develop their fluency, independence and range as they move from being a new learner to becoming an expert learner.

A modern conception of numeracy

Modern conceptions of numeracy stress the importance of effective use, not just knowledge and skills, and purpose, in making sense of use. The situatedness of numeracy, shaping its use and purpose, is stressed, as is critical engagement on the part of the numeracy 'agent', in this case, the nurse or student nurse (Coben et al., 2003; Condelli et al., 2006). With this in mind, we have adopted the following working definition of what it means to be numerate:

To be numerate means to be competent, confident, and comfortable with one's judgements on *whether* to use mathematics in a particular situation and if so, *what* mathematics to use, *how* to do it, *what degree of accuracy is appropriate*, and *what the answer means in relation to the context*. (Coben, 2000, p.35, *emphasis in the original*)

Teaching, learning and assessing numeracy for nursing

Integrating numeracy into healthcare education and training in order to support trainees before they join the profession presents challenges in that nursing lecturers may not recognize the numeracy in nursing or know how to teach it. At the same time, adult numeracy tutors are unlikely to know the numeracy requirements of the healthcare context; there is in any case a low base in terms of the numbers of trained, experienced adult numeracy tutors and numeracy has until recently been somewhat overshadowed by literacy in the Scottish 'adult literacies' field (Coben, 2005). We suggest that a team approach, with healthcare professionals and adult numeracy specialists working together on contextualised numeracy teaching and assessment, may offer a way forward, alongside the development of training, continuing professional development (CPD), teaching, learning and materials geared to healthcare professionals' numeracy needs.

Given the continued focus upon widening entry to the healthcare professions, and the impetus for adopting non-traditional routes, it has been argued that demanding minimum levels of numeracy, as determined by school qualifications, would impact greatly on recruitment, and indeed, could be prejudicial to many potentially competent staff (Hutton, 1998). Nonetheless, recent NMC consultation work on general entry requirements has focused upon a review of whether further guidance should be provided on minimum entry criteria, including numeracy standards. We suggest that a *formative* testing of numeracy skills at entry to programme would provide a baseline for developing competence. There is firm evidence that formative assessment improves learning (Black & Wiliam, 1998).

Nursing numeracy is strongly situated in the nursing/healthcare context and, as Williams and Wake point out, mathematical processes tend to become crystallised in ‘black boxes’ shaped by workplace cultures, with instruments, rules and divisions of labour tending to disguise or hide mathematics. They argue that training programmes need to be better aligned with the needs of students (Williams & Wake, 2007). In particular, nurses’ skills need to be sufficiently robust to cope with the stress, anxiety and time pressures of nursing, given that nursing is safety-critical activity: they need to be numerate under pressure.

Teaching and assessment for numeracy in nursing should be able to generate: independence; good critical judgment (e.g., on how accurate to be, when to estimate, within what tolerance and why); proficiency in practice; and accountability to relevant stakeholders. Numeracy for nursing must have high use value *and* high exchange value (Coben, 2006). It must be integrative, i.e., it must incorporate the mathematical, cultural, social, emotional and personal aspects of each individual in a particular context (Maguire & O'Donoghue, 2003). Assessment for numeracy in nursing must be formative (for effective learning) and summative (for accountability)²² (Wiliam & Black, 1996).

The nursing profession must recognise the relative importance of numeracy in the whole context of practice in order to ensure safety and quality in such key aspects of nursing practice as: drug administration and prescribing; fluid balance calculation; support for patients’ nutritional needs; calculation of intravenous fluid requirements/rates; calculations related to weight and body mass index (BMI); nursing administration; plotting and recording data; and understanding relevant research and evidence.

Good practice principles for numeracy benchmark assessment

We have developed the following research-based criteria for the features of an effective numeracy benchmark assessment tool. Such a tool should be reliable, valid and capable of recreating the complexity of nursing numeracy in an authentic assessment environment. Specifically, it should be:

- *Realistic*: Evidence-based literature in the field of nursing numeracy (Hutton, 1997; Weeks, 2001) strongly supports a realistic approach to the teaching and learning of calculation skills, which in turn deserve to be tested in an authentic environment. Questions should be derived from authentic settings. A computer based programme of simulated practice in drug calculations, formative testing, with feedback on the nature of errors made, has been shown to develop competency in medication dosage calculation, which can be also demonstrated in the clinical areas (Weeks, Lyne, & Torrance, 2000). Exposure of students to real- world situations is recommended (Weeks, 2001).
- *Appropriate*: The assessment tool should determine competence in the key elements of the required competence (OECD, 2005; Sabin, 2001).

²² “An assessment is defined as serving a formative function when it elicits evidence that yields construct-referenced interpretations that form the basis for successful action in improving performance, whereas summative functions prioritise the consistency of meanings across contexts and individuals.” (Wiliam & Black, 1996, p.537).

- *Differentiated*: There should be an element of differentiation between the requirements for each of the branches of nursing (Hutton, 1997).
- *Consistent with adult numeracy principles*: The assessment should be consistent with the principles of adult numeracy learning teaching and assessment, having an enablement focus (Coben, 2000).
- *Diagnostic*: The assessment tool should provide a diagnostic element, identifying which area of competence has been achieved, and which requires further intervention (Black & Wiliam, 1998). Thus it should “provide information to be used by students and teachers that is used to modify the teaching and learning activities in which they are engaged in order better to meet student needs. In other words, assessment is used formatively to ‘keep learning on track’”. (Wiliam, 2006).
- *Transparent*: The assessment should be able to demonstrate a clear relationship between ‘test’ achievement and performance in the practice context (Weeks, Lyne, Mosely, & Torrance, 2001).
- *Well-structured*: The assessment tool should provide:
 - a unique set of questions with a consistent level of difficulty;
 - a structured range of complexity; and
 - the assessment should take place within a defined framework, at points by which students can be effectively prepared, while allowing time for supportive remediation. (Hodgen & Wiliam, 2006)
- *Easy to administer*: the assessment should provide the opportunity for rapid collation of results, error determination, diagnosis and feedback (Black & Wiliam, 1998).

Examples of assessment items

Examples of items from the exemplar assessment tool we have developed for this project are given below (*Figure 1*). The tool has been designed by Keith Weeks and Norman Woolley as part of the Authentic World® program. This program is based on a constructivist-centred design drawn from the work of Piaget (1983), Bruner (1975), and von Glasersfeld (1987). Its aim is to facilitate the students’:


- visualisation or ‘seeing’ of the elements of the dosage calculation problem as manifested in clinical practice;
- mapping of these elements onto the word-based formulae and number-based equations used to solve the problem.


The constructivist education process forms part of a cognitive apprenticeship framework (Collins, Brown, & Newman, 1990). This framework:

- models the expert problem solving processes that may be obscured from students in the classroom and clinical settings;

- facilitates authentic diagnostic assessment of a student's dosage calculation ability using interactive representations of the syringes, etc., used to measure and administer medication dosages in clinical practice. (Weeks, 2007)

As a first step, we shall pilot the exemplar assessment tool to determine its validity and reliability.





User Name **nefs07a102**

Institute **NHS Education for Scotland**

[Log Out](#)

[Back to Menu](#)

Question 5 of 30

Extract the relevant information from the prescription chart and the medication product label. Click NEXT when you are ready to administer the dose.

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REGULAR PRESCRIPTION MEDICINES			DATE
MEDICINE (Approved Name)		START DATE	ADMINISTRATION TIMES
Furosemide		18/6/2007	06:00
DOSE	ROUTE	SPECIAL INSTRUCTIONS	
40mg	ORAL	-	-
DOCTOR'S SIGNATURE		PHARMACY SUPPLY	
Dr. Jones		A. Mann	



NEXT

Authentic  World

User Name

nefs07a102

Institute

NHS Education for Scotland

Log Out

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Question 5 of 30

Drag the correct dosage into the medicine pot, then click DONE.

1

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BACK

DONE

Figure 1: Examples of online assessment of numeracy for nursing from Authentic World®

Conclusion

We suggest that a benchmark for nursing numeracy urgently needs to be established in order to ensure consistency across education providers in meeting the requirements of all stakeholders, be they providers of education, the regulator, employers or the students themselves.

The benchmark needs to consider the levels of numeracy competence identified above and to include a strong element of process as well as outcome, based on available research evidence.

We are developing such a benchmark, together with its associated assessment tool, in our project for NHS Education Scotland, outlined above. To this end, we are working collaboratively with stakeholders across the education and service sector in Scotland to pilot items on medication dosage calculation. In the process, we are also interrogating and developing our theoretical understanding of numeracy learning and assessment for nursing.

This work moves NHS Scotland to the forefront of the nursing numeracy agenda and provides a real opportunity to establish a UK (and potentially also an international) benchmark for nursing competence in drug calculation at point of registration. Ultimately, such benchmarking is a matter for agreement by the profession; the Healthcare Commission's recommendations, if implemented, may expedite this at a multidisciplinary team (MDT) level. We hope our work will inform the debate and we look forward to reporting on the progress of the project at future conferences.

Acknowledgement

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References

- Black, P., & Wiliam, D. (1998). Inside the Black Box: Raising standards through classroom assessment. *Phi Delta Kappan*, 80(2), 139-148.
- Bruner, J. S. (1975). *Toward a Theory of Instruction*. Cambridge: Belknap/Harvard.
- Coben, D. (2000). Numeracy, mathematics and adult learning. In I. Gal (Ed.), *Adult Numeracy Development: Theory, research, practice* (pp. 33-50). Cresskill, NJ: Hampton Press.
- Coben, D. (2005). *Adult Numeracy: Shifting the focus. A Report and Recommendations on Adult Numeracy in Scotland*. Edinburgh: Learning Connections, Scottish Executive.
- Coben, D. (2006). What is specific about research in adult numeracy and mathematics education? *Adults Learning Mathematics - An International Journal*, 2(1), 18- 32.
- Coben, D., Colwell, D., Macrae, S., Boaler, J., Brown, M., & Rhodes, V. (2003). *Adult Numeracy: Review of research and related literature*. London: National Research and Development Centre for Adult Literacy and Numeracy (NRDC).
- Collins, A., Brown, J. S., & Newman, S. E. (1990). Cognitive apprenticeship: Teaching the crafts of reading, writing, and mathematics. In L. B. Resnick (Ed.), *Knowing, Learning, and Instruction: Essays in honor of Robert Glaser* (pp. 453-494). Hillsdale, NJ: Lawrence Erlbaum.
- Commission for Healthcare Audit and Inspection. (2007). *The Best Medicine. The management of medicines in acute and specialist trusts*. London: Commission for Healthcare Audit and Inspection
- Condelli, L., Safford-Ramus, K., Sherman, R., Coben, D., Gal, I., & Hector-Mason, A. (2006). *A Review of the Literature in Adult Numeracy: Research and conceptual issues*. Washington, DC: American Institutes for Research.
- Department of Health. (2001). *Building a Safer NHS for Patients. Implementing an organisation with a memory*. London: Department of Health.
- Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. *Journal of Education and Work*, 14(1), 133-156.
- FitzSimons, G. E., & Coben, D. (2007). Adult numeracy for work and life: Curriculum and teaching implications of recent research. In R. Maclean & D. Wilson (Eds.), *UNESCO-UNEVOC International Handbook of Technical and Vocational Education and Training. Bridging Academic and Vocational Education*. Dordrecht: Springer.
- Hodgen, J., & Wiliam, D. (2006). *Mathematics Inside the Black Box: Assessment for learning in the mathematics classroom*. London: nferNelson.
- Hutton, B. M. (1997). *The Acquisition of Competency in Nursing Mathematics*. Unpublished PhD, University of Birmingham, Birmingham.
- Hutton, B. M. (1998). Do school qualifications predict competency in nursing calculations? *Nurse Education Today*, 18(1), 25-31.
- Hutton, B. M. (2004). What do we mean by competency in calculation? Paper presented at the Conference *A Calculated Risk: Numeracy needs in Healthcare*, Murrayfield Stadium, Edinburgh, 19th November, 2004. Retrieved 8 October, 2007. from <http://www.nes.scot.nhs.uk/multi/numeracy/documents/Meriel.ppt>
- Kanes, C. (2003). Strategies for developing flexible learning. In J. Stevenson (Ed.), *Developing Vocational Expertise*. Sydney: Allen & Unwin.
- Maguire, T., & O'Donoghue, J. (2003). Numeracy concept sophistication - an organizing framework, a useful thinking tool. In J. Maaß & W. Schölglmann (Eds.), *Learning Mathematics to Live and Work in Our*

- World. ALM-10. *Proceedings of the 10th International Conference on Adults Learning Mathematics in Strobl (Austria)* (pp. 154-161). Linz, Austria: ALM and Johannes Kepler Universität Linz.
- NES Numeracy Working Group. (2006). *Identifying and Supporting the Numeracy Needs of Healthcare Staff in Scotland (Consultation)*. Edinburgh: NHS Education Scotland (NES).
- NHS/QIS. (2006). *Safe Today - Safer Tomorrow. Patient Safety: Review of Incident and Near-Miss Reporting. Full Report*. Edinburgh: National Health Service Quality Improvement Scotland.
- NMC. (2004). *Standards of Proficiency for Pre-registration Nursing Education*. London: Nursing and Midwifery Council.
- NMC. (2006). *Essential Skills Cluster Consultation: Medicines management (Consultation Document)*. London: Nursing and Midwifery Council.
- NMC. (2007a). *Essential Skills Clusters (ESCs) for Pre-registration Nursing Programmes. Annexe 2 to NMC Circular 07/2007*. London: Nursing and Midwifery Council.
- NMC. (2007b). *Guidance for the Introduction of the Essential Skills Clusters for Pre- registration Nursing Programmes. Annexe 1 to NMC Circular 07/2007*. London: Nursing and Midwifery Council.
- NMC. (2007c). *Introduction of Essential Skills Clusters for Pre-registration Nursing Programmes (pdf)*. Retrieved 8 October, 2007, from <http://www.nmc-uk.org/aArticle.aspx?ArticleID=2558#skills>. London: Nursing and Midwifery Council.
- NPSA. (2003). *A report on the Pilot Data Audit undertaken by the NPSA. In NPSA Business Plan 2003-04*. London: National Patient Safety Agency.
- NPSA. (2006a). *Safe Foundations: Patient safety education modules for junior doctors*. from <http://www.npsa.nhs.uk/press/display?contentId=5186>. London: National Patient Safety Agency.
- NPSA. (2006b). *Safety in Doses: Medication safety incidents in the NHS. The fourth report from the Patient Safety Observatory (No. PSO/4)*. London: National Patient Safety Agency.
- OECD. (2005). *The Definition and Selection of Key Competencies. Executive Summary*. Paris: Organisation for Economic Cooperation and Development.
- Piaget, J. (1983). *Piaget's theory*. In W. Kessen (Ed.), *Handbook of Child Psychology* (Vol. 1). New York: Wiley.
- Sabin, M. (2001). *Competence in practice-based calculation: Issues for nursing education. A critical review of the literature. Occasional Paper 3*. London: Learning and Teaching Support Network (LTSN) Centre for Health Sciences and Practice.
- SEHD. (2003). *Learning from Experience. How To Improve Safety For Patients In Scotland. A Consultation Paper*. Edinburgh: SEHD.
- Smith, J. (2004). *Building a Safer NHS for Patients – Improving medication safety*. London: Department of Health Publications.
- Tusting, K., & Barton, D. (2006). *Models of Adult Learning*. Leicester: NIACE.
- von Glasersfeld, E. (1987). *The Construction of Knowledge: Contributions to Conceptual Semantics*. New York: Intersystems Publications.
- Weeks, K. W. (2001). *Setting a foundation for the development of medication dosage calculation problem solving skills among novice nursing students. The role of constructivist learning approaches and a computer based 'Authentic World' learning environment*. Unpublished PhD, University of Glamorgan.
- Weeks, K. W. (2007). *No more 'chalk and talk': Teaching drug calculation skills for the real world. Safer Healthcare*. Retrieved 26 August, 2007, from <http://www.saferhealthcare.org.uk/IHI/Topics/ManagingChange/SafetyStories/teaching+drug+calculation+skills+for+the+real+world.htm>
- Weeks, K. W., Lyne, P., Mosely, L., & Torrance, C. (2001). *The strive for clinical effectiveness in medication dosage calculation problem solving skills: The role of constructivist theory in the design of a computer-based 'authentic world' learning environment. Clinical Effectiveness in Nursing, 5*, 18-25.
- Weeks, K. W., Lyne, P., & Torrance, C. (2000). *Written drug dosage errors made by students: The threat to clinical effectiveness and the need for a new approach. Clinical Effectiveness in Nursing, 4*, 20-29.
- Wiliam, D. (2006). *Does Assessment Hinder Learning? [Electronic Version]* from http://www.uk.etseurope.org/home-corpo-uk/news-home/?print=1&news=136&view=detail&no_cache=1.
- Wiliam, D., & Black, P. (1996). *Meanings and consequences: A basis for distinguishing formative and summative functions of assessment. British Educational Research Journal, 22(5)*, 537-548.
- Williams, J. S., & Wake, G. D. (2007). *Black boxes in workplace mathematics. Educational Studies in Mathematics, 64(3)*, 317-343.

Numeracy for Nursing: The Scope for International Collaboration

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There is widespread recognition that numeracy is a key skill for safe and effective professional practice in nursing. Yet despite research studies in various countries which reveal a lack of proficiency within both the student population and amongst registered nurses, there is no international consensus on the nature and scope of numeracy for nursing, or on ways of improving the situation. In this paper I present a brief overview of research and development on numeracy for nursing in several countries. I then outline work in progress on two inter-disciplinary research projects in the UK as a way to explore the scope for international collaboration on numeracy for nursing. I conclude that the time appears to be ripe for collaboration on an international comparative study drawing together and carrying forward research and development in numeracy for nursing: We have much to learn from each other.

Introduction

Numeracy for nursing has a “manifest disaster criterion” (Nokes, 1967). Poor numeracy can be life threatening for the patient. Numeracy is implicated in potential and actual disasters (ISMP, 2008) and in safe and effective practice in nursing. Nurses’ numeracy manifestly matters: to patients, to nurses themselves, to their employers, to the public and to nurse educators. However, studies in various countries reveal a lack of proficiency amongst both students and registered nurses (Bindler & Bayne, 1991; Brown, 2006; Grandell-Niemi, Hupli, Puukka, & Leino-Kilpi, 2006; Jukes & Gilchrist, 2006), and efforts to remediate the situation (Johnson & Johnson, 2002; Sandwell & Carson, 2005; Weeks, Lyne, & Torrance, 2000; Wright, 2005). Widespread concern about nurses’ numeracy finds periodic expression in alarming headlines such as “A third of new nurses fail simple English and maths test” (*Daily Telegraph*, 5th August, 2006), and there are examples of efforts to remediate the situation through education and training. However, as yet there is no explicit consensus on the nature and scope of numeracy for nursing, or on ways of improving the situation: Numeracy for nursing is still poorly-understood and under-developed.

In this paper I begin by presenting a brief overview of research and development on numeracy for nursing in several countries. I then outline work in progress on two inter-disciplinary research projects in the UK as a way of exploring the scope for international collaboration on numeracy for nursing.

So What Do I Mean by Numeracy for Nursing?

Numeracy may be considered as a specific competence for nursing, so before proceeding further it is necessary to define what is meant by ‘competence’ and ‘numeracy’ in this context.

The holistic notion of competence set out in the Organisation for Economic Cooperation and Development’s (OECD) report on *The Definition and Selection of Key Competencies* (known as *DeSeCo*) is useful here. *DeSeCo* defines competence as:

the ability to successfully meet complex demands in a particular context. Competent performance or effective action implies the mobilization of knowledge, cognitive and practical skills, as well as social and behavior components such as attitudes, emotions, and values and motivations. (OECD, 2005, p. 2)

There are many competing definitions of numeracy (Coben et al., 2003), some of which, including the following definition, align with the OECD notion of competence:

To be numerate means to be competent, confident, and comfortable with one’s judgements on *whether* to use mathematics in a particular situation and if so, *what* mathematics to use, *how* to do it, *what degree of accuracy is appropriate*, and *what the answer means in relation to the context*. (Coben, 2000b, p. 35, emphasis in the original)

Many aspects of nursing call for competence in numeracy in these terms, including calculations and judgments involved in tasks such as: monitoring patients’ vital signs and fluid balance; measuring weight and height; nutrition, including infant feeding and monitoring the nutrition levels of elderly and frail patients; demographic profiling; the organization of healthcare work, including staffing and budget calculations; and the area that most people think of when considering numeracy or mathematics for nursing: medication dosage calculation (Johnson & Johnson, 2002; Pirie, 1987). A Venn diagram representing medication dosage calculation as an integral element of the intersecting areas of numeracy, healthcare numeracy and medicines management is shown in Figure 1, below; other areas of numeracy for nursing could be visualized in a similar way.

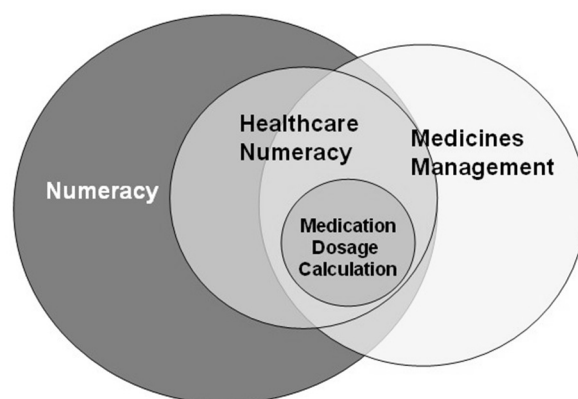


Figure 1. Drug calculation as an aspect of numeracy, healthcare numeracy and medicines management (Sabin et al., 2008).

As nursing practice incorporates the use of new technologies, the nature of numeracy for nursing is changing. In nursing, as in other areas of life, much mathematics is invisible (Coben, 2000a; Noss & Hoyles, 1996; Zevenbergen, 1996). New technology has arguably in some ways ‘demathematised’ some aspects of nursing practice, or at least removed the need for some calculations. For example, the use of personal digital assistants (PDAs) is expanding in nursing, particularly at the point of care (Greenfield, 2007) and medicines may be dispensed in standard dosages; these have the potential to reduce calculation error. However, such new nursing practices also call for numeracy on the part of the nurse: Estimation and checking strategies, for example, are vitally important for judgements as to whether a machine is correctly calibrated, or whether the correct dose of a drug has been prescribed.

Research on Numeracy for Nursing

Research on numeracy for nursing is still a new area despite pioneering work by Susan Pirie and others in the 1980s (Dexter & Applegate, 1980; Pirie, 1987). There is growing interest and research activity, much of it small-scale, such as evaluations of local initiatives. A literature review by Sabin (2001) confirms that competence in mathematical calculation skills required for clinical nursing practice is widely regarded as a pre-requisite to nurse registration. However, many studies find a lack of proficiency amongst both students and registered practitioners. Sabin examines the strength of the evidence linking achievement in calculation tests with subsequent clinical practice, alongside the demographic and cultural variables that may influence learning, teaching and assessment. He reviews the nature and role of mathematics learning within pre-registration nurse education programmes and in clinical practice. He also reviews the value of context in mathematics learning in professional settings and critiques the rationale for outcomes-based approaches to assessment since these may fail to identify, and may potentially stifle, the development and application of knowledge. On the basis of the review, he advocates an integrated approach to facilitating mathematical knowledge and application in practice and makes recommendations for future practice in the UK, as follows:

1. Early identification of individual numeracy skills should be made [...].
2. University-based teaching and learning should employ a range of approaches including workbooks, CAI [computer-assisted instruction], study groups and lectures, identifying and focusing on the key components identified by Pirie (Pirie, 1987) and others.
3. Experiential learning in clinical practice should be supported by linking specific clinical activities with calculation learning and practice.
4. Assessment of ‘competence’ in calculation [...] needs to examine mathematical knowledge acquisition, and its application in practice, rather than a narrow assessment of clinical performance.
5. Numeracy should have the same status as other equally important components of professional practice.

6. The elements of mathematics understanding required to address the evaluation and analysis of clinical and statistical data in research should be integrated into pre-registration programmes; particularly those that result in a degree-level award.
7. Remedial programmes should be supported by university-wide facilitation.
8. Just as the mathematics in nursing practice is not disengaged from its context, neither are the students. Acknowledgement of the role of demographic issues such as age, gender, class and ethnicity in approaches to learning and teaching practice calculation is long overdue.
9. Future studies should be focused towards the development of a standardised, structured framework for learning, teaching and assessment that can be employed across the UK nursing education field.
10. The development of a framework, within which education and service can facilitate and assess practitioners' calculation abilities, is needed.

(Sabin, 2001, pp. 9–10)

The International Picture

Nurse Shortage and Drop-Out from Training

Some examples of research and development published since Sabin's review indicate that numeracy for nursing continues to be an issue of concern in many countries, often against the background of nurse shortages. For example, there is a national shortage of registered nurses (RNs) in the United States of America which is predicted to continue to grow over next twenty years, with a more severe shortage in some States than others. The US Department of Health and Human Resources Health Resources and Services Administration (HRSA) states that:

to meet the projected growth in demand for RN services, the U.S. must graduate approx 90% more nurses from U.S. nursing programs. (HRSA, 2004, p. 10)

There is also concern over drop-out rates in nurse training. For example, amongst Hispanic nursing students in the USA:

National research shows a 30% drop-out rate for Hispanics in nursing programs. This has been directly tied to economic hardship issues that involve the need to support the family and an educational preparation that may not have emphasized the knowledge and skills needed to succeed in the selected program. (Hispanic Times Magazine, 2001)

This situation is reflected in the UK, also: "More than a quarter of the UK's student nurses dropped out of their courses in 2006" (BBC News 9th April, 2008, <http://news.bbc.co.uk/1/hi/health/7337259.stm>). It is not known to what extent numeracy may be a factor in recruitment and retention problems in nurse training but it is likely that it has some bearing on this situation.

Establishing the Extent and Nature of Nursing Students' Numeracy

Research studies aiming to establish the extent and nature of nursing students' numeracy include explorations of the mathematical competencies of baccalaureate degree nursing students in the USA. For example, Allen and Papas (1999) assessed entering nursing students' mathematics competencies and identified and arranged additional customized support for those who needed additional help. Drug dosage calculation is a particular concern and Brown found that a sample of nursing students in the North East region of the USA, even with the use of calculators, were unable to complete a medication examination with at least 85% accuracy within a predetermined time limit. She concluded that although medication dosage calculation errors are cited as one of the most frequently occurring types of error in medication administration, they are seen as one of the most preventable (Brown, 2006).

Improving Nursing Students' Numeracy

Other studies evaluate attempts to improve nurses' and nursing students' competence in numeracy. Numeracy is often built into Nurse Education programmes. For example, in the USA, an evaluation of the efficacy of a teaching strategy in improving beginning nursing student learning outcomes was undertaken by Rainboth and DeMasi (2006). The students who received the intervention had statistically significantly higher scores on the major medication calculation examination than the students in the control group. The intervention group students were satisfied with the method and outcome, a finding that tends to be duplicated elsewhere in evaluations of other such interventions. In Australia an evaluation of an innovation by a teaching team who developed a Safe Administration of Medicines (SAM) website was undertaken by Behrend and colleagues (Behrend, Clark, Hall, & Hill, 2006). The site includes 'Maths for Nurses' learning resources which identify the key numerical concepts and provide an explanation of these concepts, together with examples and clinical quizzes to ensure skills are contextualised in workplace practice <http://www.unisanet.unisa.edu.au/sam/>. Behrend *et al.*'s evaluation of this site found that successful outcomes had been achieved through web-based on-line numeracy learning. At Christchurch Polytechnic Institute of Technology (CPIT) in New Zealand, Dodds has created another online programme of support for numeracy for nursing (Dodds, 2006) [http://www.cpit.ac.nz/subjects/mathematics and statistics/programmes and courses](http://www.cpit.ac.nz/subjects/mathematics_and_statistics/programmes_and_courses) and there are other examples of such programmes from around the world, including the UK-based Authentic World® <http://www.authenticworld.co.uk/>.

In Australia, Galligan and Pigozzo (2002) have researched the process of assisting nursing students to solve drug calculation problems using metacognition and error analysis. They found that "nursing students who have difficulties with drug calculations do demonstrate metacognitive and cognitive skills" however, they have identified gaps in these skills. On the basis of their analysis, they have developed strategies involving planning, predicting and identifying errors, in order to enhance students' problem-solving abilities.

Also in Australia, Gillies (2004) has compared two methods for teaching drug calculation: on the one hand, traditional formula-based teaching methods and on the other hand building on students' existing mathematical problem-solving skills. On the basis of analysis using quantitative measures, the formula-based approach appeared more effective. However, she

found that alternative teaching methods may be more effective in increasing students' confidence and achieving better long-term recall and transfer of skills.

There is concern about equity and inclusion of minority groups. For example, in Finland, the basic mathematical proficiency and the medication calculation skills of graduating nursing students in Finland were studied by Grandell-Niemi and her colleagues. They looked at how students experienced the teaching of medication calculation. They aimed to find out whether these experiences were associated with various background factors and the students' medication calculation skills. They established that the students found it hard to learn mathematics and medication calculation skills. Overall their mathematical skills were inadequate, with one-fifth failing the medication calculation test. There was a positive correlation between students' grades in mathematics before starting nurse training and their skills in medication calculation (Grandell-Niemi, Hupli, & Leino-Kilpi, 2001).

In New Zealand, Gibson-van Marrewijk is conducting a project investigating factors impacting on student completion, retention, and achievement rates for science modules in applied health programmes, with particular attention to Maori students (<http://www.tlri.org.nz/projects/2005/>).

In the UK, recent research includes: analysis of the concept of competence in numeracy for nursing (Hutton & Gardner, 2005); a study of techno-mathematical aspects of pediatric nursing practices (Hoyle, Noss, & Pozzi, 2001); development of local support initiatives and materials (Starkings, 2003); and evaluation of teaching and learning interventions (Hall, Jones, Hilton, Davies, & MacDiarmid, 2005; Wright, 2006).

Approaches to Teaching, Learning and Assessment of Nurse Education

As yet there has been no systematic national (nor yet international) survey of the ways in which nursing students are educated in numeracy. Anecdotally, it would appear that these approaches range along a continuum from the teaching of decontextualised arithmetic, through the teaching of formulae commonly used in nursing, to problem-solving approaches and task-based activities involving simulation of practice, through to fully contextualized approaches situated directly in practice, known colloquially in the UK as 'sitting by Nelly'. In the case of numeracy for nursing, 'sitting by Nelly' involves working alongside an experienced nurse while he or she undertakes tasks involving numeracy. Any given nurse education programme may offer combinations of these approaches, which may be delivered in the classroom, online, or on the ward or other healthcare setting.

A full review of these approaches is outside the scope of this paper but we know that it cannot be assumed that students will readily transfer knowledge from the classroom to the practice situation (Eraut, 2003; Evans, 1999; Guile & Young, 2003). 'Sitting by Nelly' is not necessarily the answer either, since 'Nelly' may have become so habituated to the numeracy required in her role that it has become "common sense" (Coben, 2000a)—that is, self-evident to an experienced worker but not to a novice. An example of the sort of problem that may arise is shown in Figure 2. The experienced nurse (on the right) talks her colleague through a medication dosage calculation: "We need Aminophylline 200 milligrams... It comes as 250 milligrams in 10ml. Therefore we need to give 8ml ... OK?". She probably believes she is

being clear and helpful but her inexperienced colleague has no idea what she is talking about and is too embarrassed to ask. An opportunity for learning has been lost and the experienced nurse may be unaware that there is any problem.

Two current inter-disciplinary research projects in the UK are seeking to address some of the issues outlined above. These are: 'Medication Dosage Calculation: a benchmark assessment for nursing' (NHS).²³

Education Scotland); and 'Numeracy for Nursing'²⁴ (King's College London). Both projects are based on the notions of numeracy as a competence for nursing outlined above. They are being conducted against the background of the new (from September 2008) requirement by the UK Nursing and Midwifery Council (NMC²⁵) for nursing students to achieve 100% in a test of "numeracy in practice" before they will be allowed to register as nurses (NMC, 2004). However, there are currently no national standards for teaching or assessment of numeracy during pre-registration nurse training, so tests are likely to vary in validity and reliability. This means the tests may not be measuring what they are intended to measure (i.e., they may be invalid) in terms of content (domain- or subject-specific); construct (indicating an internal trait, attribute, or process); or criterion (factors that can be related to an observable outcome). Reliability implies freedom from measurement errors and consistency between measurements across time, situations and raters. Reliability is necessary, but not sufficient, for validity. In an area with a manifest disaster criterion such as numeracy for nursing, assessment must be both valid and reliable.

²³ 'Medication Dosage Calculation: A benchmark assessment for nursing' (2007–09). The team was brought together by Mike Sabin, NHS Education for Scotland and Scottish Government and comprises, in addition to Mike Sabin: Dr Keith Weeks and Norman Woolley of University of Glamorgan and Authentic World®; Dr Carol Hall, University of Nottingham; Professor Diana Coben, King's College London; Dr Meriel Hutton, Consultant and Senior Visiting Research Fellow, King's College London; and Dr David Rowe, University of Strathclyde. The project is funded by NHS Education for Scotland (NES) <http://www.nes.scot.nhs.uk/>.

²⁴ 'Numeracy for Nursing at King's College London' (Jan-Sept., 2008). Principal Investigators: Professor Diana Coben, and Dr Jeremy Hodgen, with Sherri Ogston-Tuck and Dr Meriel Hutton; conducted in collaboration with the Florence Nightingale School of Nursing and Midwifery and funded by the Department of Education and Professional Studies Research Committee, King's College London.

²⁵ NMC is the UK regulatory body for nursing and midwifery <http://www.nmc-uk.org/>.



Figure 2. Numeracy in the workplace: nursing.

Two Current Projects on Numeracy for Nursing

Medication Dosage Calculation: A Benchmark Assessment for Nursing

This ongoing interdisciplinary study aims to create a national benchmark for numeracy for nursing in Scotland against which numeracy for nursing may be assessed, initially at point of registration but potentially thereafter at other stages of nurse preparation and in practice. A robust competence benchmark will allow practitioners to demonstrate achievement, universities to demonstrate effective learning and teaching strategies and employers to support governance and patient safety. If we succeed in creating a benchmark for numeracy for nursing accepted by the profession and other stakeholders, we believe it will be the first of its kind anywhere in the world.

The project is rooted in constructivist and socio-cultural approaches to numeracy for nursing and builds on work by members of the project team (Coben, 2000b; Hall et al., 2005; Hutton,

2000; Rowe & Mahar, 2006; Sabin, 2006; Weeks, Lyne, Mosely, & Torrance, 2001) . Initially we are focusing on drug dosage calculation, addressing issues of parity, scope and level in assessing numeracy skills for successful calculation of medication dosages by nurses when they qualify. In the first phase of the study we developed an evidence-based benchmark assessment tool utilising interactive computer simulations that approximate to real world practice, based on the following criteria—such a tool should be:

Realistic:

Evidence-based literature in the field of nursing numeracy (Hutton, 1997; Weeks et al., 2001) strongly supports a realistic approach to the teaching and learning of calculation skills, which in turn deserve to be tested in an authentic environment. Questions should be derived from authentic settings. A computer based programme of simulated practice in drug calculations, formative testing, with feedback on the nature of errors made, has been shown to develop competency in medication dosage calculation, which can be also demonstrated in the clinical areas (Weeks et al., 2000). Exposure of students to real-world situations is recommended (Weeks, 2001).

Appropriate:

The assessment tool should determine competence in the key elements of the required competence (OECD, 2005; Sabin, 2001).

Differentiated:

There should be an element of differentiation between the requirements for each of the branches of nursing (Hutton, 1997).

Consistent with adult numeracy principles:

The assessment should be consistent with the principles of adult numeracy learning teaching and assessment, having an enablement focus (Coben, 2000b).

Diagnostic:

The assessment tool should provide a diagnostic element, identifying which area of competence has been achieved, and which requires further intervention (Black & Wiliam, 1998). Thus it should “provide information to be used by students and teachers that is used to modify the teaching and learning activities in which they are engaged in order better to meet student needs. In other words, assessment is used to ‘keep learning on track’” (Wiliam, 2007).

Transparent:

The assessment should be able to demonstrate a clear relationship between ‘test’ achievement and performance in the practice context (Weeks et al., 2001).

Well-structured:

The tool should provide:

- a unique set of questions with a consistent level of difficulty; o a structured range of complexity; and

the assessment should take place within a defined framework, at points by which students can be effectively prepared, while allowing time for supportive remediation. (Hodgen & Wiliam, 2006)

Easy to administer:

the assessment should provide the opportunity for rapid collation of results, error determination, diagnosis and feedback (Black & Wiliam, 1998).

(Coben, Hall, et al., 2008, pp. 96–97)

The next phase (currently underway) seeks to evaluate this tool and compare it with assessment of the same competencies in a practical setting, using task-based activities.

Progress reports on the project have been presented at national and international conferences (Coben et al., 2008; HEA, 2006; Sabin et al., 2008) and published in articles (Coben, 2007).

Numeracy for Nursing at King's College London

Meanwhile, an exploratory investigation of aspects of teaching, learning and assessment of numeracy for nursing is underway in the Florence Nightingale School of Nursing and Midwifery (FNSNM) King's College London undergraduate/Diploma Nursing programme. Like other university Schools of Nursing in the UK, FNSNM has developed its own numeracy teaching, learning and assessment programme. This programme is being evaluated in this interdisciplinary project; in particular, we are:

- analysing existing data from online numeracy assessment of FNSNM Nursing undergraduate/Diploma students;
- critically evaluating numeracy assessment instruments and procedures used by FNSNM and recommending improvement as appropriate, in order to establish the validity and reliability of FNSNM assessment of numeracy for nursing;
- characterising the approach to teaching, learning and assessment of numeracy for nursing in FNSNM with a view to developing future studies, including international comparative studies.

We have conducted an initial analysis of the difficulty and coverage of assessment items and we are currently reviewing formative and summative assessment processes and materials and analysing the results of summative assessments of FNSNM nursing students' mathematical knowledge. The project will be reported on at the 9th Annual Interdisciplinary Research Conference, Transforming Healthcare through Research Education and Technology, School of Nursing and Midwifery, Trinity College Dublin: 5th-7th November, 2008.

We envisage that this project will complement the 'benchmark' project in Scotland since it entails an analysis of a different set of assessment items in terms of the scope and nature of numeracy for nursing implicit in them. We hope that these projects may contribute to the establishment of a sound basis for teaching learning and assessment of numeracy for nursing in the UK and elsewhere in the world.

The Scope for International Collaboration in Research and Development on Numeracy for Nursing

Following on from these projects, and building on Sabin's (2001) review of research and the other studies outlined above, there appears to be considerable scope for international collaboration on research and development in numeracy for nursing.

Debate on the nature and scope of numeracy for nursing should be facilitated through existing national and international networks such as ALM (Adults Learning Mathematics—A Research Forum, www.alm-online.net/), FINE (European Federation of Nurse Educators, www.fine-europe.eu/Organisation.htm) and other international, national and regional healthcare and nursing and nurse education organizations and mathematics and numeracy education fora. This should raise awareness of issues in numeracy for nursing and alert practitioners and others to the current lack of agreement on what numeracy is required for nursing and the variation in standards and content likely to be found in assessments of numeracy for nursing.

If a benchmark for numeracy for nursing can be successfully established in Scotland and is accepted by the profession and by nurse educators, employers and others, the standards expected of qualified nurses in other countries and at other stages in nurses' education and subsequent careers (newly-qualified; experienced; etc.) could be compared with the benchmark. The validity and reliability of assessments of numeracy for nursing used in various countries and institutions could also be compared with the benchmark. Teaching, learning and assessment programmes in numeracy for nursing could be compared in terms of their fitness for purpose, fitness for practice and fitness for award. The complexity and potential difficulty of both the mathematical and nursing content could be compared.

As a first step, an international comparative study of numeracy for nursing is in preparation. This will investigate: relevant aspects of regulatory frameworks and employment contexts; the numeracy nurses need to be safe and effective; the nature and scope of numeracy education for nursing, including case studies of educational interventions in numeracy for nursing. The study will include an exploration of what adult numeracy educators, in collaboration with nurses and nurse educators, can do—and are doing—to help turn numeracy for nursing from a poorly-understood area of concern to a well-understood beacon of good practice. The time is surely ripe for collaboration on an international comparative study drawing together and carrying forward research and development in numeracy for nursing: we have much to learn from each other.

References

- Allen, S., & Papas, A. (1999). Enhancing math competency of baccalaureate students. *Journal of Professional Nursing*, 15(2), 123–129.
- Behrend, M., Clark, R., Hall, C., & Hill, P. (2006). Online numeracy resources in the context of Safe Administration of Medicines (SAM). In M. Horne & B. Marr (Eds.), *Connecting voices in adult mathematics and numeracy: Practitioners, researchers and learners. Proceedings of the Adults Learning Mathematics (ALM) 12th annual international conference jointly with the Australian Council for Adult Literacy (ACAL) and in cooperation with the Australasian Bridging Mathematics Network (BMN)* (pp. 61–68). Melbourne: ACU National.
- Bindler, R., & Bayne, T. (1991). Medication calculation ability of Registered Nurses. *Journal of Nursing Scholarship*, 23(4), 221–224.

- Black, P., & Wiliam, D. (1998). Inside the black box: Raising standards through classroom assessment. *Phi Delta Kappan*, 80(2), 139–148.
- Brown, D. L. (2006). Can you do the math? Mathematic competencies of baccalaureate degree nursing students. *Nurse Educator*, 31(3), 98–100.
- Coben, D. (2000a). Mathematics or common sense? Researching invisible mathematics through adults' mathematics life histories. In D. Coben, J. O'Donoghue & G. E. FitzSimons (Eds.), *Perspectives on adults learning mathematics: Research and practice* (pp. 53–66). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Coben, D. (2000b). Numeracy, mathematics and adult learning. In I. Gal (Ed.), *Adult numeracy development: Theory, research, practice* (pp. 33–50). Cresskill, NJ: Hampton Press.
- Coben, D. (2007). When numeracy is in the eye of the beholder: Benchmarking numeracy for nursing. *Numeracy Briefing*, (11), 16–17.
- Coben, D., Colwell, D., Macrae, S., Boaler, J., Brown, M., & Rhodes, V. (2003). *Adult numeracy: Review of research and related literature*. London: National Research and Development Centre for Adult Literacy and Numeracy (NRDC).
- Coben, D., Hall, C., Hutton, B. M., Rowe, D., Sabin, M., Weeks, K., et al. (2008). Numeracy for nursing: The case for a benchmark. In T. Maguire, N. Collieran, O. Gill & J. O'Donoghue (Eds.), *The changing face of adults mathematics education: Learning from the past, planning for the future. Proceedings of ALM-14, the 14th international conference of Adults Learning Mathematics—A Research Forum (ALM)* (pp. 88–102). Limerick: University of Limerick in association with ALM.
- Dexter, P., & Applegate, M. (1980). How to solve a math problem. *Journal of Nursing Education*, 19(2), 49–54.
- Dodds, K. G. (2006). *Safety in numbers—Essential calculations for nurses*. Wellington, NZ: Dunmore Press.
- Eraut, M. (2003). Transfer of knowledge between education and the workplace. In H. P. A. Boshuizen (Ed.), *Expertise development: The transition between school and work* (pp. 55–72). Heerlen: Open Universiteit Maastricht.
- Evans, J. (1999). Building bridges: Reflections on the problem of transfer of learning in mathematics. *Educational Studies in Mathematics*, 39, 23–44.
- Galligan, L., & Pigozzo, R. (2002). Assisting nursing students solve drug calculation problems using metacognition and error analysis. *Literacy and Numeracy Studies*, 12(1), 45–62.
- Gillies, R. (2004). Numeracy for nurses: The case for traditional versus non-traditional methods for teaching drug calculation. In I. Putt, R. Faragher & M. McLean (Eds.), *Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 255–262). Townsville: Mathematics Education Research Group of Australasia.
- Grandell-Niemi, H., Hupli, M., & Leino-Kilpi, H. (2001). Medication calculation skills of graduating nursing students in Finland. *Advances in Health Sciences Education*, 6(1), 15–24.
- Grandell-Niemi, H., Hupli, M., Puukka, P., & Leino-Kilpi, H. (2006). Finnish nurses' and nursing students' mathematical skills. *Nurse Education Today*, 26(2), 151–161.
- Greenfield, S. (2007). Medication error reduction and the use of PDA technology. *Journal of Nurse Education*, 46(3), 27–31.
- Guile, D., & Young, M. (2003). Transfer and transition in vocational education: Some theoretical considerations. In T. Tuomi-Grohn & Y. Engeström (Eds.), *Between school and work: New perspectives on transfer and boundary crossing* (pp. 63–81). New York: Pergamon Press.
- Hall, C., Jones, C., Hilton, D., Davies, S., & MacDiarmid, I. (2005). *A study to evaluate first year student nurses' use of a national numeracy resource to develop key skills for nursing practice*. London: Higher Education Academy.
- HEA. (2006). Supporting maths difficulties workshop summary. Higher Education Academy Health Sciences and Practice Newsletter, 19, 8.
- Hispanic Times Magazine. (2001). \$265,000 grant to A&M International to address Hispanic RN shortage - Artículo Breve. *Hispanic Times Magazine*.
- Hodgen, J., & Wiliam, D. (2006). *Mathematics inside the black box: Assessment for learning in the mathematics classroom*. London: Nelson.
- Hoyles, C., Noss, R., & Pozzi, S. (2001). Proportional reasoning in nursing practice. *Journal for Research in Mathematics Education*, 32(1), 4–27.
- HRSA. (2004). *What is Behind HRSA's Projected supply, demand, and shortage of Registered Nurses?* Washington, DC: US Department of Health and Human Resources Health Resources and Services Administration.
- Hutton, B. M. (1997). *The acquisition of competency in nursing mathematics*. Unpublished doctoral dissertation, University of Birmingham.
- Hutton, B. M. (2000). Numeracy must become a priority for nurses. *British Journal of Nursing*, 9(14), 894.
- Hutton, B. M., & Gardner, H. (2005). Calculation skills. *Paediatric Nursing*, 17(2), 1–19.

- ISMP. (2008). Seven lapses add up to tragedy. *NurseAdvise-ERR*, 6(5), 1–2.
- Johnson, S. A., & Johnson, L. J. (2002). The 4 Cs: A model for teaching dosage calculation. *Nurse Educator*, 27(2), 79–83.
- Jukes, L., & Gilchrist, M. (2006). Concerns about numeracy skills of nursing students. *Nurse Education in Practice*, 6(4), 192–198.
- NMC. (2004). *Standards of proficiency for pre-registration nursing education*. London: Nursing and Midwifery Council.
- Nokes, P. L. (1967). *The professional task in welfare practice*. London: Routledge & Kegan Paul.
- Noss, R., & Hoyles, C. (1996). The visibility of meanings: Modelling the mathematics of banking. *International Journal of Computers in Mathematics Learning*, 1(1), 3–30.
- OECD. (2005). *The definition and selection of key competencies. Executive summary*. Paris: Organisation for Economic Cooperation and Development.
- Pirie, S. (1987). Nurses and mathematics: Deficiencies in basic mathematical skills among nurses. *Development and evaluation of methods of detection and treatment*. London: Royal College of Nursing.
- Rainboth, L., & DeMasi, C. (2006). Nursing students' mathematic calculation skills. *Nurse Education Today*, 347–353.
- Rowe, D., & Mahar, M. (2006). Construct validity. In T. Wood & W. Zhu (Eds.), *Measurement theory and practice in kinesiology* (pp. 9–26). Champaign, IL: Human Kinetics.
- Sabin, M. (2001). Competence in practice-based calculation: Issues for nursing education. A critical review of the literature. *Occasional Paper 3*. London: Learning and Teaching Support Network (LTSN) Centre for Health Sciences and Practice.
- Sabin, M. (2006). NHS Education for Scotland: Supporting numeracy skills in the Scottish healthcare workforce Retrieved 25 August, 2007, from http://www.nes.scot.nhs.uk/multi/documents/Numeracy_NES_Update.pdf
- Sabin, M., Hall, C., Coben, D., Rowe, D., Weeks, K., Woolley, N., et al. (2008, February). *Establishing an assessment benchmark in nursing: The example of medication dosage calculation*. Paper presented at the RCN Partners in Practice 2008, Blackpool, 22nd February.
- Sandwell, M., & Carson, P. (2005). Developing numeracy in child branch students. *Paediatric Nursing*, 17(9), 24–6.
- Starkings, S. (2003). Drug calculation and the mathematics required for nursing. *MSOR Connections*, 3(4), 46–49.
- Weeks, K. W. (2001). *Setting a foundation for the development of medication dosage calculation problem solving skills among novice nursing students. The role of constructivist learning approaches and a computer based Authentic World' learning environment*. Unpublished doctoral dissertation, University of Glamorgan, Pontypridd.
- Weeks, K. W., Lyne, P., Mosely, L., & Torrance, C. (2001). The strive for clinical effectiveness in medication dosage calculation problem solving skills: The role of constructivist theory in the design of a computer-based 'Authentic World' learning environment. *Clinical Effectiveness in Nursing*, 5(1), 18–25.
- Weeks, K. W., Lyne, P., & Torrance, C. (2000). Written drug dosage errors made by students: The threat to clinical effectiveness and the need for a new approach. *Clinical Effectiveness in Nursing*, 4(1), 20–29.
- Wiliam, D. (2007). Keeping learning on track: Formative assessment and the regulation of learning. In F. K. J. Lester (Ed.), *Second handbook of mathematics teaching and learning* (pp. 1053–1098). Greenwich, CT: Information Age Publishing.
- Wright, K. (2005). An exploration into the most effective way to teach drug calculation skills to nursing students. *Nurse Education Today*, 25(6), 430–436.
- Wright, K. (2006). An investigation to find strategies to improve student nurses' maths questions. *British Journal of Nursing*, 13(21), 1280–1287.
- Zevenbergen, R. (1996). The situated numeracy of pool builders. *Critical Forum*, 4(2), 34–46.

Behind the Headlines: Authentic Teaching, Learning and Assessment of Competence in Medication Dosage Calculation Problem Solving in and for Nursing

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This article addresses a key issue for mathematics educators preparing students for work: how should teaching, learning and assessment be designed to meet the mathematical demands of the workplace, especially when those demands are safety-critical? We explore this question through a discussion of our interdisciplinary research on numeracy for nursing, focusing in particular on the characterization and authentic assessment of competence in medication dosage calculation problem solving, where errors can and do cause death.

Introduction

Medication errors by nurses make headlines. An error can have disastrous consequences for the patient, for example: ‘Mother-of-four dies after blundering nurse administers TEN times drug overdose’ (Daily Mail Reporter, 2011), and for the nurse; in a recent extreme case this led to suicide: ‘Nurse’s suicide highlights twin tragedies of medical errors’ (Aleccia, 2011).

Concern in the nursing profession and the wider public, fuelled by headlines such as these, has led to a growing number of studies, both of qualified nurses, for example, Grandell-Niemi et al. (2003) in Finland and Hoyles et al (2001) in the UK, and of nursing students, for example, in the USA (Rainboth & DeMasi, 2006), the UK (Coben et al., 2010; Jukes & Gilchrist, 2006) and Australia (Eastwood, Boyle, Williams, & Fairhall, 2011).

Medication errors, characterised as adverse drug events (ADEs), have been categorized by Bates et al. (1995) in terms of whether actual or potential harm from medicines is caused to the patient. Where harm is manifested in the patient ADEs are classified as:

- Preventable: errors in prescribing, dispensing, calculating, preparing or administering the drug.
- Non-preventable: for example, where the patient is correctly administered the medication for the first time but subsequently has an adverse drug reaction that could

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not have been predicted by current technologies.

- Harm that can be minimised: for example, when, following correct administration, a drug rises to toxic levels in the blood; harm can be minimised by careful monitoring of blood levels of the drug.

Types of ADEs are shown in Figure 1, below.

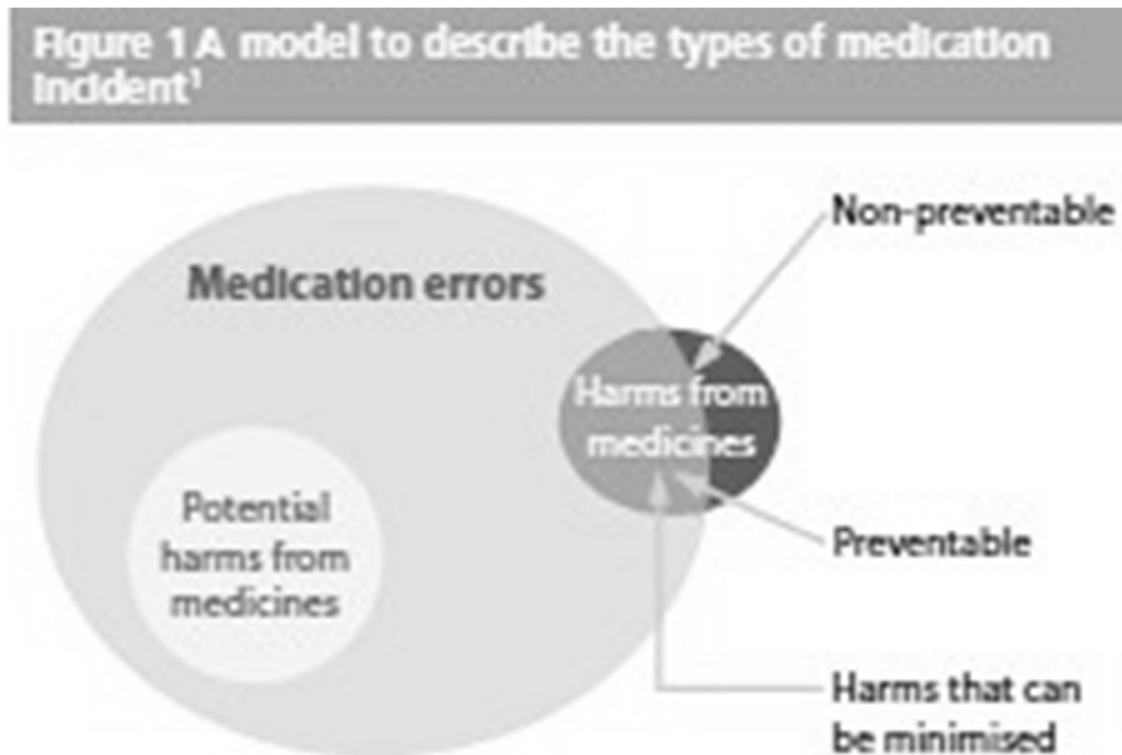


Figure 1: A model to describe the types of medication incident (NPSA, 2009, p. 6)

Our focus in this article is on the development and assessment of the vocational skills, knowledge and understanding required to minimize preventable errors by nurses in prescribing, dispensing, calculating, preparing or administering drugs; we characterize this area as medication dosage calculation problem-solving (MDC-PS). The jury is still out on the part played by nurses' numeracy skills in such errors. Indeed a recent review found insufficient evidence to suggest that medication errors are caused by nurses' poor calculation skills, although the author concludes that more research is required into calculation errors in practice (Wright, 2010). We believe that calculation skills are only part of the picture, as shown in the definition of numeracy and the model of competence in MDC-PS set out below (Figures 3 and 4). We developed the model as members of the NHS Education for Scotland (NES) interdisciplinary Expert Numeracy Reference Group (hereinafter: the Reference Group), building on earlier work outlined below. The Reference Group undertook research on the 'Benchmark Assessment of Numeracy for Nursing: Medication Dosage Calculation at Point of Registration' project commissioned by NES (Coben, et al., 2010); this work is at the heart of this article.

The outcomes of the NES research programme (Coben, et al., 2010; Sabin et al., 2013) were reported to the UK regulatory body, the Nursing and Midwifery Council (NMC) in 2010. These outcomes subsequently part-informed the construct and content of the MDC-PS competencies within the NMC's Essential Skills Cluster (ESC) for Medicines Management (NMC, 2010, p. 32) (see Table I).

Table I: MDC-PS competencies within the NMC ESC for Medicines Management

Essential skills cluster: Medicines management¹ The newly qualified graduate nurse should demonstrate the following skills and behaviours. They should be used to develop learning outcomes for each progression point and for outcomes to be achieved before entering the register.			
33 People can trust the newly registered graduate nurse to correctly and safely undertake medicines² calculations.			
First progression point	Second progression point	Entry to the register	Indicative content
1 Is competent in basic medicines calculations (*) relating to: <ul style="list-style-type: none"> • tablets and capsules • liquid medicines • injections including: <ul style="list-style-type: none"> • unit dose • sub and multiple unit dose • SI unit conversion. 		2 Is competent in the process of medication-related calculation in nursing field involving: <ul style="list-style-type: none"> • tablets and capsules • liquid medicines • injections • IV infusions including: <ul style="list-style-type: none"> • unit dose • sub and multiple unit dose • complex calculations • SI unit conversion. 	Numeracy skills, drug calculations required to administer medicines safely via appropriate routes including specific requirements for children and other groups.

In addition, when commenting on the MDC-PS competency model emerging from the NES research (see below), the NMC referred to the work of the Reference Group, stating in their 'Advice and Supporting Information for Implementing NMC Standards for Pre-Registration Nursing Education' that:

We have considered some new work in relation to calculations that has informed our approach to dealing with the numerical assessment in relation to the ESC on medicines management. The issues around standards of numeracy competence in the health workforce have been addressed by a team commissioned by NHS Education for Scotland. Programme providers may wish to take the following information into account when determining assessment criteria:

- An ESC assessment strategy for medication-related calculation that demonstrates competency across the full range of complexity, the different delivery modes and technical measurement issues.
- Assessment that takes place in a combination of the practice setting, computer lab and simulated practice that authentically reflects the context and field of practice.
- Diagnostic assessment that focuses on the full range of complexity, identified at each stage, and recognizing the different types of error (conceptual, calculation, technical measurement) which can then be linked to support strategies. (NMC, 2011, pp. 60-61)

The NMC statement falls short of directly advising programme providers to take the information into account when determining assessment criteria, saying only that they "may wish to take the [...] information into account". Nevertheless, this is an important step towards the adoption in the UK of a more comprehensive, evidence-based approach to the development of professional competence in numeracy for nursing.

The Development of Professional Competence in Nursing

The *raison d'être* of professional nursing education programmes is to facilitate the development of practitioners who demonstrate professional competence in the following domains: cognitive competence ('knowing that' and 'knowing why'); functional competence ('knowing how' and skills); ethical competence (the embodiment of a professional value system); and personal competence (the ability to apply these competences in different practice situations) (European Communities, 2005; Weeks, Hutton, Coben, Clochesy, & Pontin, 2013). However, the traditional organisation of nursing (and most other) traditional vocational education systems has typically created an artificial distinction between the teaching and assessment of knowledge and the teaching and assessment of professional practice and values. These systems have in turn created an artificial theory-practice gap and an artificial knowledge-performance gap that has largely separated the teaching and learning of the professional body of knowledge from the teaching and learning of professional know-how and skilled performance (Lum, 2009; Weeks, Sabin, Pontin, & Woolley, 2013). This problem is complicated by the common practice of articulating competence and assessment problems in a word-based form and then requiring the interpretation and measurement of such abstract and descriptive competence statements in authentic practice environments.

When the professional know-how and skilled performance involves mathematics, as in MDC-PS, the problem is further exacerbated by the tendency for mathematical processes to become invisible in the workplace: "crystallised in 'black boxes' shaped by workplace cultures" as Williams and Wake (2007) put it. Other exacerbating factors include: problems in the transfer of learning between the classroom and the workplace (FitzSimons & Coben, 2009); incoherent nursing numeracy assessment regimes and invalid test items applied in high stakes testing (Coben, Hodgen, Hutton, & Ogston-Tuck, 2009); and problems with word problems in mathematics (Verschaffel, Greer, & De Corte, 2000). When the professional practice is safety-critical, as in MDC-PS, any disjuncture between theory and practice and between knowledge and performance may have serious consequences.

To address this central problem within the domain of MDC-PS in and for nursing in this paper we:

- Present the definition of numeracy and related criteria operationalized in our research;
- Review the problem of articulating MDC-PS competence in a word-based form;
- Propose a model that reflects the integration of the three competence sub-domains of MDC-PS (conceptual, calculation and technical measurement competence) and that articulates both the word-based definitions of 'knowing that and knowing why' and the authentic virtual representations of the 'knowing how' of MDC-PS competence that need to be manifested by the registered nurse in clinical practice; and
- Illustrate the essential design features of an authentic virtual learning and diagnostic assessment environment, based on the principle of authentic assessment, that facilitates the measurement of MDC-PS cognitive competence constructs (professional knowledge) and synthesizes these with the enculturation and sensitization of the nursing student to the MDC-PS functional competence (professional know-how and skilled performance) requirements of professional clinical nursing practice. This is aimed at bridging both the theory-practice and knowledge-performance gaps.
- Discuss the implications of our analysis for vocational mathematics education beyond nursing.

Conceptualization of Numeracy for Nursing

The Reference Group was mindful of the terminological confusion and contestation around adults' use and learning of mathematics, often characterized as numeracy (Coben & O'Donoghue). Nonetheless, we needed to define the domain within which we laboured. We decided to adopt the following generic definition of what it means to be numerate:

To be numerate means to be competent, confident, and comfortable with one's judgments on *whether* to use mathematics in a particular situation and if so, *what* mathematics to use, *how* to do it, *what degree of accuracy* is appropriate, and *what the answer means* in relation to the context. (Coben, 2000, p. 35, emphasis in the original)

This definition was chosen because numeracy is characterized as the exercise of judgment with respect to specific issues in relation to the demands and affordances of a given context, in this case: nursing. It allowed us to explore issues of competence in MDC-PS and to delineate what we did and not consider to be numeracy issues in that context. Our formulation of the relationship between the domains involved in our analysis (numeracy; healthcare numeracy; medicines management; and medication dosage calculation) is given below (Figure 2).

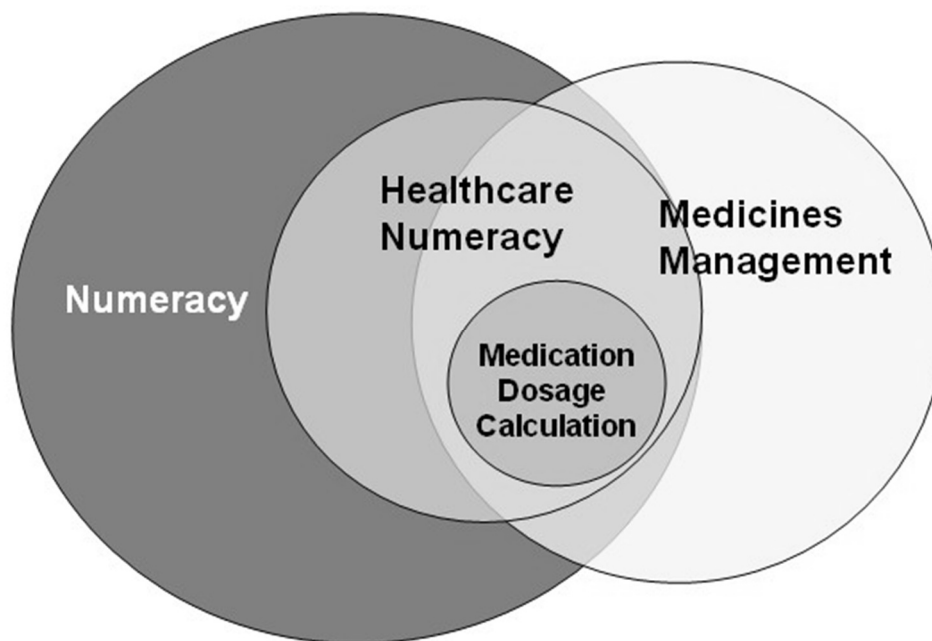


Figure 2: Medication dosage calculation in the context of numeracy and medicines management (Coben, et al., 2010, p. 13)

Evidence-based criteria for the assessment of numeracy for nursing

On the basis of the above conceptualization of numeracy for nursing, the Reference Group developed the following research-based criteria for the features of an effective nursing numeracy benchmark assessment tool, consistent with Gulikers *et al.*'s five-dimensional framework for authentic assessment (Gulikers, Bastiaens, & Kirschner, 2004). Gulikers identified five dimensions that inform the design and articulation of assessment environments:

1. Task: Students should be exposed to authentic tasks that involve integration of knowledge, skills and attitudes. The tasks should be meaningful and relevant to the student and should reflect the full range of complexity, domains of practice and structure of the tasks as encountered in the real practice setting.
2. Physical context: The tasks should be learned and assessed in physical contexts that are as congruous as possible with the real physical practice setting; and should be undertaken and assessed using the typical tools available in the setting and along similar time-frames available to undertake the real tasks.
3. Social context: The social context of the practice setting should be as closely aligned as possible in respect of the individual or groups of professionals typically engaged in problem-solving and undertaking the tasks.
4. Criteria: Assessments should be centred on criterion-referenced outcomes, should be based on the criteria used in professional practice and should be realistic and transparent in respect of the processes and outcomes expected in the practice setting.
5. Form/result: Competence should be demonstrated and measured in respect of professionally relevant results that are observable and subject to multiple indicators of learning.

Work towards a benchmark assessment of numeracy for nursing undertaken by the Reference Group focused on the ‘task’, ‘physical context’ and ‘social context’ design features of authentic assessment environments, with student performance data measured against the ‘criteria’ and ‘form/result’ dimensions of Gulikers’ framework. The framework was employed for evaluating the authenticity and construct validity of a proposed benchmark assessment tool and associated assessment environment for MDC-PS in higher education institution (HEI) and practice settings. The framework was also used to evaluate nursing students’ perceptions of congruence between the authentic assessment environment and medication dosage problem-solving and computation requirements in practice settings (Coben, et al., 2010).

The Reference Group proposed the following evidence-based criteria for the assessment of numeracy for nursing supported by Gulikers’ (2004) framework. Such assessment should be:

Realistic: Evidence-based literature in the field of nursing numeracy (Hutton, 1997; Weeks, 2001) strongly supports a realistic approach to the teaching and learning of calculation skills, which in turn deserve to be tested in an authentic environment. Questions should be derived from authentic settings. A computer-based programme of simulated practice in drug calculations, formative testing, with feedback on the nature of errors made, has been shown to develop competency in medication dosage calculation, which can also be demonstrated in the clinical areas (Weeks, Lyne, & Torrance, 2000). Exposure of students to real-world situations is recommended (Weeks, 2001).

Appropriate: The assessment tool should determine competence in the key elements of the required competence (OECD, 2005; Sabin, 2001).

Differentiated: There should be an element of differentiation between the requirements for each of the branches of nursing (Hutton, 1997).

Consistent with adult numeracy principles: The assessment should be consistent with the principles of adult numeracy learning teaching and assessment, having an enablement focus (Coben, 2000).

Diagnostic: The assessment tool should provide a diagnostic element, identifying which area of competence has been achieved, and which requires further intervention (Black & Wiliam, 1998). Thus it should “provide information to be used by students and teachers that is used to modify the teaching and learning activities in which they are engaged in order better to meet student needs. In other words, assessment is used formatively to ‘keep learning on track’” (Wiliam, 2006).

Transparent: The assessment should be able to demonstrate a clear relationship between ‘test’ achievement and performance in the practice context (Weeks, Lyne, Mosely, & Torrance, 2001).

Well-structured: The assessment tool should provide:

- a unique set of questions with a consistent level of difficulty;
- a structured range of complexity; and
- the assessment should take place within a defined framework, at points by which students can be effectively prepared, while allowing time for supportive remediation (Hodgen & Wiliam, 2006).

Easy to administer: the assessment should provide the opportunity for rapid collation of results, error determination, diagnosis and feedback (Black & Wiliam, 1998). (Coben et al., 2008, pp. 96-97)

The Problem of Articulating MDC-PS Competence in a Word-based Form

Word-based competence rubrics (such as that in Table 1, above), statements and regulatory body advice are common features of traditional competence-based education and training programmes. However, in questioning whether competence can be described and communicated through language in accurate and unequivocal terms, Lum states that

the entirety of present policy-making and practice in education, not only in the UK but increasingly elsewhere, is underpinned by the assumption that such descriptions are possible. It is extraordinary that these strategies have gained such widespread acceptance and been afforded such unqualified official approval while the very assumption upon which they are based seems hardly to have received any attention. What makes this all the more remarkable is that there would appear to be profound and irrevocable difficulties with the idea that competence can be specified in clear and precise terms.

(Lum, 2009, p. 76)

Ultimately, this difficulty in accurately defining and describing particularly functional competence (know-how and skilled performance) in a word-based form, results in the potential for variable interpretation of the required competence in vocational and practice-based professions like nursing. For example, returning to the NMC descriptors of competence in MDC- PS (Table 1), without a shared and unified understanding of ‘conceptual’, ‘calculation’ or ‘technical measurement’ competence, or ‘unit dose’ and ‘sub and multiple

unit dose' calculations, etc., misinterpretation or variable interpretation by educators and students can (and does) occur. To counter this problem within the MDC-PS competence domain (and other vocational mathematics domains) we have proposed that both 'knowing that' and authentic virtual 'knowing how' representations of MDC-PS competence can be modeled to illustrate the fundamental principles of how competent MDC-PS should be manifested in authentic practice environments.

Authentic Assessment and Modeling and Measurement of MDC-PS Competence in Virtual and Practice-based Environments

Mueller (2005, p. 2) defined authentic assessment as "A form of assessment in which students are asked to perform real-world tasks that demonstrate meaningful application of essential knowledge and skills". In the nurse education context it is not necessarily practicable, appropriate or ethical to ask students to perform real-world tasks in the real world with real patients. Nevertheless, students must "demonstrate meaningful application" of the "essential knowledge and skills" of MDC-PS if they are to practise safely and effectively as professional registered nurses. The solution to this conundrum may be found through authentic assessment undertaken in a safe, controlled environment, in which assessments are designed to be truly representative of performance in the field and assessment criteria seek to evaluate essentials of performance against well-articulated performance standards. The hallmark of authentic assessment environments is their capacity to measure the meaningful application of cognitive competence (knowledge) and functional competence (know-how and skills) in realistic contexts, together with the provision of a rubric (diagnostic framework) against which to measure performance (European Communities Education and Culture, 2008; Sabin, et al., 2013). Above all, "within reasonable and reachable limits, a real test replicates the authentic intellectual challenges facing a person in the field" (Wiggins, 1989, p. 706).

Authentic learning and competency assessment environments that meet these criteria support the bridging of the theory-practice and knowledge-performance gaps (Boud, 1990; Coben, et al., 2010; Gulikers, et al., 2004; Lowes & Weeks, 2006; Weeks, 2001; Weeks, et al., 2001; Weeks, et al., 2000; Weeks & Woolley, 2007). This is critical in the domain of MDC-PS and other vocational mathematics domains where assessment schedules must reliably and validly measure the construction, synthesis and meaningful application of the mathematical and measurement knowledge, problem-solving and professional skills that underpin safe and effective professional practice.

For the purposes of extrapolating and operationalizing a definition of dosage calculation competence based on the above definition of numeracy and consistent with the criteria set out above, the Reference Group adapted a competency model derived from an initial premise described by Weeks, Lyne and Torrance (2000) and elaborated by Authentic World Ltd²⁸. Figure 3 illustrates a competence model that provides generic definitions for the three sub-elements of MDC-PS competence (conceptual, calculation and technical measurement); and Figure 4 illustrates an analogous competence model that reflects exemplar computer-generated iconic representations of the requirements for solving an injection-based dosage calculation problem. Comparison of these two models highlights the difficulty noted by Lum (2009) in accurately describing competence in precise detail in a word-based form; and

²⁸ <http://www.authenticworld.co.uk/>

shows how analogous computerized iconic modeling adds essential pictorial detail for the student in both constructing schemata (internal cognitive representations of an individual's world) and learning competent MDC-PS practice, and for the educator and clinician who need to assess competence requirements against a defined rubric (e.g., see Table I).

Figures 3 and 4 articulate the interrelationship between the three sub-elements that combine to form competence in medication dosage calculation problem-solving. The confluence (central white section) of the model highlights how all three sub-elements must be practised concomitantly in order to achieve a correct dosage or rate solution. Conversely, an uncorrected error in one or more of the three sub-elements of the competency model WILL result in a medication dosage calculation error in clinical nursing practice. Consequently, we argue that it is imperative that nursing students are supported in the development of schemata and competence in MDC-PS and undertake systematic diagnosis of cognitive and functional competence in authentic assessment and clinical practice environments (Sabin, et al., 2013; K. W. Weeks, et al., 2013).

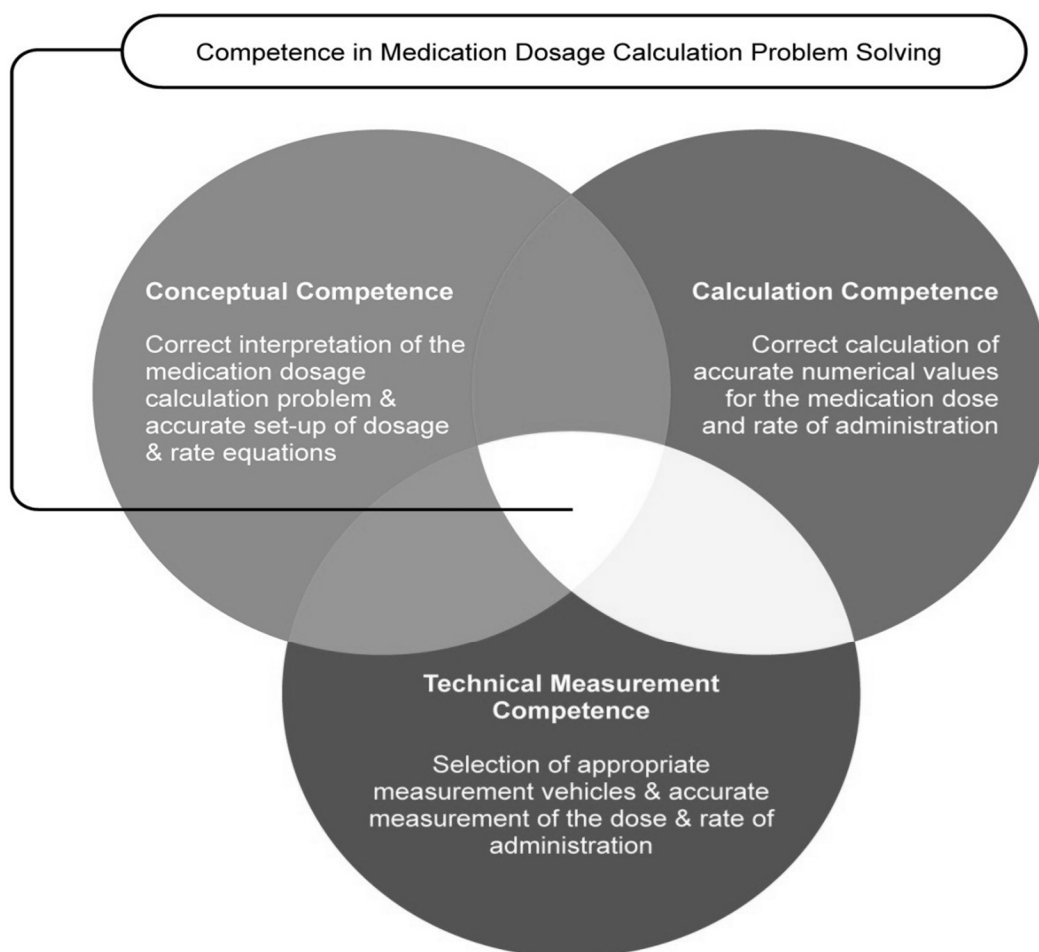


Figure 3: MDC-PS Competency Model (generic word-based definitions)

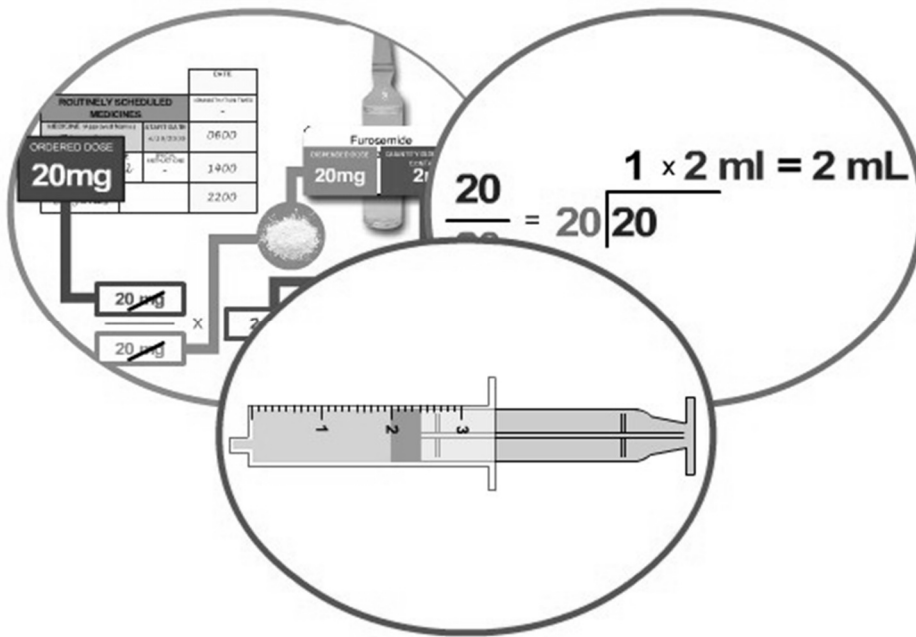


Figure 4: MDC-PS Competency Model (example iconic representations for an injection-based dosage calculation problem) (©Authentic World Ltd)

Knowing That, Knowing Why and Knowing How

Weeks, Hutton, Coben, Clochesy and Pontin (2013) describe and illustrate the design features of an authentic virtual MDC-PS learning and diagnostic assessment environment. The central theoretical perspectives that informed the design of the Authentic World safeMedicate and eDose authentic virtual learning and diagnostic assessment environments are illustrated in Figure 5. In addition to our focus on competency modeling we provide a short summary of these perspectives here.

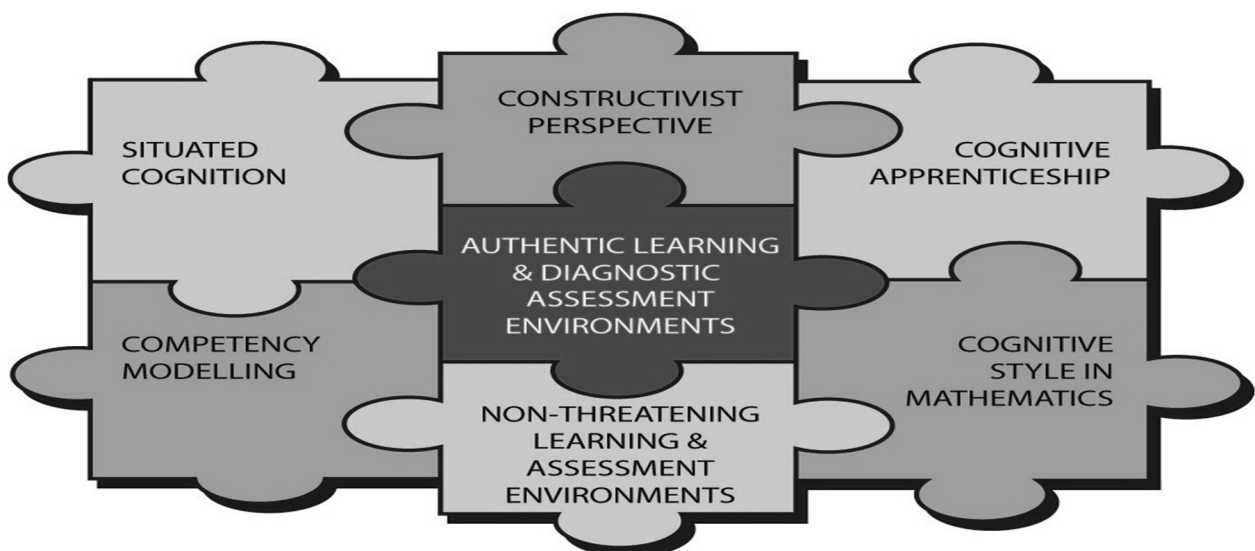


Figure 5: Authentic learning and diagnostic assessment environment pedagogical design features

The design of the virtual environments is underpinned by a central constructivist and situated cognition perspective that actively engages learners in knowledge construction, and their active engagement with the context-bound artefacts (functional objects) and tools of the social and clinical environments within which the professional knowledge is to be applied. These perspectives, together with the articulating cognitive apprenticeship, cognitive style in mathematics and non-threatening features of the learning and assessment environments, are fully explored by Hutton, Coben, Clochesy, and Pontin (2013).

This theory of learning lies in stark contrast to traditional didactic transmission methods of nurse education that not only fail to engage the learner actively in knowledge construction, but also largely rely on the use of abstract word problems to describe the clinical and social features of medication dosage problems and to assess cognitive competence. Figure 6 illustrates a typical word-based ‘complex’ essential skills medication dosage calculation problem in this genre.


<p>Prescribed dose, Aminophylline 200 mg, Dispensed dose, Aminophylline 250 mg/10ml. What volume should be drawn for injection?</p>

Figure 6: A typical word problem used to assess medication dosage calculation problem-solving ability

Word problems of this type are highly structured and formalized, unlike “inherently ambiguous and open-ended” authentic challenges (Wiggins, 1989, p. 706). They inform the learner of the ‘prescribed dose’ and the ‘dispensed dose’, etc., a luxury not afforded to the registered nurse in clinical practice. In reality, as illustrated in our competency model (Figures 3 and 4) the competent registered nurse is required to understand conceptually and interpret this numerical information from prescription charts and medication ampoule labels, etc., to calculate an accurate numerical value for the dose and to perform an accurate technical measurement of the dose in an appropriately selected measurement vehicle (in this case a syringe). Hence, in contrast to the word-based problem, Figure 7 illustrates an example from the authentic diagnostic assessment environment that represents the same medication dosage problem as that described in words in Figure 6.

The example illustrates the presentation of the problem in an authentic form, together with diagnostic feedback of conceptual competence, calculation competence and technical measurement competence. Macdonald et al (2013) further inform this premise and illustrate examples of competence assessment for ‘unit dose’, ‘multiple-unit dose’, ‘sub-unit dose’ and ‘conversion of SI unit’ MDC-PS calculations. Note that the images in Figure 7 are screenshots and do not show the dynamic nature of the assessment, whereby students select appropriate vessels, drag and drop pill icons into containers and ‘pull up’ liquid medicine into a syringe. These dynamic iconic computerized models provide exemplar benchmarks for competent performance meeting the requirements of the NMC ESC hierarchical rubric illustrated in Table I.

REGULAR PRESCRIPTION MEDICINES		DATE
MEDICINE (Approved Name) <i>Aminophylline</i>		ADMINISTRATION TIMES <i>06:00</i>
DOSE <i>200mg</i>	ROUTE <i>I.V.</i>	START DATE <i>1/5/2012</i>
DOCTOR'S SIGNATURE <i>Dr. Jones</i>		PHARMACY <i>A. Mann</i>



Aminophylline
250mg in 10mL

DIVISION

$\frac{200}{250}$

$\frac{\text{mg}}{\text{mg}}$

x

10


$\frac{\text{mL}}{\text{mL}}$


=


8


$\frac{\text{mL}}{\text{mL}}$


NEXT


1mL 


10mL 

50 units 

2mL 

20mL 

100 units 

5mL 

DIVISION

$\frac{200}{250}$

$\frac{\text{mg}}{\text{mg}}$

x

10

$\frac{\text{mL}}{\text{mL}}$

=


8

$\frac{\text{mL}}{\text{mL}}$

NEXT

CORRECT ANSWER

$$\frac{200}{250} \frac{\text{mg}}{\text{mg}} \times 10 \frac{\text{mL}}{\text{mL}} = 8 \frac{\text{mL}}{\text{mL}}$$



Conceptual Competence

YOUR ANSWER

$\frac{200}{250}$

$\frac{\text{mg}}{\text{mg}}$

x


10

$\frac{\text{mL}}{\text{mL}}$

=

8

$\frac{\text{mL}}{\text{mL}}$



Calculation Competence

Technical Measurement Competence

Figure 7: Computerized iconic model illustrating conceptual, calculation and technical measurement cognitive competence diagnostic feedback for a typical essential skills ‘complex dose’ calculation

Weeks, Higginson, Clochesy & Coben (2013) have explored a grounded theory of schema construction for medication dosage calculation problem-solving (MDC-PS). Within this theory, and as illustrated in this paper, we have stressed the importance of the student actively engaging in and with the medication dosage artefacts (functional objects) within which essential numerical and measurement information is embedded (e.g., prescription charts, medication ampoule labeling, syringes, etc.). Students engage in what Treffers (1987) calls “horizontal mathematization”, i.e., the mathematization of contextual problems, in which they come up with mathematical tools which can help to organize and solve a problem located in a real-life situation.

Of critical note, our findings suggest that ‘seeing’ and interacting with these context-bound physical artefacts, and their iconic representations, rather than the mere description of the dosage problem in a word-based form, is an essential factor for many nursing students in the construction of accurate schemata for MDC-PS and for subsequent development of competence in MDC-PS.

Similarly, Lum noted that in terms of developing schemata and vocational capability there is an imperative for moving beyond descriptions and engaging in ‘learning to see’ the vocationally relevant features of the world:

The important point here is not so much the difficulty we have putting these things into words (important though that is) so much as the idea that we are surrounded by features of the world, be they physiognomies, facial expressions, functional objects, meaningful behaviours, all of which we must learn to see. In coming to recognize features of the world around us, many of the capacities we develop we will have in common with others. Importantly, however, a considerable part of what it is to be vocationally capable consists in being able to apply schemata which enable us to ‘see’ those features of the world which are relevant and perhaps even unique to a particular vocational role. (Lum 2009, p. 102)

We argue that promoting the simultaneous ‘seeing’ of the context-bound physical artefacts (functional objects) of medication dosage calculation problems and supporting the construction of semantic connections with abstractions in the form of dosage calculation equations and word-based competency statements, both supports accurate schemata construction and improves our shared and unified understanding of competency requirements in vocational mathematics. In the absence of advancing this process we are unlikely to close the theory-practice and knowledge-performance gaps created by traditional vocational education systems.

Implications for Vocational Mathematics Education beyond Nursing

In this final section we review the implications of our work on MDC-PS for vocational mathematics education beyond nursing. We begin by summarising the implications for education and training of recent research on adult numeracy for work and life set out by FitzSimons and Coben in their entry in the *UNESCO-UNEVOC International Handbook of Technical and Vocational Education and Training* (FitzSimons & Coben, 2009) and then consider what our work adds to this picture.

FitzSimons and Coben state that:

1. Numeracy skills cannot be isolated and taught out of context.
2. The expectation that numeracy skills will be directly transferred from classroom to

workplace or applied unproblematically is unsustainable. Rather, mathematical knowledge must be transformed into partly or potentially new context-specific knowledge, integrating experience-based judgment with the social and cultural norms of the workplace.

3. The training environment should employ simulation techniques.
4. Students should be aware of the variability between training and workplace environments and the inherent variability of workplace tasks.
5. There is no optimal site of learning for work (viz. the academy or the workplace).
6. Integrated curricula, problem-based pedagogies, and the development of generic skills of communication and reflection, as well as problem solving, modelling situations, planning, and teamworking skills are needed, rather than linear, hierarchical curricular models, unrelated to students' lives (FitzSimons, 2002).
7. Normatively, mathematics education for the workplace should be intended to enhance the knowing of workers both subjectively and objectively, so that as individuals they may be empowered as 'knowledge producers' as well as 'knowledge consumers', i.e., to be technologically, socially, personally and/or democratically numerate.
8. Mathematics should be presented in contexts that make sense to the learner.
9. Qualitative and quantitative information should be valued together with an ethical approach to people and the environment.
10. Learning how to question critically and to listen, as part of enhanced communication skills, should be valued, particularly with regard to data handling and interpretation.
11. Following Wake and Williams (2001), there should be an emphasis on:
 - a) using relatively 'low level' mathematics in quite complex situations and contexts;
 - b) encouraging experiences of a diversity of conventions and methods,
 - c) having students experience activities where the mathematics is embodied in context and to use artefacts with which they have become familiar;
 - d) preparing students to transform their existing mathematical knowledge to make sense of activities in unfamiliar workplace situations;
 - e) having students design spreadsheet programmes for modelling and for the recording, processing and analysis of data; and
 - f) making students aware that there are many and varied ways to solve any problem.
2. In addition, Hoyles et al. (2002) recommend:
 - a) an ability to perform paper and pencil calculations and mental calculations as well as calculating correctly with a calculator;
 - b) calculating and estimating (quickly and mentally), including understanding percentages;
 - c) multi-step problem solving;
 - d) use of extrapolation;
 - e) recognising anomalous effects and erroneous answers when monitoring systems;
 - f) communicating mathematics to other users and interpreting the mathematics of other users;

- g) developing an ability to cope with the unexpected; and
 - h) a sense of complex modelling, including understanding thresholds and constraints.
- (summarised from FitzSimons & Coben, 2009)

We endorse these points. Our work adds comprehensive breadth and depth specifically on numeracy for nursing, while drawing attention to the vital importance of authentic assessment, supported by authentic pedagogies, in all vocational mathematics education. We believe our work offers a model of vocational mathematics education which others may wish to develop in other work contexts.

Finally, as has been noted above, the Reference Group is interdisciplinary; it comprises expert researchers who are qualified nurses and nurse educators as well as non-nurses: experts in adult numeracy education, quantitative research methodologies and psychometrics. Such interdisciplinary research collaborations are unusual in the field of vocational mathematics education but in our experience there is much to be gained from them.

References

- Aleccia, J. (2011, 6/27/2011 8:39:55 AM ET). Nurse's suicide highlights twin tragedies of medical errors. Retrieved 7th October, 2012, from http://www.msnbc.msn.com/id/43529641/ns/health-health_care/t/nurses-suicide-highlights-twin-tragedies-medical-errors/ - UHFM3Rgri00
- Bates, D. W., Cullen, D. J., Laird, N., Petersen, L. A., Small, S. D., Servi, D., et al. (1995). Incidence of adverse drug events and potential adverse drug events. Implications for prevention. *JAMA*, 274(1), 29-34.
- Black, P., & Wiliam, D. (1998). Inside the Black Box: Raising standards through classroom assessment. *Phi Delta Kappan*, 80(2), 139-148.
- Boud, D. (1990). Assessment and the promotion of academic values. *Studies in Higher Education*, 15(1), 101-111.
- Coben, D. (2000). Numeracy, mathematics and adult learning. In I. Gal (Ed.), *Adult Numeracy Development: Theory, research, practice* (pp. 33-50). Cresskill, NJ: Hampton Press.
- Coben, D., Hall, C., Hutton, B. M., Rowe, D., Sabin, M., Weeks, K., et al. (2008). Numeracy for nursing: The case for a benchmark. In T. Maguire, N. Colleran, O. Gill & J. O'Donoghue (Eds), *The Changing Face of Adults Mathematics Education: Learning from the past, planning for the future. Proceedings of ALM-14, the 14th International Conference of Adults Learning Mathematics - A Research Forum (ALM), held at the University of Limerick, Ireland, 26-29 June, 2007* (pp. 88-102). Limerick: University of Limerick in association with ALM.
- Coben, D., Hall, C., Hutton, B. M., Rowe, D., Weeks, K., & Woolley, N. (2010). *Research Report: Benchmark Assessment of Numeracy for Nursing: Medication dosage calculation at point of registration*. Edinburgh: NHS Education for Scotland (NES).
- Coben, D., Hodgen, J., Hutton, B. M., & Ogston-Tuck, S. (2009). High stakes: Assessing numeracy for nursing. *Adult Learning*, 19(3-4), 38-41.
- Coben, D., & O'Donoghue, J. (in press) Adults learning mathematics: An emerging field. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education*. Heidelberg, Germany: Springer.
- Daily Mail Reporter. (2011, 23 February 2011). Mother-of-four dies after blundering nurse administers TEN times drug overdose. *Daily Mail*.
- Eastwood, K. J., Boyle, M. J., Williams, B., & Fairhall, R. (2011). Numeracy skills of nursing students. *Nurse Education Today*, 31(8), 815-818.
- European Communities. (2005). *The European Qualifications Framework for Lifelong Learning (EQF) Glossary*. Luxembourg: Office for Official Publications of the European Communities.
- European Communities Education and Culture. (2008). *The European Qualifications Framework for Lifelong Learning (EQF)* Luxembourg: Office for Official Publications of the European Communities.
- FitzSimons, G. E. (2002). *What Counts as Mathematics? Technologies of power in adult and vocational education* (Vol. 28). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- FitzSimons, G. E., & Coben, D. (2009). Adult numeracy for work and life: Curriculum and teaching implications of recent research. In R. Maclean & D. Wilson (Eds), *UNESCO-UNEVOC International Handbook of Technical and Vocational Education and Training. Bridging Academic and Vocational Education* (Vol. 6, pp. 2731- 2745). Dordrecht: Springer.

- Grandell-Niemi, H., Hupli, M., Leino-Kilpi, H., & Puukka, P. (2003). Medication calculation skills of nurses in Finland. *Journal of Clinical Nursing*, 12(4), 519–528.
- Gulikers, J. T. J., Bastiaens, T. J., & Kirschner, P. A. (2004). A five-dimensional framework for authentic assessment. *Educational Technology Research and Development*, 52(3), 67-85.
- Hodgen, J., & Wiliam, D. (2006). *Mathematics Inside the Black Box: Assessment for learning in the mathematics classroom*. London: nferNelson.
- Hoyles, C., Noss, R., & Pozzi, S. (2001). Proportional reasoning in nursing practice. *Journal for Research in Mathematics Education*, 32(1), 4-27.
- Hoyles, C., Wolf, A., Molyneux-Hodgson, S., & Kent, P. (2002). *Mathematical Skills in the Workplace. Final Report to the Science, Technology and Mathematics Council. Foreword and Executive Summary*. London: Institute of Education, University of London; Science, Technology and Mathematics Council.
- Hutton, B. M. (1997). *The Acquisition of Competency in Nursing Mathematics*. Unpublished PhD, University of Birmingham, Birmingham.
- Jukes, L., & Gilchrist, M. (2006). Concerns about numeracy skills of nursing students. *Nurse Education in Practice*, 6(4), 192-198.
- Lowes, L., & Weeks, K. (2006). Researching your own clients/students. In D. Allen & P. Lyne (Eds), *The Reality of Nursing Research: Politics, practices and processes* (pp. 132-147). London: Routledge.
- Lum, G. (2009). *Vocational and Professional Capability: An epistemological and ontological study of occupational expertise*. London: Continuum International Publishing Group Ltd.
- Macdonald, K., Weeks, K. W. & Moseley, L. (2013). 'Safety in Numbers 6: Tracking pre-registration nursing students' cognitive and functional competence development in medication dosage calculation problem- solving: the role of authentic learning and diagnostic assessment environments.' *Nurse Education in Practice* 13(2): e66-77.
- Mueller, J. (2005). The authentic assessment toolbox: Enhancing student learning through online faculty development. *Journal of Online Teaching and Learning*, 1(1), 1-7.
- NMC. (2010). Final: Standards for Pre-registration Nursing Education – Annexe 3. Essential Skills Clusters (2010) and Guidance For Their Use (Guidance G7.1.5b) (pp. 41). London: Nursing and Midwifery Council.
- NMC. (2011). Advice and Supporting Information for Implementing NMC Standards for Pre-Registration Nursing Education. London: Nursing and Midwifery Council.
- OECD. (2005). *The Definition and Selection of Key Competencies. Executive Summary*. Paris: Organisation for Economic Cooperation and Development.
- Rainboth, L., & DeMasi, C. (2006). Nursing students' mathematic calculation skills. *Nurse Education Today*, 26(8), 347-353.
- Sabin, M. (2001). *Competence in practice--based calculation: Issues for nursing education. A critical review of the literature. Occasional Paper 3*. London: Learning and Teaching Support Network (LTSN) Centre for Health Sciences and Practice.
- Sabin, M., K. W. Weeks, et al. (2013). 'Safety in numbers 5: Evaluation of computer-based authentic assessment and high fidelity simulated OSCE environments as a framework for articulating a point of registration medication dosage calculation benchmark. *Nurse Education in Practice*, 13(2): e55–e65.
- Treffers, A. (1987). *Three Dimensions. A model of goal and theory description in mathematics instruction. The Wiskobas project*. Dordrecht: Kluwer Academic Publishers.
- Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making Sense of Word Problems*. Lisse, NL: Swets and Zeitlinger Publishers.
- Wake, G. D., & Williams, J. S. (2001). *Using College Mathematics in Understanding Workplace Practice: Summative report of the research project funded by the Leverhulme Trust*. Manchester: University of Manchester.
- Weeks, K. W. (2001). *Setting a foundation for the development of medication dosage calculation problem solving skills among novice nursing students. The role of constructivist learning approaches and a computer based 'Authentic World' learning environment*. Unpublished PhD, University of Glamorgan, Pontypridd.
- Weeks, K. W., R. Higginson, Clochesy, J. M. & Coben, D. (2013). 'Safety in Numbers 7: Veni, vidi, duci: A grounded theory evaluation of nursing students' medication dosage calculation problem-solving schemata construction.' *Nurse Education in Practice* 13(2): e78-87.
- Weeks, K. W., Hutton, B. M., Coben, D., Clochesy, J. M., & Pontin, D. (2013). Safety in numbers 2: Competency modelling and diagnostic error assessment in medication dosage calculation problem-solving. *Nurse Education in Practice*. 13(2): e23-32.

- Weeks, K. W., Lyne, P., Mosely, L., & Torrance, C. (2001). The strive for clinical effectiveness in medication dosage calculation problem solving skills: The role of constructivist theory in the design of a computer-based 'authentic world' learning environment. *Clinical Effectiveness in Nursing*, 5(1), 18-25.
- Weeks, K. W., Lyne, P., & Torrance, C. (2000). Written drug dosage errors made by students: The threat to clinical effectiveness and the need for a new approach. *Clinical Effectiveness in Nursing*, 4(1), 20-29.
- Weeks, K. W., Sabin, M., Pontin, D., & Woolley, N. (2013). Safety in numbers: An introduction to the Nurse Education in Practice series. *Nurse Education in Practice*, 13(2): e4-e10.
- Weeks, K. W., & Woolley, N. (2007). *Innovative and Authentic Assessment of Medication Dosage Calculation Problem Solving Skills, via Engagement and Interaction with an Authentic World Learning Environment*. LTO Innovation Grant 2005-06 Report. Pontypridd: University of Glamorgan.
- Wiggins, G. (1989). A true test: Toward more authentic and equitable assessment. *Phi Delta Kappan*, 70(9), 703- 713.
- Wiliam, D. (2006). Does Assessment Hinder Learning? *ETS Europe Breakfast Seminar, 11th July* Retrieved 10th August, 2008, from http://www.uk.etseurope.org/home-corpo-uk/news-home/?print=1&news=136&view=detail&no_cache=1
- Williams, J. S., & Wake, G. D. (2007). Black boxes in workplace mathematics. *Educational Studies in Mathematics*, 64(3), 317-343.
- Wright, K. (2010). Do calculation errors by nurses cause medication errors in clinical practice? A literature review.

Counting or Caring: Examining A Nursing Aide's Third Eye Using Bourdieu's Concept Of Habitus

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Abstract

This article is derived from analysis of observations and an interview with, Anita, a nursing aide, who was followed in her work in a semi-emergency unit in Sweden. Based on an analysis of this information, it is suggested that the process of going from school to a workplace can be viewed as a transition between different mathematical activities, which involve and require learning. Although it is easy to see transitions occurring between different contexts, they may also occur within the boundaries of a workplace and be connected to critical moments in the execution of work tasks. Adopting a social critical perspective, this article initiates a discussion about the transitions between potentially mathematical activities in work and how the values given to these different activities can be understood. It is further suggested there is difficulty in recognizing some activities in work, because, often, they are over-shadowed by other competences and components needed in work, such as caring.

Key words: workplace mathematics; capital; habitus; transition; nursing aide

Introduction

Adults' mathematics learning takes place in a wide range of settings throughout life. Notwithstanding this, many of us have school mathematics as a reference for learning mathematics. The focus on mathematics within the context of schooling may make it difficult for one to detect and understand what mathematics may become in other spheres in life. In this article, I investigate potentially mathematical activities in a nursing aide's work using Bourdieu's (1992, 2000) concept of habitus²⁹, while taking into account that my previous experience as a vocational teacher of mathematics has influenced what I am able to see and understand.

The feeling of having a limited understanding of mathematics in workplaces, and a curiosity to learn more about this has been the driving force for my work. I became aware of my limited understanding of workplace mathematics when I worked as a mathematics

²⁹ Habitus is at its most basic definition humans' dispositions to act in the social world.

teacher in vocational education and prepared tasks on intravenous drips. The students were to become nursing aides and I produced tasks on different drip speed, drip size, and concentrations of active substances. Having received help from a nurse, the tasks seemed realistic, with proper substances and so on. I was very happy with the tasks, until the students reacted with stress, anxiety and fear. almost crying, they asked me if tasks such as these were really the responsibility of nursing aides'. I tried to calm them down and apologized several times for giving them inappropriate tasks, but I was left with a feeling of how limited I was in understanding the practice they were heading for.

Furthermore, I became more and more in doubt about school mathematics as being useful outside of school, although its relevance is often justified in this way (Dowling, 2005). The differences between mathematics taught in schools compared to mathematics in workplaces may have consequences for how adults' competence in workplaces is regarded (Gustafsson & Mouwitz, 2008, see also Björklund Boistrup & Gustafsson, in press). In addition, Wake (2013) suggests that workers sometimes do not even consider that what they are doing is related to mathematics, but rather to a goal-directed, workplace activity.

It is obvious that the use of mathematics in the work of nursing aides is situated in a certain context, influenced by many factors, such as the workplace organization, the well-being of patients, and possibly also the relationships that humans have with mathematics. Trying not to diminish these important aspects of the work situation, while remaining focused on the potentially mathematical activities, my aim is to consider how the transitions between different potentially mathematical activities can be understood through Bourdieu's, concepts of *habitus* and *capital* (Bourdieu, 1992, 1996, 2000, 2004). In so doing, I ventured to investigate if it is possible to do two things: (1) capture the mathematical knowledge frequently labelled as tacit, and (2) identify what may be gained and lost in different transitions humans make when moving between contexts.

Mathematics in work

Activities involving mathematics in workplaces are not easily described, as they are often connected to the use of the technology and routines (e.g. FitzSimons, 2013; Hoyles, Noss, Kent, & Bakker, 2010; Jorgensen Zevenbergen, 2010; Wake & Williams, 2007; Wedege, 2000, 2004a). Consequently, the mathematics in these activities has been discussed as being "*black-boxed*", both socially and technically (Wake & Williams, 2007). By this, they mean that the use of technology and automation has created a distinct genre of mathematics. With the increased use of technology, mathematics becomes more implicit, and hence technically black-boxed. Moreover, different groups and staff members in the workplace have different norms and rules, and this division of labour creates a social black-boxing. The notion of black-box derives from Latour (1993), and his networks of people, objects and ideas seemingly function as a whole, in which certain parts become invisible. Mathematics in work has also been labelled as tacit, with the possibilities for making it explicit considered difficult although not impossible (FitzSimons, 2002). These difficulties in identifying the mathematics may lead to a gap between adult learners' perspectives on learning mathematics, compared to those of education policy makers and employers (Evans, Wedege, & Yasukawa, 2013). FitzSimons (in press) found that curriculum and vocational numeracy education mostly was about content knowledge, and hence based on narrow assumption of what vocational

students may need. Instead of perceiving simple operations as sufficient for those students, FitzSimons further suggests that there is a need for a more holistic approach. With this approach not only the conceptual understanding is considered but also the creativity required in the workplace, and she notes:

However, in the workplace, as elsewhere in society, problems are ever-evolving and the development of new knowledge – locally new if not universally new – is an essential requirement for completing the task at hand within constraints of time and/or money, so workers often find themselves in ‘unthinkable’ territory, creating new knowledge. (FitzSimons, in press, p.1)

Hence, there is a complexity surrounding the mathematics involved in workplace activities. Another complexity is that researchers use both mathematics and numeracy to describe activities in the workplace. In this article I avoid this sometimes value-based distinction. Instead I use the term mathematics in a wide sense when focusing on the transitions between different potentially-mathematical activities of nursing aides.

Mathematics for nurses

Similar to the case with other workplaces, previous studies have shown that the mathematics used by nurses is strongly interwoven with the practice of nursing, its routines and other important considerations (Coben, 2010; Pozzi, Noss, & Hoyles 1998). Consequently, there may be differences for nurses in what they learnt in their formal education and what they actually do. For example, Pozzi, et al. (1998) described how nurses considered that “knowing the drug” was a better safeguard against errors, than using an algorithmic calculation suggested in teaching text for nurses. The safety of patients is crucial in this kind of work, and mistakes in an emergency unit can have serious and fatal consequences (Coben, 2010). In such a context, numeracy is about being confident, competent, and comfortable in deciding whether to use mathematics and how (Coben, 2010).

Although the work of nurses has some similarities to that of a nursing aide, the fact that nursing aides do not have medical responsibility indicates that there are also some differences. For example, there are differences in the social and historical conditions around these professions and the division of labour involved. These kinds of differences are important because they may have an impact on Williams and Wake’s (2007) social black-boxing. Evertsson (1995) claims that the history of the profession of nursing aides is, to a large extent, neglected and over-shadowed by a focus on nurses. Caring institutions and hospitals reflect the power structures in society and in the educational system, with regard to class and gender, for example (Evertsson, 1995).

A sociomathematical approach and Bourdieu’s concepts of habitus

In the sociomathematical approach (Wedeg, 2004b), mathematics is related to more than mathematics as an academic discipline, and also it takes into account mathematics as a social phenomenon and school subject (Wedeg, 2004b). Examples of sociomathematical research interests are peoples’ relation to mathematics, and the function of mathematics education in society (Wedeg, 2004b). Both humans and general structures are in focus in this approach, and in particular the interplay between *general* structures and *subjective* meaning is highlighted. The tension between humans and societal structures is captured by Wedeg

(2004b, 2010), who makes a distinction between *demands* made on humans, for example in school or as requirements for getting a job, and the mathematics *developed* by humans in certain practices.

Following Wedege's (2004b, 2010) suggestion, I consider general structures of the workplace as having certain mathematical demands on nursing aides, but also that the nursing aides themselves develop their own mathematical competence. This individual creativity may be crucial when facing difficult tasks or demanding tasks (FitzSimons, in press). By this, I do not mean that official demands made on workers are necessarily more or less important than the developed competences, rather they are complementary. One example of a study accounting for both the demanded mathematics and what is developed by workers, could be given by the findings of Pozzi, et al. (1998). They compare the common algorithmic procedures and formulas suggested for drug administration in teaching texts for nurses, with what nurses actually do. The formal and assumed calculations in the teaching texts for nurses are examples of the demanded mathematics. The authors further observed how the nurses instead used the specific concentration and more for each drug as the basis for their calculations. One example of such a calculation was '*doubling it and put an extra zero*', which was the calculation used for a certain drug (Pozzi, et al., 1998, p.110).

This way of finding alternative arithmetic methods by "knowing the drug", as described by Pozzi, et al. was also the safeguard against errors. This provides an illustration of the developed mathematics. In this article my focus is the transition between the demanded and the developed mathematics. The urge to capture the knowing to be found in those transitions has also guided my theoretical choice. In the framework of Bourdieu the mutual interplay between humans and society is captured in the concepts of *habitus*. Habitus is a system of dispositions, defined by Bourdieu (1992) as:

The conditions associated with a particular class of conditions of existence produce habitus, systems of durable, transposable dispositions, structured structures predisposed to function as structuring structures, that is, as principles which generate and organize practices and representations that can be objectively adapted to their outcomes without presupposing a conscious aiming at ends or an express mastery of the operations necessary in order to attain them. Objectively 'regulated' and 'regular' without being in any way the product of obedience to rules, they can be collectively orchestrated without being the product of the organizing action of a conductor. (p. 53)

Thus, the habitus is to be found firstly in the conditions of existence, which commit humans to the social structures, sometimes without objective and conscious goal orientation. Moreover, habitus is a complex system of dispositions for acting in the social world, transposable but at the same time durable and carrying collective features. In Wedege's (1999) research, a woman's habitus was shown to have influenced her dispositions towards mathematics, and her dispositions for seeing herself as mathematically competent. The woman was born in a saddler's family in Denmark at the beginning of the previous century, and failed in school mathematics. This outcome was seen as normal for a girl at this time. Later success in mathematics and involvement in mathematics at work and during leisure time, could not completely overcome how the woman perceived herself with regard to mathematics. With this case Wedege provides an example of the complexity of habitus

carrying features of both class and gender, not as stereotypical labels but rather inscribed in the person as natural features in a specific context. It is also clear in this woman's case how habitus is durable yet transposable or changeable, as she never fully ceased to see her failures in mathematics. If mathematics is the foundation of the sociomathematical approach (Wedegé; 2004b, 2010), then, as discussed in the next section, Bourdieu's different *capitals* act as the link between humans beings and social structures in a framework via habitus.

Bourdieu's concepts of capital in relation to habitus

Bourdieu (1992, 1996, 2000, 2004), described different forms of capital, such as economic, cultural, and symbolic and saw them as the link between the individual and the social world. Humans act in the social world to convert one form of capital to another, according to which form is valued in the particular social space (Broady, 1998). Cultural capital has to do with education, as a consideration of both upbringing and the educational system. How the cultural capital is valued may differ according to different cultures and school systems. In Bourdieu's work, the French system was in focus, and so his discussion of the impact of cultural capital may not be valid in another context or at different points in time. However, as Williams (2012) notes that this kind of capital has an exchange value. By this, he means that a particular mark or degree in school mathematics becomes an entrance ticket to certain jobs or further education. Consequently, cultural capital can be considered the formal education for nursing aides in mathematics. Social capital is the social relations or contacts and can also give humans a kind of interest rate on their educational capital. The symbolic capital refers to what is valued in a certain social space. This kind of capital can grow into the body and become part of our habitus, sometimes unconscious and invisible, even to ourselves. For example, when we just know what to do in a given situation, often out an obvious necessity and make use of our embodied symbolic capital (Bourdieu, 2000). Corporal mechanisms and mental schemata in a person's habitus can even erase the distinction between the physical and the spiritual world (Wacquant, 2004).

In this study, the focus will be on habitus, cultural capital, and symbolic capital. The concepts of Bourdieu have also been used for earlier studies concerning mathematics in work, and were found to be useful tools for the theorization of the world of work, and how mathematical dispositions may promote or hinder workers (Zevenbergen Jorgensen, 2010). The concepts of Bourdieu were in this study useful for understanding the younger workers' skills, instead of seeing the young workers as having limited numeracy. Zevenbergen Jorgensen found significant differences in habitus, and also how these differences created tensions between old and young workers based on their ways of seeing and enacting numeracy. The younger generations' habitus were to a larger extent influenced by digital technology, while the older had more manual arithmetic frames of reference. Bourdieu does not explicitly mention mathematics as a form of capital. In his later work, he did emphasise how there was a shift from Latin to mathematics as a selection tool in the educational system (Williams, 2012). The importance given to mathematics in the education system influences its value. What is considered as important and relevant with regard to mathematics also changes over time as shown by Zevenbergen Jorgensen (2010). Therefore, it is likely that different mathematical activities are valued differently and hence hold various amounts and forms of capital. This is important in a study about nursing aides, where the hierarchical workplace organisation as a practical and rational matter, may conceal other power relations

(Evertsson, 1995). These relations affect the valuing and attention paid to certain activities. The focus in this article is potentially mathematical activities, and when nursing aide may need to make *transitions* between these in critical situations.

Transitions

In transitioning from school to work, for example, it is important to recognise the transformation and creation of new relations between knowledge and social activities, and how this could contribute to an understanding of mathematics in the workplace (Wake, 2013). Meaney and Lange (2013) see transitions between contexts as always involving learning, with contexts being defined as systems of knowledge enacted in social practices. The notion of transition could also be seen as a way around the issue of transfer (Beach, 1999). Beach claims that transfer derives from educational psychology and refers to cognitive matters. From a purely cognitive approach, transfer is seen as relatively unproblematic (Evans, 1999). Evans notes that from a situated perspective, transfer instead should be considered impossible. Beach (1999) suggested an alternative stance from a sociocultural viewpoint, namely that of consequential transitions, which means transitions that are reflected upon from a sociocultural perspective. Thus, the social and historical context of the activity is taken into account as well as the artefacts involved. Beach also identifies several forms of transition. In this article, I make use of Beach's lateral and encompassing transitions. The former occurs when individuals move between contexts such as school and work, and the latter when change occurs within the boundaries of a social activity.

Encompassing transitions I suggest can be related to the moving between the demanded and developed mathematics in the sociomathematical approach. By this I mean that the official ways to handle a work task are related to the demanded mathematics. Workers also develop complementary ways of completing the potentially mathematical tasks. In other words it is likely that workers in general, and nursing aides in particular, need to make transitions between what is demanded and what they develop. I find it important to shed light on the dichotomy between developed and demanded, as the transition between these has and holds learning opportunities. I suggest that this can be done by understanding different ways of being engaged in potentially mathematical activities, the transitions between them, and the value the activities are given.

For this purpose I have chosen the concept of habitus (Bourdieu, 1992, 2000), firstly because of its possibility to grasp the interplay between individuals and structures. Secondly, my reason for choosing habitus and capital is the fact that habitus has a clear corporal component and different capitals can grow into the body. Hence, there is a possibility that the body becomes itself a black box. It is important to try to understand the significance of the bodily understanding when people transition between the demanded mathematics and the developed. This I see as crucial for reducing the gap between adult learners' perspectives on learning mathematics and those of education policy makers and employers. The incorporation of capitals is also connected to learning. Bourdieu (2000) describes learning as a durable bodily change (see also Wacquant, 2004). Habitus as a theoretical choice calls for methodological explanation and justification.

Methodology

My intention with this small scale case study (Bryman, 2008) is to understand and construe the transitions made within the boundaries of a workplace not familiar to me. This is done through my interpretation of the work of Anita (a pseudonym). As a matter of reflexivity (Hammersly & Atkinson, 1995; Malterud, 2001), my lack of previous experience was used as an advantage. What was obvious for a person with Anita's long experience was not at all evident to me. This made it possible to pinpoint tacit knowing. Anita is a nursing aide, with more than twenty years of experience, both in her home country in Eastern Europe and in Sweden. The empirical part of the study was conducted with inspiration from ethnography (Hammersly & Atkinson, 1995). However, in a study of this format it is not possible to provide the descriptive thickness normally associated with ethnography. In addition, there is the ethical dilemma of construing another person's habitus. Bourdieu (2000, p. 128) writes: "Even among specialists of the social sciences, there will always be those who will deny the right to objectify another subject and to produce its objective truth."

Access was facilitated by a research team member having personal contact with a nurse. From this contact we were introduced to a physician, also head of the ward. He gave us permission to enter the ward with a video-camera. On the ward the nursing aide in charge picked a colleague for us to follow. First our intention was to follow the nursing aide in charge, but she wanted, as she said, to give this opportunity to a colleague of hers. This she told us was because the nursing aides were so rarely paid attention. Two video-recorded visits were made in a hospital in Sweden 2012, each lasting for about an hour. These were then transcribed. After an initial analysis, an informal interview was held with Anita. Having in mind that that mathematics in the workplace might be black-boxed, both technically and socially, the topics of the interview were to a large extent introduced by Anita. She talked much about how the profession of nursing aides had developed from formerly being about assisting nurses to nowadays being what she called "its own profession". This is aligned with Evertsson's (1995) historical analysis of the profession overshadowed by a focus on nurses.

The interview was tape-recorded and partly transcribed. The sound was of good quality except a short part which was difficult to hear as Anita and I watched the video together. In qualitative research the reliability is often referred to as dependability (Bryman, 2008), and the data loss when we watched the video was compensated by what was gained by Anita explaining what had happened during the observation. As the interview was conducted a couple of months after the observation, looking at the video were also crucial for refreshing our memory. Another way of ensuring dependability was to look at the video together with researchers in the team before conducting the interview. From the individual case of Anita alone it is not possible to make any generalisations, frequently labelled as transferability in qualitative research (Bryman, 2008; Malterud, 2001). With the concepts of Bourdieu it is, however, possible to connect the individual case to the social structures in society. This is aligned with the methodology proposed by Salling Olesen (2012). He notes that workers or groups of workers invest their body and soul, knowledge and commitment when entering a workplace, but they do so against the background of a life history that is a part of a wider societal context (Salling Olesen, 2008, 2012). About, using an individual case Salling Olesen (2012) claims:

It is to use this individual case to theorize learning as an aspect of the social practice, a moment in a subjective life history embedded in the symbolic and social environment, and contributing to societal processes of reproduction as well as innovations. (p. 5)

This methodological view – taking the connection between individuals and society into account – is aligned with the framework of Bourdieu. The possibility for including the bodily manifestation of habitus was facilitated by the use of video and the opportunity to watch it several times. Thereafter, it was possible to raise questions about issues not understandable by the observation alone. My intention with carrying out firstly observations and then the interview was to grasp the complexity of habitus, in which my own habitus, with its own connection to mathematics education, was also considered. Therefore, I have tried to be attentive to and reflect on my own relation to mathematics, and to school. School mathematics will have influenced our perceptions of mathematics; both regarding what should be included as mathematics, and also as a personal relation and experience of it. Thus, our understanding of mathematics and emotions related to it are likely to be connected to the school mathematics incorporated into our habitus (Lundin, 2008).

The analysis makes use of the sociomathematical concepts of demanded and developed mathematics. By this, I refer to the demanded mathematics as what is required in this kind of work, in relation to the mathematical activities that are developed in work. I start with a description of the observation, then I analyse what could be seen as demanded. Then the interview with Anita is described. The analysis is supported by the concepts of habitus and capital, and the notion transition.

Visiting Anita at work

The first meeting with Anita was made at the semi-emergency unit where she works. To blend in with the environment, those of us in the research team had to wear the same white clothes as the staff. I followed and observed Anita, while a research colleague was video-recording. This made it possible for me to ask questions in order to understand what was going on, which had to be done in a manner that did not disturb the work.

During this first visit, Anita was monitoring patients or “*tog kontrollør*” (which means “took controls” in English) on the patients, as said it is described on the ward. The patients were connected to digital supervision monitors. Controls, she told us, were made every four hours and included collection of the physiological parameters: respiratory rate, heart rate, blood pressure, temperature, urine output, and alertness. Different values of these parameters were given colours and scores on a chart. The chart (see Figure 1) was coloured outwards, from green in the middle (0), then yellow (1), orange (2), and, finally, red (3) at either end. The red columns indicated the most critical values. A total score ≥ 5 required immediate attentions from a doctor and the emergency team. There is also an additional text in the chart, which says that deep concerns about a patient or acute deterioration are other reasons for contacting the doctor and the emergency team.

During the control of one patient, a doctor was summoned to take a blood sample for a blood gas analysis. The blood gases are connected to several of the physiological parameters in the chart, but give another kind of description of the patient’s condition. The doctor arrived quickly and took a blood sample and disappeared after a few minutes. Anita then

took the sample to a digital laboratory where the analysis was performed and automatically transferred onto digital patient records. In the digital laboratory, Anita said: "This will take a minute, but one minute is a long time so I can do other things instead, so I will not wait for the test results". After this, Anita returned to other patients to encourage, console and chat, while further controls were made. None of the controls were apparent to an observer, but these were explained by Anita afterwards. The observation clearly noted that Anita was devoted to caring and comforting.

Score	3	2	1	0	1	2	3
Resp		<9		9-14	15-20	21-29	≥30
Puls/min		≤40	41-50	51-100	101-110	111-129	≥130
Syst bltr	≤70	71-80	81-100	101-199			
mp C		≤35	35.1-36	36.1-38	38.1-38.5	>38.5	
O m/h	<20 ml/h		Nytilkommen förvirring <35 ml/h	Alert	Reagerar på tilltal Stor urinprod	Reagerar på smärta	Reagerar ej

Figure 1. The coloured chart, with normal and critical values.

After having collected the values from the patients Anita took out a piece of scrap paper from her pocket (Figure 2) and typed in values in the patients’ digital hospital record. The scrap paper shows the data collected from two patients.

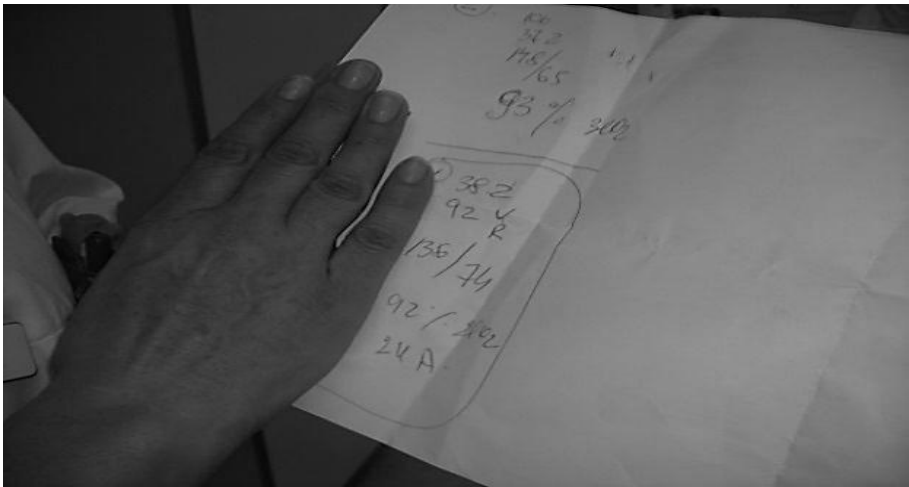


Figure 2. Anita’s scrap paper showing one patient’s values are circled.

While these routines appear to be very structured, and even have numerical and mathematical content, the nursing aides on the ward did not seem to have these perceptions. On several occasions, they said "you know" or "you feel" or even "it's the third eye". We were told, during the observation, that the "*third eye*" was an important characteristic of the nursing aides' skills. During the interview, Anita told me more about the third eye. She said that "everybody can have it, but not from the beginning," suggesting that it develops from experience. There also seemed to be a tension between the rational chart, based on numeric values, and the more elusive feeling of just knowing. This feeling is discussed later in more detail.

While the observation clearly noted that Anita was devoted to caring and comforting, it was not noted that she also, for instance, counted breaths minute. This became clear during the interview, when she gave an example of how she is counting breaths. It was also visible on her paper, which she showed us after she had taken the controls. The respiratory rate or breaths per minute is noted as "24 A" on the paper in Figure 2. Otherwise, the explicit use of numerical values seemed absent, as these were probably not very interesting for the patients. Only once did Anita explicitly talk about values, and it was to convince a patient about her recovery: "Your values are much better today than they were yesterday, so much better".

Analysis of the demands

After having visited Anita at work, and also interviewing her, it was obvious that taking the controls on the patients was one of her regular and important work tasks. The coloured chart with its columns and scores can be considered as part of the explicit and demanded mathematics. So, from the sociomathematical viewpoint, handling the chart can be seen as one mathematical requirement for nursing aides. The coloured chart was used for facilitating the judgment of a patient's condition, and to identify patients at risk of catastrophic deterioration. In order to take the different parameters measured into account, these are given different scores, and a total score of five or more is defined as a risk, which needs attention from a doctor. Understood in mathematical terms, nursing aides need skills in:

- ☐ Reading a chart
- ☐ Understanding distribution of values, facilitated by colours
- ☐ Comparing values
- ☐ Adding values

This could be considered as basic mathematics by for example a mathematics teacher. By giving the values different scores which need to be added, it can be reduced to simple calculations. This work can be considered as needing only a limited amount of cultural capital, or education.

The interview with Anita makes clear that "17 is not always 17"

When I met Anita we had a conversation about working as a nursing aide. With her many years of experience, also in different countries, she had a lot to tell. Due to her experience

and by having a mother tongue other than Swedish she also volunteers as an interpreter on the ward. This she told me has a certain value for doctors and of course for patients, in a semi-emergency unit where quick decisions are crucial. Over and above this she talked about her presence as having a calming influence on immigrant patients because they felt confident with her. She had difficulties in explaining this but phrased it as: "She is like us". The conversation also covered much about workplace education but did not turn to mathematics, which I wanted to understand more about it in relation to this workplace. (The Swedish original transcript is given in brackets)

Maria: *I was thinking about this technical, technical education, and such. All the things you do with the tests, reading the monitors, using the charts, and so on. To me it seems somehow like mathematics.* (Jag tänkte på det där, det där med teknisk utbildning och sånt. Alla saker du gör med tester, avläsa monitorer, använda tabeller och sånt. För mig verkar det på något sätt som matematik.)

Anita: *Yes, it is!* (Ja, det är det!)

Maria: *But I don't know... if I think about school... how could school provide this education?* (Men jag vet inte... om jag tänker på skolan... hur skulle skolan kunna utbilda för detta?)

Anita: *Ah, okay you mean like that ... one should have basic mathematical skills, absolutely, like percentages, one should have the basic knowledge but nobody will ask about sine and cosine, nobody will ... but I have learnt it and everybody has but for our profession I mean that we need the basic stuff. It has to do with percentages, addition and subtraction. That is necessary but I take for granted that everybody knows that.* (Ah, okej, du menar så... man ska ha, man ska ha grundläggande matematiska kunskaper, absolut, som procent, man ska ha grundläggande kunskap men ingen kommer att fråga efter sinus och cosinus, det kommer ingen att göra, men jag kan det och alla kan det men för vårt yrke är det vi behöver det grundläggande. Det har att göra med procent, addition och subtraktion. Detta är nödvändigt men det tar jag för givet att alla kan.)

It worth noting that Anita considered trigonometry as something that everybody has learnt. As I perceived that there was something, from my own school mathematical habitus "vaguely mathematical" in her work, I tried to find out more about it:

Maria: *It is difficult to explain, but all the judgments you make and all the priorities you have, and the fact that you do it differently...like when you read from the monitor and taking the pulse manually.* (Det är svårt att förklara men alla bedömningar du gör och alla prioriteringar, och just det att du gör det annorlunda...till exempel när du avläste monitorn och tog pulsen samtidigt.) **Anita:** *I think it is normal and obvious. Such thing cannot be learnt in school.* (Jag tycker att det är normalt och självklart. Sånt kan man inte lära sig i skolan.)

Maria: *For you it is obvious but for me coming from school it is very interesting.* (För dig är det självklart men för mig som kommer från skolan är det väldigt intressant.)

Anita: *For example I count the respiratory rate. All people breathe differently and when they are ill even more differently. Such things you don't learn ... so I count the respiratory rate of one patient and get 17 let's say 17 but I have learnt that this patient has 17 because s/he is ill. I can judge that, I can judge that the other has 17 because s/he really doesn't feel well, yet another has 17 because s/he is hyperventilating, and that one has 17 by pretending in order to get more morphine than s/he has already got, and that one has 17 because...* (Till exempel så räknar jag andningsfrekvensen. Alla människor andas olika och när de är sjuka ännu mer annorlunda. Såna saker lär du inte dig i skolan...så jag räknar andningsfrekvensen hos en patient och får 17, säg 17, men jag har lärt mig att hon har 17 därför att hon är sjuk. Jag kan avgöra det. Jag kan avgöra att en annan har 17 därför att han eller hon verkligen inte mår bra, och en annan har 17 för att han eller hon hyperventilerar, och den har 17 för att den låtsas för att få mer morfin än den redan har fått, och den har 17 för att...)

Maria: *I think it is really interesting that 17 can mean so many things compared to school, where it means 17.* (Jag tycker verkligen att det är intressant att 17 kan betyda så olika för i skolan betyder det ju 17.)

Anita: *This is something completely different, and everybody here would have told you exactly the same.* (Detta är något helt annat och vem du än skulle fråga så skulle du få samma svar.)

Being a bit confused by 17 not being 17, I return to this issue again:

Maria: *This I find really interesting ... even this you are saying about the respiratory rate. Because you mean that we have different lungs and you cannot know how much oxygen a breath contains neither measure the volume of the lungs, so this is replaced by a feeling...that you feel what 17 means in this case.* (Det här är ju verkligen intressant...även detta du säger med andningsfrekvensen. För att du menar att vi har olika lungor och du kan inte veta hur mycket syre varje andetag innehåller eller mäta lungornas volym, så detta ersätter du med en känsla, att du ser och känner vad som menas med 17 för just den patienten.)

Anita: *Yes, it is the same as with the woman I just looked after. On our ward we have a machine that is connected to the patient and from that I have learnt to read how much air that is getting into the lungs.* (Ja, det är samma som med den kvinnan jag nyss tittade till. På vår avdelning har vi en maskin som kopplas till patienten och där vi har lärt oss att avläsa hur mycket luft som kommer in i lungorna.)

Maria: *Aha... (Aha...)*

Anita: *Such things are learnt here in work* (Såna saker lär man sig här på arbetet.)

Maria: *This machine...now I must try to understand...this machine can measure what you have a feel for?* (Alltså den här maskinen...nu måste jag försöka förstå...den här maskinen mäter det du känner på dig?)

Anita: *Yes, something like that.* (Ja, någonting sånt.)

Maria: *It is actually quite...* (Det är ju faktiskt ganska....)

Anita: *Yes, something like that...* (Ja, någonting sånt...)

This extract of the interview with Anita suggests that there is more going on than merely the collection of the patients' values and comparison of these with the values on the coloured chart. Instead, Anita has developed an experienced-based abstract feeling for the patients' condition – a third eye. As an example of this feeling she takes 17 breaths per minute to illustrate what differences in meaning an isolated and discrete figure can have.

Analysis of the developed mathematics and the “third eye”

Taking the respiratory rate as an example, the chart gives concrete and decontextualized values. This I suggest is in contrast to what Anita says about counting to 17, as an abstract value related to many other parameters as, for example, the depth of each breath, or about a patient's simulation of illness. When the rate of breaths is connected to, for example, the volume of lungs, the depth and the pressure, it is closer to a function of oxygen saturation of the blood than to discrete and concrete values. Anita's explanation of counting to 17 does not explicitly refer to mathematics, although many different parameters and the relations between them are taken into account. Instead I suggest that it refers to “the feeling” the nursing aides have, which they also label as “the third eye”.

I interpret the “third eye” as what is developed from a sociomathematical perspective. So, having the third eye could be seen as a symbolic and also embodied capital shared by competent nursing aides, a disposition to understand the patients' conditions and act accordingly. In critical situations there is no time for reasoning. Instead, having the habitus of a skilled nursing aide involves the corporal or sensual component of the “third eye”, allowing for judgements and decisions. Furthermore, relating the respiratory rate to many different parameters requires a higher level of abstraction than just calculating a total sum of 5. By this I do not mean that one form of knowing is preferable to the other, but rather how both forms of knowledge can be seen as complementary.

What is, instead, interesting to note is the transition between the explicitly demanded mathematics on the chart and the developed but elusive feeling. This transition I suggest requires a habitus with the “third eye,” but also the demanded mathematics as cultural capital, and hence formal education. Moreover, it requires confidence and some power to, if needed, go against the coloured chart. The mathematics demanded in this case is rather basic, but facilitates the workplace routines and probably increases patient security. However, it seems crucial to be competent, confident, and comfortable about how to and when to use mathematics, as Coben (2010) suggests in her definition of numeracy for nurses. I suggest that the skill of making these transitions should be paid more attention, and also the connection to reasoning and to being critical in general.

Discussion

Although mathematics in work is different from school, it could be relevant to consider the similarities between the mathematics demanded in work, such as the coloured chart, and school mathematics. In doing so it is also necessary to pay attention to the transitions that have to be made in critical situations. An example of this is when Anita compared her

capacity, or her “third eye,” to a number on the chart. Therefore, it would be a mistake to consider the explicit demanded mathematics as what is needed purely in terms of school mathematics. In connection to this it is also worth mentioning the limitation that my school mathematical habitus places on understanding what is actually going on. My focus when doing observations was on the chart and technical tools. It was not until I had the conversation with Anita that I became aware of what else was going on.

It would be naïve to believe that this activity can easily be contextualized in school mathematical tasks. Furthermore, the development of a “third eye” could not possibly happen in school, as Anita noted. A third-eye is certainly not gained from so-called real world problems. A misplaced contextualization can even make the task less accessible for several reasons. It may be that it makes students worried because it is not in line with the work they are heading for, as was the case with my intravenous drip task. Another risk is that the contextualization restricts or overshadows the mathematical content, which gets less space and probably becomes insufficient for vocational students. Therefore, the benefit of contextualized tasks should be further investigated, although it is still important to take into account the complexity in work and influencing factors others than mathematics.

Viewing learning as a bodily change (Bourdieu 2000, Wacquant, 2004) the embodied knowledge that Anita developed as a feeling of what 17 means in a particular case is, to some extent, related to mathematics and a crucial competence in Anita’s work. The symbolic capital of the “third eye” grows into and becomes a part of the body and senses and thus a part of habitus, as a form of bodily knowing. However, it is less likely to be acknowledged than the explicit use of mathematics found in the demands and in the chart. So, ironically, in this work transitioning from a lower level of abstraction to a higher leads to a loss of the visible need for cultural capital. The embodied knowledge is rendered invisible as it becomes a part of an experienced nursing aide’s habitus with the “third eye” as an important characteristic. The third eye is consistent with what Wacquant (2004) writes about corporal and mental schemata of habitus erasing the distinction between the physical and the spiritual world. The difficulties in detecting this knowing, together with the historical subordination of nursing aides, may lead to an assumption that abstractions or mathematical reasoning are neither needed nor used by nursing aides.

When observing the semi-emergency unit with a video camera, I could not by any means perceive that Anita was focusing on counting breaths per minute and judging the rate in relation to other conditions. What I perceived was, instead, how she cared for the patients and gave them comfort. I suggest that this could also be seen as an example of the black-boxing, mentioned by Williams and Wake (2007). She was apparently doing both caring and counting simultaneously in order to make the patient feel comfortable while she was counting. This is also aligned with common requirements in a workplace where conceptual knowledge and creativity are mutually dependant when completing work tasks (FitzSimons, in press).

Making the assumption about the profession of nursing aides as being mostly about caring is misleading. Instead, I see a need to further investigate the kind of knowing that is frequently labelled as tacit, or as being black-boxed either technically or socially. It is important to take into account how the profession of nursing aides has been viewed in the

past, how it has developed and is viewed in our current society. Certainly, the transitions between the chart and the “third eye” require a particular competence and experience. From a sociomathematical viewpoint, it is an act of balancing between the demanded mathematics and the developed. Seen as a reflected transition these acts of balancing require learning. If vocational students are provided with short courses, or restricted curricula, it will have serious consequences. These will not only affect the possibilities of gaining access to higher education, but may also lead to the presumption that the work force is easily educated and replaceable. This is just the opposite of what Anita explains about her work.

What is happening in the transition between the demanded mathematics and the developed in terms of corporal understanding needs to be further researched. I believe that it is also highly relevant to consider the transitions adult learners have undertaken, such as, for example, moving from school to work, or moving between other kinds of contexts such as different countries. An example is when Anita refers to trigonometry as something that everyone knows, but which is different to how I view it. It seems as if this cultural capital gets lost in her transition to a new country. This could also be seen in relation to how Anita volunteers as interpreter on the ward. Whether she knows trigonometry, or not, is not of interest in this work. Instead, she had made use of her language skills, and probably gained a position on the ward through doing so. There also seem to be parts of her habitus shared by immigrant patients who feel comfortable and secure with Anita.

The question of whether it is common or rare to know trigonometry is however hanging in the air. It is not possible to know retrospectively if this knowledge could have been an advantage in the Swedish education system. Some parts of habitus or certain capitals may be lost or rendered invisible in different transitions, and others may instead be rendered visible. The question is who benefits from what is gained and lost in these transitions, and the overall question that I think needs further investigation is: What can we learn from the different transitions learners make, and how can these be related to mathematics? For this purpose, the concepts of capital and habitus, and, more specifically, the changes habitus undergoes in transitions, could be useful as analytical tools. An underlying question is if the label tacit knowledge is more relevant for work than for school, or if it could be that certain forms of knowing are silenced?

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References

- Beach, K. D. (1999). Consequential transitions: A sociocultural expedition beyond transfer in education. *Review of Research in Education*, 28, 46-69.
- Björklund Boistrup, L. & Gustafsson, L. (In press). Construing mathematics-containing activities in adults' workplace competences: Analysis of institutional and multimodal aspects. *Adults Learning Mathematics: An International Journal*. Special issue.
- Bourdieu, P. (1992). *The logic of practice*. (New ed.) Cambridge: Polity.
- Bourdieu, P. (1996). *The rules of art: genesis and structure of the literary field*. Cambridge: Polity Press.
- Bourdieu, P. (2000). *Pascalian meditations*. Stanford, Calif.: Stanford University Press.
- Bourdieu, P. (2004). *Science of science and reflexivity*. Oxford: Polity.
- Broady, D. (1998). Kapitalbegreppet: Som utbildningssociologiskt verktyg (Sketronhäften, 15). Uppsala: Forskningsgruppen för utbildnings- och Kultursociologi.15.
- Bryman, A. (2008). *Social research methods*. (3. ed.) Oxford: Oxford University Press.
- Coben, D. (2010). At the sharp end of education for an ethical, equitable and numerate society: Working in a safety-critical context – Numeracy for nursing. In U. Gellert, E. Jablonka & C. Morgan (Eds.), *Proceedings of the Sixth International Mathematics Education and Society Conference*.(pp.9-22). Berlin, Germany: Freie Universität.
- Dowling, P. (2005). *The sociology of mathematics education: Mathematical myths pedagogic texts*. London: Routledge FALMer.
- Evans, J. (1999). Building bridges: Reflections on the problem of the transfer of learning in mathematics. *Educational Studies in Mathematics*, 39 (1-3), 23-44
- Evans, J., Wedege, T., & Yasukawa, K. (2013). Critical perspectives on adults' mathematics education. In Clements, K.; Bishop, A. J.; Keitel, C.; Kilpatrick, J.; Leung, F. K. S (Eds.), *The Third International Handbook of Mathematics Education* (pp. 203-242). Dordrecht: Springer.
- Evertsson, L. (1995). Vårdbiträden och undersköterskor – en ouppmärksam professionshistoria, I: Johansson, S (red.) *Sjukhus och hem som arbetsplats. Omsorgsyrken i Norge, Sverige och Finland*. Stockholm & Oslo: Bonnier Utbildning & Universitetsförlaget.
- FitzSimons, G.E. (2002). *What counts as mathematics?: Technologies of power in adult and vocational education*. Boston: Kluwer Academic Publishers.
- FitzSimons, G. E. (2013). Doing mathematics in the workplace: A brief review of selected recent literature. *Adults Learning Mathematics: An International Journal*, 8(1), 7-19.
- FitzSimons, G. E. (in press). Learning mathematics in and out of school: A workplace education perspective. In U. Gellert, J. Giménez, C. Hahn, & S. Kafoussi (Eds.), *Ways into mathematics: A C.I.E.A.E.M. sourcebook*.
- Gustafsson, L. & Mouwitz, L. (2008). *Validation of adults' proficiency: Fairness in focus*. Göteborg: Nationellt centrum för matematikutbildning, Göteborgs Universitet.
- Hammersley, M. & Atkinson, P. (1995). *Ethnography: Principles in practice*. (2. ed.) London: Routledge.
- Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for techno-mathematical literacies*. New York: Routledge.
- Latour, B. (1993). *We have never been modern*. Cambridge, Mass.: Harvard Univ. Press.
- Lundin, S. (2008). *Skolans matematik: en kritisk analys av den svenska skolmatematikens förhistoria, uppkomst och utveckling - The mathematics of schooling : a critical analysis of the prehistory, birth and development of Swedish mathematics education*. Diss. Uppsala : Uppsala universitet, 2008. Uppsala.
- Malterud, K. (2001). Qualitative research: Standards, challenges, and guidelines. *Lancet*, 358, 483-488.
- Meaney, T. & Lange, T. (2013). Learners in transition between contexts. In K. Clements, A. J. Bishop, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Third international handbook of mathematics education*. (pp. 169-201). Dordrecht: Springer.
- Pozzi, S., Noss, R., & Hoyles, C. (1998). Tools in practice, mathematics in use. *Educational Studies in Mathematics*, 36(2), 105-122.
- Salling Olesen, H. (2008). Workplace learning. In P. Jarvis (Ed.), *The routledge international handbook of lifelong learning* (pp. 114-128). London: Routledge Mental Health.
- Salling Olesen, H. (2012). The societal nature of subjectivity: An interdisciplinary methodological challenge. *Forum Qualitative Sozialforschung / Forum: Qualitative Social Research*, 13, Art.4.
- Wacquant, L.J.D. (2004). *Body & soul: notebooks of an apprentice boxer*. Oxford: Oxford University Press.
- Wake, G. (2013) Making sense of and with mathematics: the interface between academic mathematics and mathematics in practice. *Educational Studies in Mathematics*, (under review).
- Wedge, T. (1999). To know or not to know: Mathematics, that is a question of context. *Educational Studies in Mathematics*, 39(1/3), 205-227.

- Wedge, T. (2000). *Matematikvide nog teknologiske kompetencer hos kortudannede voksne*. MFUFA-tekst 381. (Doctoral thesis). Roskilde: Roskilde Universitetscenter.
- Wedge, T. (2004a). Mathematics at work: Researching adults' mathematics-containing competences. *Nordic Studies in Mathematics Education*, 9(2), 101-122.
- Wedge, T. (2004b). Sociomathematics: Researching adults' mathematics in work. In J. Maaß and W. Schlöglmann (Eds.), *Learning Mathematics to Live and Work in our World. Proceedings of the 10th International Conference on Adults Learning Mathematics* (pp. 38-48). Linz: Universitätsverlag Rudolf Trauner
- Wedge, T. (2010). Ethnomathematics and mathematical literacy: People knowing mathematics in society(keynote speech.) In C. Bergsten, E. Jablonka & T. Wedge (Eds.), *Mathematics and mathematics education: Cultural and social dimensions* (pp. 31-46). Linköping: Svensk Förening för Matematikdidaktisk Forskning, Linköping universitet.
- Williams, J., & Wake, G. (2007) Black boxes in workplace mathematics. *Educational Studies in Mathematics*, (64), 317-343.
- Williams, J. (2012). Use and exchange value in mathematics education: contemporary CHAT meets Boudieu's sociology. *Educational Studies in Mathematics*, 80, 57 – 72.
- Zevenbergen Jorgensen, R. (2010). Young workers and their dispositions towards mathematics: Tensions of a mathematical habitus in the retail industry. *Educational Studies in Mathematics*, 76, 87-100.

‘Don’t Ask Me about Maths – I Only Drive the Van’

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Abstract

The Mathematics experience in the classroom is disciplined by the curriculum and the expected learning outcomes, and taught with a terminal assessment in mind. Depending on the subject area and the course objectives, related topics are introduced in a sequence that suits the very practical considerations of the time available and the mixture of ability amongst the students and the teacher’s commitment and capacity to engage with the subject. The level of sophistication of the mathematics increases incrementally from the fundamentals towards the most abstract concepts to meet the expectations associated with the calibre of the academic award sought. Such circumstances stand in stark contrast with the experience of mathematics outside the classroom, to the extent that many people perceive themselves as not having any use for, or facility with, mathematics of any kind. A recent National Survey in Ireland (Keogh, 2013) designed to hear what people at work had to say about their use for mathematics, confirmed the use/denial paradox, i.e. that mathematics are generally important but personally irrelevant. Curiously, the same respondents self-reported high degrees of reliance on numerate behavior (Keogh, Maguire, & O’Donoghue, 2012). This paper recounts the numerate behaviour of a delivery van driver, and makes connections to the broad range of mathematics concepts upon which he relied and about which he was entirely oblivious. The key message in this paper is that work practice underpinned by mathematical thinking and behaving, however elaborate it might be, is dismissed routinely as something other than mathematics. Nevertheless, the opportunity to view his own set of mathematical knowledge and skills, however acquired, revised the van driver’s opinion of himself, such that he invited the researcher to tell his wife that he was not stupid.

Key words: adult numeracy, numerate behaviour, mathematics concepts

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Introduction

In keeping with the theme of the conference, this paper draws a contrast between mathematics as it is taught and learned inside the classroom and its role in the outside world, where it may become enmeshed in the complex business of life, living, community and work. This is not to suggest that the classroom construct is any less ‘real’ than the so-called real world, but rather to underline that it is a different reality. A simple example may serve to illustrate. Tom’s work requires him to visit a number of customers that are scattered throughout Dublin’s South Inner City. The quantity, variety and configuration of the customers change every day, as do their specific needs. The challenge is to do this as efficiently as possible to save time or money or to meet the conditions of some other operational metric. Inside the classroom, this is viewed as the classical Travelling Salesman Problem (TSP).

The ‘Virtual’ Travelling Salesman Problem

The ‘virtual’ TSP is the focus of the classroom. It is based on the conceit that a travelling salesman must visit a quantity of points, fixed in geography, by the most efficient tour; efficiency being a function of a common cost factor, e.g., fuel consumption, that is shared at least pairwise, Figure 1.

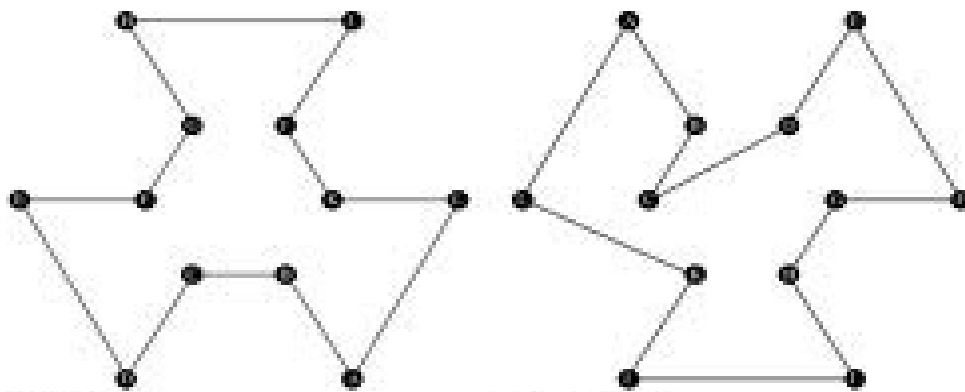


Figure 1. Alternative ‘tours’ visiting 12 points

Shown are two ways of touring 12 points, however, the number of possible solutions is given by the formula $(n-1)!$ which is an almost inconceivable 40 million tours, or thereabouts. Typically, other methods are used to determine a set of better solutions, among which the easiest to understand is the ‘greedy algorithm’, guiding the salesman to the next nearest neighbour, although this approach is thought to be sub-optimal. An alternative is a ‘brute force’ attack, which is to try, and try again, avoiding sub-tours, or linking locations around which the points seem to cluster. That this type of problem may be more easily solved by computing, led to the introduction of Dynamic Programming, Figure 2.

$$\begin{array}{llll}
\min & \sum_{(i,j) \in E} c_{ij}x_{ij} & & \\
\text{s.t.} & \sum_{j \in V} x_{ij} = 1 & \forall i \in V & (\text{assign_i}) \\
& \sum_{i \in V} x_{ij} = 1 & \forall j \in V & (\text{assign_j}) \\
& u_i - u_j + 1 \leq (|V| - 1)(1 - x_{ij}) & \forall (i,j) \in E, i \neq 1, j \neq 1 & (\text{mtz}) \\
& 2 \leq u_i \leq |V| & \forall i \in \{2, \dots, |V|\}, & \\
& x_{ij} \in \{0, 1\} & \forall (i,j) \in E &
\end{array}$$

Figure 2. An extract from the rationale that underpins Dynamic Programming

Dynamic programming builds on the Fibonacci Theory of sequences. It comprises an exhaustive cycle of calculations to determine the most efficient tour, depends on common ‘cost’ estimates and seeks to eliminate sub-tours. Whether this represents a useful support to the actual travelling salesman is not clear, for a variety of reasons.

The Actual Travelling Salesman Problem

Background

Tom works for a records management company, which stores up to 3 million items in a secure, fire- protected, purpose-built warehouse, with the promise to return them to the owner on demand. Every day, both in the morning and afternoon, Tom is provided with a bundle of up to twenty ‘work orders’ each detailing the workload for a specific customer.

Tom doesn’t ‘do’ maths; he drives a van

Tom’s first reaction is to assess the content of the work orders to determine whether he can pass on part of the work to a colleague on the basis of convenience. This is of limited benefit as it creates an informal obligation on him to reciprocate. Simply stated, Tom loads the van with items that have been placed in storage previously, and delivers them to order. Along the way, he must collect items from customers, return to base and unload the van, all the while adhering to the company’s Standard Operating Procedures and local traffic regulations. Tom may not know the precise distance involved in a visit to each customer, because being confined to a relatively small part of the city, distance is not the primary concern. Instead there are multiple competing factors which interact in different ways, depending on the time of year, day of the week and time of day which add layer upon layer of complexity to his task. It is in Tom’s interest to work efficiently, because he is not compensated for overtime. Superficially, there would not seem to be much evidence of numerate behaviour informed by mathematics knowledge, skills and competence, except for having to estimate the likely impact of a range of competing constraints.

Constraints

In deciding the optimal route, Tom takes account of traffic patterns that are affected by the school term, weather, temporary road closures, pedestrian ways, time-limited parking restrictions, roadworks, accidents, demonstrations, parades, and garbage collections. Wet

weather increases vehicle density, obscures visibility, and tends to clog the main arterial roads, causing Tom to choose alternative, minor roads, changing the delivery/collection sequence by bringing different customers into viable range. The side of the road on which a customer's premises is located, may influence the selection of the 'nearest neighbour' due to having to cross traffic or to navigate one-way traffic flow systems. The customer's readiness may cause him to adapt.

In addition, customers may not be ready to hand over their computer media backup at a time that suits Tom's optimum schedule. He is further constrained by having to observe the customer's standard operating procedures, their security arrangements, and parking facilities. Even the type of building can influence the time it takes to complete a delivery/collection, by whether it is equipped with an elevator. The loading sequence of the van has to be informed by Tom's solution to the TSP, such that the last items loaded will be the first to be unloaded, while allowing space for collections and load stability. Were the items in boxes to topple over, their contents could become intermingled creating a significant amount of otherwise unnecessary work for a colleague.

Tom's numerate behaviour informed by mathematics

On closer examination, Tom would seem to 'do' the following mathematics:

Space and Shape:

- Capacity, weight distribution, stability, identification, access to buildings

Chance:

- Traffic density, delay, time, opportunity, parking, late amendments, unexpected requests

Quantity and Number:

- Reconcile items listed on work orders to items collected or delivered, time calculations

Pattern and Relationships:

- Code recognition, sequence, security procedures
- Network mathematics

While it is relatively straightforward to identify the provenance of such workplace behaviour, a comparison of the contexts in which they are realised may be revealing, see Table 1.

Table 1. Comparison of mathematics inside and outside the classroom

Comparison of Mathematics Inside and Outside the Classroom	
<i>Inside the Mathematics Classroom</i>	<i>Outside the Mathematics Classroom</i>
Complicated	Complex
Well defined content	Poorly defined content
Complete information	Incomplete information
Clear assessment criteria	Retrospective assessment

Consistent ‘right’ answer	Least ‘worse’ answer
Stable and predictable	Volatile
Low level risk	Potentially catastrophic risk
Personal responsibility	Material consequences for error

The term, ‘complicated’ is used here in the sense of being hard to understand, but completely replicable, whereas ‘Complex’ reflects the range of known and unknown ways in which components interact, with outcomes that are unpredictable and unlikely to be precisely repeatable.

Classroom content is defined by curriculum, complete within its own parameters, stable and consistent, with associated assessment mechanism, whereas the World outside is seldom so. Participating in a mathematics class carries low levels of risk, with most of the responsibility for the expected outcome being borne by the student and the extent of his/her engagement and persistence. This stands in stark contrast with the world of work especially, where risk and consequences can have a profound effect on livelihood and life itself.

Implications for Mathematics Teaching and Learning for Adults

The normal use of the term ‘transfer’ connotes a transplanting, or the act of moving someone or something to another place. There is no embedded implication for the need for change, only relocation. In this sense, the expectation of successful transferability of knowledge skills and competence from its source to destination may be problematic (Evans, 2000). The context of the mathematics classroom comprises a unique set of individuals, each unique in their own right, occupying different roles across a landscape of personalities, not simply learner or teacher, surrounded by a single discipline in which people are motivated to participate for multiple outcomes and regulated by formal and informal rules. Similarly, any context outside the classroom may be characterised in multiple dimensions, and possibly the intersection of multiple contexts. In this light, the idea of a simple transfer of knowledge, skills and competence seems very unlikely. A more realistic expectation may be the possible integration of mathematics knowledge skills and competence being re-contextualised taking account of the non-classroom paradigm.

To this end, the classroom pedagogy may need to recognise that practical constraints may apply outside the classroom that may be inconvenient when exploring a mathematical principle. Furthermore, and especially for adult learners, it should not be beyond the competence of the teacher to explain the purpose of mathematical thinking and algorithms, other than to pass an examination. Failing that, the rules and symbols of mathematics may amount to little more than a series of ‘tricks’ that amuse only the initiated, and largely irrelevant otherwise.

It may be of longer term benefit to equip students with the tools and techniques to analyze the context into which they may be expected to realise their mathematics knowledge and skills.

References

- Evans, J. (2000). *Adults’ Mathematical Thinking and Emotions: A Study of Numerate Practices* (P. Ernest Ed.). London: RoutledgeFalmer.
- Keogh, J. (2013). *Looking at the Workplace through Mathematical Eyes*. (Doctorate), Dublin Institute of Technology, ARROW.

Keogh, J., Maguire, T., & O'Donoghue, J. (2012). *A Workplace Contextualisation of Mathematics: Visible, Distinguishable and Meaningful Mathematics in Complex Contexts*. Paper presented at the 2012 Adults Learning Mathematics - A Research Forum. 19th International Conference (ALM19). Synergy - working together to achieve more than the sum of the parts. Te-Piringa - Má pango, má where, ka oti, Auckland.

CONSTRUING MATHEMATICS-CONTAINING ACTIVITIES IN ADULTS' WORKPLACE COMPETENCES: ANALYSIS OF INSTITUTIONAL AND MULTIMODAL ASPECTS *

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Abstract

In this paper we propose and discuss a framework for analysing adults' work competences while construing mathematics-containing "themes" in two workplace settings: road haulage and nursing. The data consist of videos and transcribed interviews from the work of two lorry-loaders, and a nurses' aide at an orthopaedic department. In the analysis we adopt a multimodal approach where all forms of communicative resources (e.g., body, speech, tools, symbols) are taken into account. We also incorporate the institutional norms of workplace activities into the analysis. We coordinate a multimodal social-semiotic perspective with a learning design sequence model (Selander, 2008) which makes explicit the institutional framing. Adopting this framework enables us to understand learning as communication within a domain, with an emphasis on content matters, interpersonal aspects, and roles of communicative resources and artefacts. A tentative theme, Measuring: precision through function and time, is described and we illuminate how workplace specific resources for measuring provide efficiency and function.

Key words: workplace, mathematics, competence, multimodality, learning, institutional norms, interpersonal

Introduction

An overarching aim of the project to which this paper is connected is to analyse and understand adults' mathematics-containing work competences (Wedegé, 2013). In doing this we want to investigate how we can learn from the workplace without taking assumptions of school mathematics for granted. In subsequent investigations and papers we will relate these findings to vocational education and general schooling. In this paper we propose and discuss an analytical framework for analysing mathematics-containing activities in adults' work competences where different functions of multimodal communication and institutional aspects are addressed. Two situations from our data, both video collected, will serve as a

starting point for the article. They will be briefly described in the beginning of this article, then discussed later in the article.

In one situation, we followed two lorry-loaders when they loaded a trailer. As will be shown later on, one essential resource in this task was the loading pallets on which most of the goods were positioned. We will describe how we can identify measuring in the work performed by these lorry-loaders and how our analytical framework helps us to broaden our understanding of the mathematics-containing activity. In another situation we visited a nurses' aide at an orthopaedic department. Her main responsibility was to put plaster cast on injured limbs. Measuring was also identified here and was elaborated using the framework.

Research on and approaches to workplace mathematics

In the literature concerning workplace mathematics we have distinguished a number of themes that are particularly relevant to us. They are described here and we pay extra attention to research in relation to measuring. In addition, different possible approaches for research in this field are described.

Mathematics in the workplace

Research on mathematical practices in the workplace has been carried out since at least the beginning of the 1980s: For example, the Cockcroft report (1982) which initiated several other studies.

Research on workplace mathematics has been described as a field which has passed through different phases (Bessot & Ridgway, 2000; FitzSimons, 2002, 2013; Hoyles, Noss, Kent, & Bakker, 2010; Wedege, 2010a). In the early years researchers presumed that mathematics was easily observable and visible in workplace activities, and frequently such studies resulted in (long) lists of mathematical contents described in "school mathematics" terms (Fitzgerald, 1976). Many of these studies have been criticized for having been conducted with, what has been described as a *mathematical lens* (Zevenbergen & Zevenbergen, 2009) or *mathematical gaze* (Dowling, 1996, 1998), or with a far too narrow conception of mathematics/numeracy (Harris, 1991; Noss, 1998).

Seminal works on the use of mathematics in informal workplace or everyday settings during the 1980s and 1990s draw attention to, for example, differences in strategies and cognitive structures between "school mathematics" and "out-of-school mathematics" and to the fact that schooled and un-schooled individuals perform and succeed differently in everyday and workplace practices as compared to school contexts (Lave, 1988; Nunes, Schliemann, & Carraher, 1993).

Research on workplace mathematics has, during recent years, been dominated by socio-cultural perspectives. Increasingly sensitive theoretical and methodological tools have been used to reveal the complexity of mathematical practices at work. One finding is the fact that mathematics in work is often hidden in activity, culture, social practice, and artefacts. This has been used to explain why it is so difficult to classify these mathematical practices in school-mathematical terms and, when so classified, how the complex use of mathematics in workplaces is reduced to simple computations, measurements, and arithmetic (Gustafsson & Mouwitz, 2008; Hoyles, Noss, & Pozzi, 2001; Keogh, Maguire, & O'Donoghue, 2010).

Mathematics as activity: The example of measuring

Bishop (1988) identified six pan-cultural activities which can be characterized as mathematical activities. These are: counting, locating, measuring, designing, playing, and explaining. In this paper, we focus mainly on measuring which, according to Bishop, is concerned with “comparing, with ordering, and with quantifying qualities which are of value and importance” (p. 34).

We are looking at practices which include measuring in a broad sense. Measuring is central in mathematical activities in people’s everyday lives and in workplace practices in all cultures. Several studies have shown the importance of measuring in different occupations (for an overview see, e.g., Baxter et al., 2006). Among these are studies on carpenters, carpet layers, nurses, process- and manufacturing industry workers, and so forth. Other more recent examples are Bakker, Wijers, Jonker, and Akkerman (2011) who write about the use, nature, and purposes of measurement in workplaces; a study of process improvement in manufacturing industry (Kent, Bakker, Hoyles, & Noss, 2011); a study of boat-building (Zevenbergen & Zevenbergen, 2009); and a study of telecommunication technicians (Triantafillou & Potari, 2010).

Measuring is closely linked to estimating, and the boundaries between these activities are not obvious. Adams and Harrel (2010) have, as part of a more extensive study, presented observations and interviews from four occupations, and concluded that experienced workers often replace measuring with estimation. One important conclusion is that estimation is a complex activity that is learned by experience, and is based on a different rationality from conventional school-methods for measurement which may focus on units and calculations (at least in secondary school). In this article we will use the term *measuring* linking to the concept of activity (ie doing) rather than the generic label *measurement*, to address the human activity of measuring. We also include estimating in the concept of measuring.

Adopting a subjective approach when researching adults’ competence

In the literature on mathematics in the workplace, two approaches can be identified (Wedeg, 2013). In the subjective approach, the interest lies in mathematics as part of personal needs and professional competences in working communities and in various situations. In the general approach, the interest lies in societal demands or demands made from the perspective of school mathematics. Drawing on Bernstein’s (2000) pedagogical models, performance and competence, Wedeg (2013) also identifies professional competence as construed from the workplace rather than taking school mathematics as a starting point. In the research described here, we draw on the subjective approach when we strive towards capturing the mathematics-containing activities within workers’ competences. In this article we present a tentative finding of what could be called a *theme* in professional competence within the sectors of nursing/caring and vehicle/transport. Adopting our analytical framework from this article, we are able to construe wider themes between activities in two sectors of work. These themes will in subsequent research and papers be connected to a general approach when we compare our findings to the demands made within school.

In this article, we draw on the notion of competence. Ellström (1992) describes competence as an individual's readiness for action with respect to a certain task, situation or context. Wedege (2001) concurs and opposes a view of competence as consisting of "objective" competencies defined as being independent of individuals and situations. According to Wedege (2001), competence is:

- always linked to a subject (person or institution)
- a readiness for action and thought and/or an authorisation for action based on knowledge, know-how and attitudes/feelings (dispositions)
- a result of learning or development processes both in everyday practice and education
- always linked to a specific situation context (p. 27).

The term *competence* can be further understood from two perspectives: (a) formal competence in terms of authorisation; for example, that a person has adequate education for a given position; and (b) real competence in terms of whether a person will really be able to demonstrate the abilities that are identified; for example in a particular certification (Wedege, 2001; 2003). In terms of our research interest here, the second meaning is more relevant.

Addressing the socio-political through the notion of institutional framing

We position this paper in a socio-political paradigm – paradigm is here understood according to Lerman (2006) – in mathematics education. This is connected to sociology and critical theories (Valero & Zevenbergen, 2004; see also Ernest, Greer, & Sriraman (Eds.), 2009). Mathematics incorporates means for understanding, building, or changing a society (Mellin-Olsen, 1987). Skovsmose (2005) acknowledges this (see also Jablonka, 2003; Gellert & Jablonka, 2009), whilst also stressing that mathematics does not hold any intrinsic good; instead mathematics can be used for different purposes in society and people's lives. Thus, there is a need to address the role of the use of mathematics in society and in this article we incorporate institutional aspects of workers' mathematics-containing activities.

We view the institutional context as always present. An early example of a theoretical discussion of this is given by Popkewitz (1988), who considers institutional framings as one way to address social and critical aspects in studies of school mathematics (see also Mellin-Olsen, 1987). Also, in work-places the institutional context and societal dimensions are always present (e.g., Salling Olesen, 2008). Here are included dominant discourses, the use of artefacts developed over time, the division of time, established routines, workplace structures, and authoritative rules (Selander, 2008, drawing on Douglas, 1986). A similar view is described by Bishop (1988, p. 36) when he writes about the development of units, and systems of units: "there is a clear progression, with the main idea being that of the stronger the environmental and social need the more detailed, systematic and accurate the measure". As we will show in our analysis and findings, what constitutes an accurate measuring unit may be quite different in the workplace from what is usually emphasised in school.

Institutional aspects were addressed by Wedege (2010b) when she proposed the concept of *sociomathematics*. She described sociomathematics as both a subject field combining mathematics, people, and society, and a research field. We are also inspired by FitzSimons and Wedege (2007) who adopted Bernstein's (2000) concept of horizontal and vertical discourses (see also FitzSimons, Mlcek, Hull, & Wright, 2005). Bernstein refers *Vertical discourse* to knowledge within a discipline, such as academic mathematics. This knowledge is coherent and systematic. The *horizontal discourse* refers to contextual knowledge and a relevant example for us is the context bound mathematics used and developed in the workplace. In the study by FitzSimons et. al (2005), activity theory (Engeström, 2001) was adopted as a theoretical framework, and the main findings were that mathematically straightforward skills become "transformed into workplace numeracy competence, when the complexities associated with successful task completion as well as the supportive role of mediating artefacts and the workplace community of practice are taken into account" (p. 49).

Analytical framework

In this section we present our analytical framework where a theory of communication – multimodal social semiotics (e.g., Van Leeuwen, 2005) – is coordinated with a model of a learning design sequence. Design is here understood in a broad sense, for example including both aesthetic and functional aspects. The term *coordinate* implies that the two theoretical approaches are compatible with respect to underlying assumptions (Prediger, Bikner-Ahsbahr, & Arzarello, 2008).

Learning as multimodal communication

In this article we attempt to problematise learning in order to avoid the term learning becoming a black box (Ellström, 2010). Ellström uses the term black box to refer to learning as it is in studies on innovations in workplaces. Learning is here described as a key concept, but it is not really spelled out how it is operationalised in the studies. We view learning as closely connected to human activity and understood as meaning-making towards an increased communication in the world through the communicative resources of a discipline (Selander & Kress, 2010; see also Björklund Boistrup, 2010). Learning in a work-place constitutes, at least in part the competence that the worker gains over time. This competence is not something fixed, but changes and may evolve over time. In operationalising learning, we discuss knowing that is part of workplace activities, and hence the worker's competence, rather than discussing learning as such. By using the term knowing instead of knowledge we want make clear that we do not take into account an objective knowledge "out there" to be learnt. Instead, knowing is viewed as constructed and construed in communication among humans throughout history (Foucault, 2002; see also, e.g., Delandshere, 2002; Valero, 2004b; Volmink, 1994). What valid knowing is and how it is demonstrated in communication is not set in stone. At different times throughout history, the perception of what qualifies as important knowing has changed and will continue to do so.

In this article we draw on a multimodal approach when we adopt social semiotics as part of an analytical framework (Van Leeuwen, 2005). In a multimodal approach, described by Selander (2008; see also Björklund Boistrup & Selander, 2009), all modes of communication are recognised. Communication in a multimodal perspective is not understood in the same

way as communication in a narrow linguistic perspective, focussing on verbal interaction only. Rather, all kinds of modes are taken into consideration, such as gestures, and gazes, pictorial elements and moving images, sound, and the like. Modes are socially and culturally designed in different processes of meaning-making, so that their meaning changes over time. It is also the case that “content” in one kind of configuration (e.g., as a measure on a dip stick), will not necessarily be exactly the same content in another configuration (e.g., as a number on a device for filling the oil):

Different representations of the world are not the “same” in terms of content. Rather, different aspects are foregrounded. In verbal texts we read linearly, within a time frame, whilst a drawing will be read within a space frame. And a graph does not represent a knowledge domain in the same way as numbers does [sic]. The modes that are “chosen” in a specific situation reflect the interest of the sign maker, and they are therefore not arbitrary. (Björklund Boistrup & Selander, 2009, p. 1566)

We argue for the importance of understanding multimodal communication to be able to fully understand a phenomenon such as mathematics knowing and learning in a workplace. In social semiotics, three meta-functions are often operationalized in analysis (Halliday, 2004). Halliday focused mainly on written and spoken language in his work but in this article, drawing on Van Leeuwen (2005), we adopt the meta-functions in connection with a multimodal approach. These meta-functions are: the ideational, the interpersonal, and the textual. In Morgan (2006), these functions are used with a focus on the construction of the nature of school mathematics activity. In this article, we start out with the meta-functions as used by Kress et.al. (2001; see also Björklund Boistrup & Selander, 2009). The ideational meta-function is related to human experience and representations of the world (Halliday, 2004). Here there is a possibility to address the content, the “what-question” of a communication. In this article we look for measuring activities and resources in lorry-loaders’ and nurses aides’ practices and competences. The *interpersonal* meta-function is about how language (used in a broad sense in this article) enacts “our personal and social relationships with the other people around us” (Halliday, 2004, p 29). In this article we examine the roles of measuring activities for and in relations between the people involved. The *textual* meta-function is related to the construction of a “text” and this refers to the formation of whole entities (Halliday, 2004). With a multimodal approach, the term *text* refers to multimodal ensembles which are communicatively meaningful and part of the overall pattern of the actual communication. Here we are interested in what roles resources and communicative modes play in the measuring activity.

A model for understanding learning in other-than-school contexts

We draw on a model where a multimodal approach is connected to an institutional framing (Selander, 2008; Selander & Kress, 2010): a design theoretical perspective of learning.

LEARNING DESIGN SEQUENCE

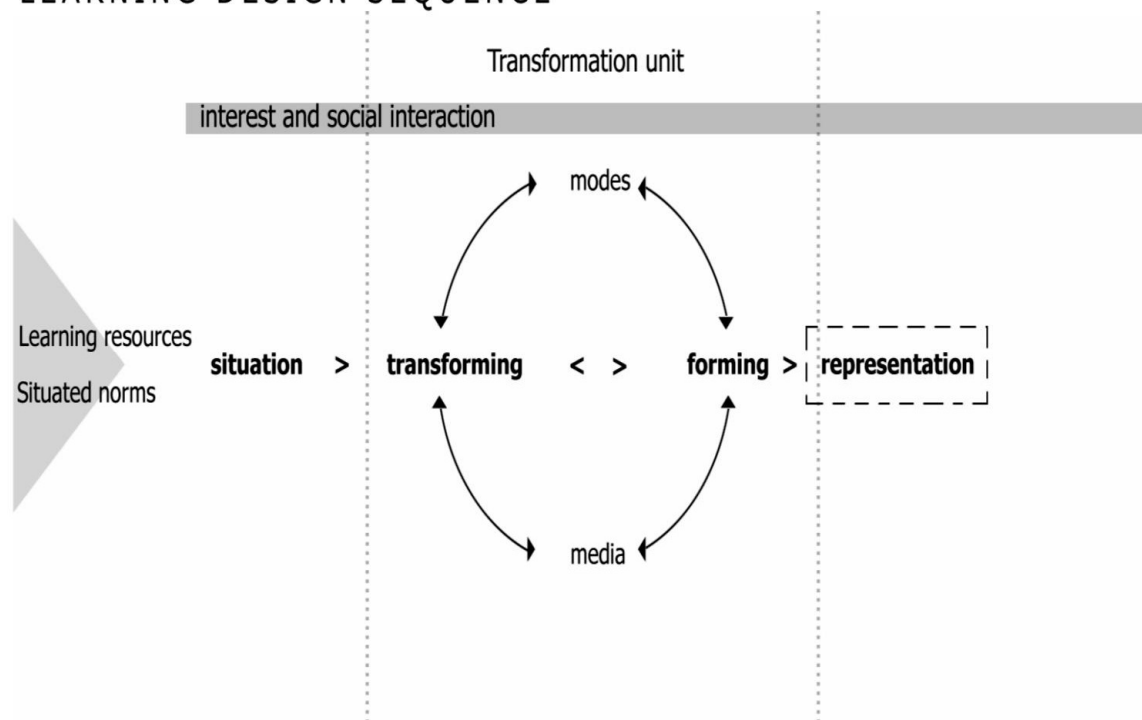


Figure 1. A learning design sequence (Selander, 2008, p. 16).

This first model (Figure 1) gives the general principles for how communication, learning, and knowing can be addressed without starting from the perspective of a school setting, but considering meaning-making and learning as something always present. The starting point, the “situation”, is here to be taken as any other-than-school setting, for example a workplace. The worker and his/her work are embedded in a social practice with different kinds of social norms and with different semiotic resources at hand. The duration of the process that the model captures can be rather short (seconds) but also longer (like hours or days). Selander (2008) writes:

In many instances we are put in situations where we try to figure out the challenge and what standpoint and action that is meaningful. It could be situations where we ask ourselves if the bus ticket still is of value or if we can swap a book, for example given as a present, for another one in the book store. In each such micro situation we also learn something about what is usual or “proper”, about restrictions and regulations etc. And there are also moments of creativity when we try out different solutions. (p. 14-15)

It could be possible to use this general learning design sequence to analyse what a person is doing at work. A person who is performing a well-known task is now and then met by an explicit learning purpose while working. It may be a situation where an innovation of some kind is needed in order to facilitate the work (Ellström, 2010). Even more explicit is the learning purpose when the person is new at her/his job. Even though the model by Selander (2008) is relevant for a study of learning and knowing mathematics at a workplace, we find

the next model more suitable for our purpose. The reason for this is that we, as the research team, change the situation when we are present, and even more when we pose questions during the filming of the activity. The model that we use as our analytical frame is the Semi-Formal Learning Design Sequence.

Semi-formal - LEARNING DESIGN SEQUENCE

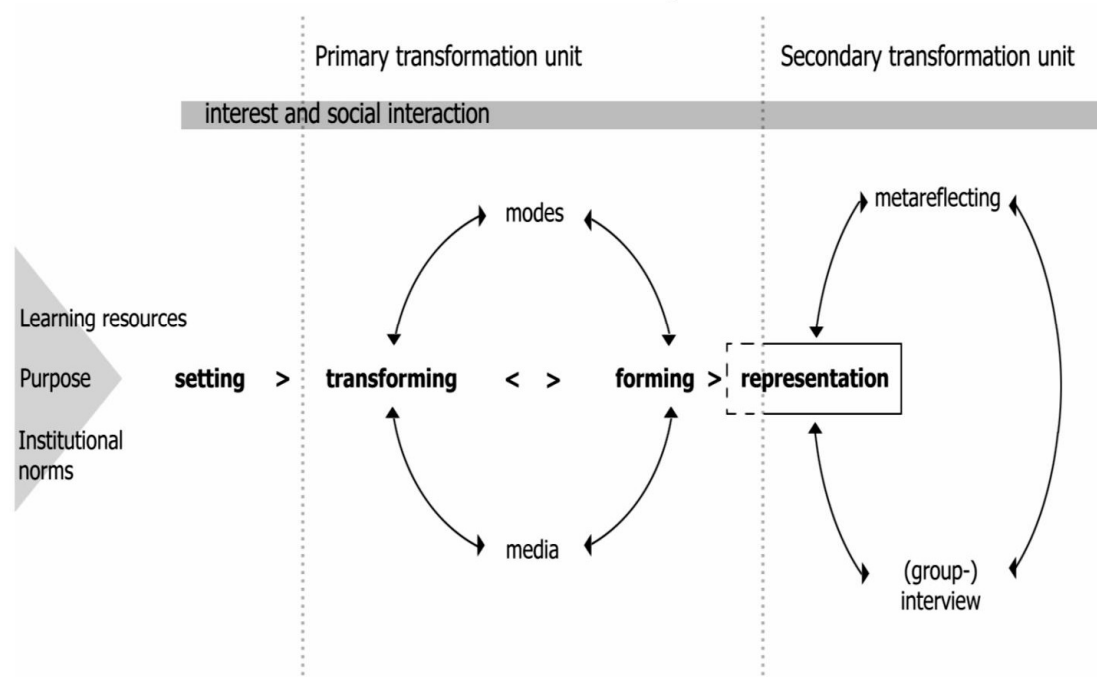


Figure 2. A learning design sequence – semi-formal (Selander, 2008, p. 17)

The idea behind the semi-formal learning sequence in Figure 2, is that the starting point is a *setting* (not a *situation*, as before) where the learner is confronted by an articulated learning purpose. In our case it is mainly the workplace that constitutes the setting where there are institutional norms affecting what is taking place and what is counted as relevant knowing. When we as researchers pose questions (see below for a description of our interviews), the worker is invited to meta-reflect on her/his work. There are then transformations taking place when the worker communicates through her/his actions, engaging with different artefacts, and then describes and explains the working process through speech and gesture, and so forth. In this sense both the primary transformation unit – the actual work – and the secondary transformation unit – answering our questions and showing us the tools and processes of the work – are going on at the same time.

Methodology

The research design of the qualitative study for which this article is written is a case study. When using the term *case study*, we draw on Yin's definition (1989):

A case study is an empirical inquiry that:

- investigates a contemporary phenomenon within its real-life context, when
- the boundaries between phenomenon and context are not clearly evident, and in which
- multiple sources of evidence are used (p. 23)

The phenomenon we are interested in here is to learn more about mathematics within the work and competences of lorry-loaders and nurses' aides. More specifically we are interested in how we can analyse the complexity of mathematics interwoven in work. Our data gathering methods consist of:

- *Videos* which were filmed at one or two visits at each work-place. We followed one worker (or two), who was doing her/his regular work, with a hand held camera for about one hour. As a back-up, we also recorded sound with additional sound recorders. In total we visited six work-places, three in each sector.
- *Apprenticeship interviews* which were performed when possible during the filming. With apprenticeship interviews we mean that we took the role of a person trying to learn the work processes that the worker was engaged in. We then posed curious questions to the worker during her/his work.
- *Photographs* which were taken during our visits with a special focus on signs, notices, artefacts, etc.
- *Interviews* which were performed after the first visit at the workplace. These have so far been performed by Maria C Johansson as part of her PhD process (Johansson, 2013; in preparation). We used excerpts from these interviews to inform our understanding of the video data.

This article is based on data from two workplaces: one road-haulage company and a plaster unit at an orthopaedic department of a hospital.

Analysis and example of findings: measuring activities

The analytical framework that we present in this article is connected to the subjective approach mentioned earlier (Wedeg, 2012). Here we pay attention to adults' work competences and the activities we can construe as being possible to connect with mathematics. Our emphasis is on learning/communication in a workplace setting and we view workers' actions as communication, as well as learning and knowing.

In the following, we utilise the analytical framework of the three meta-functions outlined above (ideational, interpersonal, textual) to describe the measuring activities as part of lorry-loaders' work competence and of nurses' aides' work competence. The kind of measuring that we focus on in this analysis is what Bishop (1988, p. 34) labels "quantifying

qualities which are of value and importance.” We also use the Learning Design Sequence model above, in how we view the presence of the institutional framing. The three meta-functions are actually interwoven and it is an analytical construction to tease them apart. This may, in a systematic and structured way, bring forth findings that we otherwise would not capture. This also causes the same “events” to turn up more than once in the analysis, but with different emphasis.

Lorry-loaders

At the road-carrier company, we visited lorry-loaders, one of whose tasks was to load trailers according to specifications provided in written forms. The form was developed by administrative staff in the office. While we were there, one trailer was loaded using forklifts, and in discussions we were told about the written loading form (see Figure 3) which specified, for example, the number of pallets, the weights of the goods, the companies’ names for delivery (these names have been deleted in the photograph), and where different pallets were intended to be unloaded. In the first excerpt, the two lorry-loaders are talking with two persons from the research project before they start to load one trailer. One of them, Con (pseudonym), describes how they decide whether to load the trailer in one or two layers (Excerpt 1). The transcripts are made multimodally. In Excerpt 1, we identify Time, Speech (what people say and how they say it), Body (what people do including resources and artefacts), and Gaze (where people look).

Time	Speech	Body	Gaze
03:11	<i>The ideal is that you can get it [the trailer] fully loaded. But for this trailer we load now, well, we, I have seen the loading form before.</i>	Nods now and then. Points at trailer. Two hands 20 cm apart [marking the top and bottom of form].	Looks at research staff.
	Det optimala är ju att man kan få den fullastad. Men som denna trailern vi lastar nu. Så har vi, jag, sett lastlistan innan.*		
03:21	<i>Then I know that we can load it without the dual goods. Dual goods means two pallets on top of one another.</i>	Moves hands up and down. Right hand above left hand with a distance between them.	Looks at research team.
	<i>Då vet jag att vi kan lasta den utan att dubbla godset. Dubbla gods det är två pallar på varandra.*</i>		

03:27	<i>Then we utilize the whole space to make the safest solution possible.</i>	Moves flat hands from center and out [indicating a surface].
	Då utnyttjar vi hela utrymmet för att göra en så säker lösning som möjligt.*	

Excerpt 1. Lorry Loader describing the loading of a trailer. *Speech in Swedish.

Later on, when the workers started loading, the use of the pallets becomes clearer. Either each pallet was positioned “horizontally” in the trailer, like this: . In this case there is room for two pallets beside each other along the trailer’s width. If instead they were positioned vertically there was room for three. This was also explained in a communication after the trailer was finished loading. Con explained how the size of the pallets, 800 mm x 1200 mm, makes this possible. In Figure 3 the pallets in the trailer are shown and it is also possible to get a glimpse of the loading form that Con describes and shows with his hands early in Excerpt 1.



Figure 3. Images from the road-carrier company.

In the following, we describe our analysis where we operationalise the social semiotic meta-functions and where we also coordinate with the learning design sequence by Selander (2008). The concepts from the Learning Design Sequence (Figure 2) are in italics and the analysis is mainly organised through the meta-functions.

- Ideational meta-function: We analysed the data from the lorry-loaders, looking for human experiences and representations of the world (the content, the what-question) in relation to the measuring we could construe. We then construed a measuring activity in the *institutionally framed setting* where the lorry-loaders used the loading pallets (i.e. *resources*) as measuring units for the actual goods to be carried. Here the

workers did not use the measuring means and units normally used in school, such as using a measuring tape to find out the two lengths in centimetres, and then calculate the area.

- **Interpersonal meta-function:** When analysing the data from the lorry-loaders for personal and social relationships, we were able to capture how the informal measuring activity via the pallets entailed their involvement in the process on behalf of the customer, and also gave a certain amount of control to the loaders. Our assumption is that the use of pallets as measuring resources saves time, which in the end lowers the cost for the customer. This may be seen as one *purpose* with the use of the pallets. The pallets were also communicative *resources* for the two lorry-loaders who, ALMost without any talking, communicated on how to position the pallets on the trailer when carrying them on the forklifts. When Con told the research team about his work we could identify engagement and an *interest* in making clear what he meant and generally in his work. This analysis is based on his speech and the many gestures. During this *meta-reflection* there were many *transformations* between speech and gestures.
- **Textual meta-function:** When looking at the multimodal text that was communicated to us as visitors through actions, speech, gestures, etc., we analysed the roles of, in this case, the informal measuring activity through the *resources* of the pallets. Our finding is that the pallets took the role of facilitating the measuring, as they provided a measuring function in themselves as well as a means for efficiency and effectiveness. Another *resource*, the written form, made the measuring activity visible for people involved. As shown in Excerpt 1, we could identify how there are *transformations* between different communicative *modes* which also *forms* the activity. One *transformation* goes from the written loading form to the loading process. This *transformation* concerns both *media* (from written form to physical activity) and *modes* (from writing in words and symbols to speech, body movements, and gaze). During the loading, Con ticks off the things that are loaded, an activity which constitutes a new *transformation*.
- **Institutional norms:** The loading form is normally used at this workplace and *formed* the situation. In this workplace, its use is a long-standing tradition. The pallets are standardised according to the transport sector regulations.

Nurses' aide (plastering)

In the orthopaedic department of a hospital, we visited a nurses' aid who specialised in plastering. During our visit, she put plaster on an arm and hand of a patient who had an injury to his thumb. In this situation we were mainly silent and the chat was between the nurses' aide and the patient. For this example we have chosen only to present pictures. In Figure 4 some details from the room where it took place are shown. It is also possible to see how the nurses' aide rolls out dry plaster wrap on the arm. The analysis mainly is focused on this action.

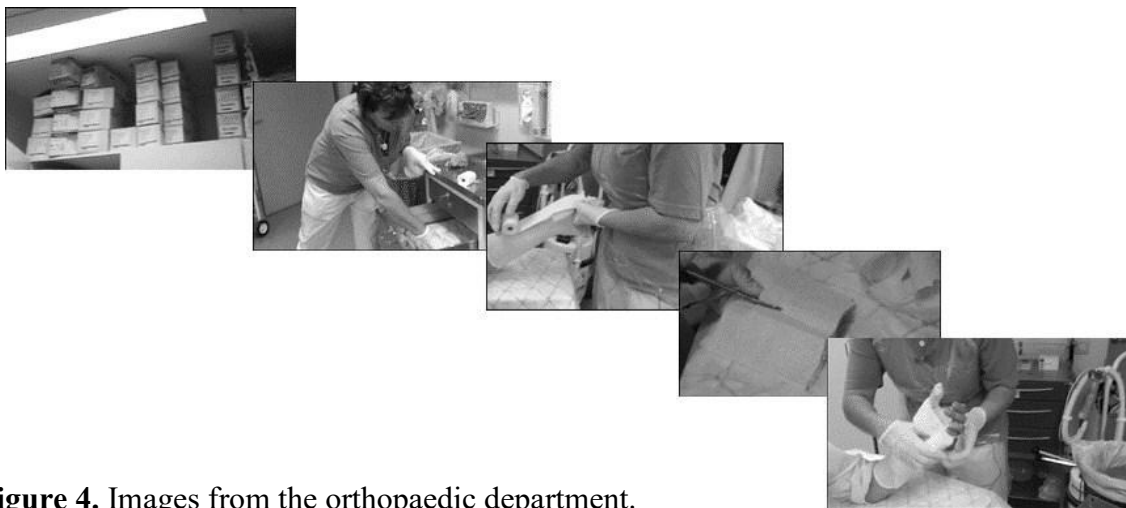


Figure 4. Images from the orthopaedic department.

In the following we describe our analysis of measuring activity from the work performed by this nurses' aide. Similar to the previous section, the concepts from the learning design sequence (Selander, 2008) are in *italics* and the analysis is mainly organised through the social semiotic meta-functions.

- Ideational meta-function: We were able to construe a measuring activity where the *setting* was a room that was *designed* for plastering. There were boxes with different kinds of plaster stored on shelves and there were appropriate tools present (*resources*). Prior to the actual plastering process, the nurses' aide measured up with the dry plaster wrap directly on the patient's arm. The aide then used the first measuring as a unit and made repeated folds based on this unit before finally adhering it to the patient's arm. The *resource* for measuring here is the plaster itself.
- Interpersonal meta-function: This plastering activity is very important with respect to the patient (in the healing process). Measuring directly on the arm may then be the most accurate.

It should also look neat and tidy (caring about the patient). The nurses' aide described the procedure of plastering to the patient as she worked. This also seemed to act as a cALMing function and simultaneously gave her an opportunity for *meta-reflection* on her activity. Here we could identify *interest* and *interaction*.

- Textual meta-function: The plaster has several roles here. The main function was to stabilise the arm and hand during the healing process. Moreover, it fulfilled a measuring function, and its correlation to the length of the arm was part of the function. The *transformations* took place both during the *primary* and *secondary transformation unit*. In the *primary transformation unit*, one example is where a specific distance on the arm was *transformed* from the body to the plaster (*resource*) by the nurses' aide when she measured up. This unit was then *transformed* to a longer piece of plaster during the repeated folds. In the *secondary transformation unit*, there were *transformations* from *modes* such as body and artefacts into speech when the nurses' aide explained the process to the patient.
- *Institutional norms*: Methods for plastering are *designed* together at this workplace. Some may be general between hospitals, and some are specific to this work-place. Speed is important: Another patient is waiting, but the long-term function for this patient is the highest priority.

A general theme of measuring activity: Precision through function and time

Here we connect the two cases described above and we construe a general measuring activity between the two sectors of vehicle and transport, and of nursing and caring, which we expect to be found in many workplaces within these sectors.

Ideational. This measuring activity is an alternative to school-traditional precision measuring with tools. The worker uses “rough” measuring units. At a first glance it seems like function, result, and/or time is superior to precision. At a second glance we interpret that the accurate precision for the loading process or healing process is accomplished *through* this workplace specific measuring activity. “Rough” in this case does not contradict that the method is well adapted to the situation and that accuracy is judged by the situational needs and constraints/restrictions.”

Interpersonal. In this activity we captured relationships between the worker and the workplace and (in)directly the customer (company or patient). Ethical considerations are that it is important to do a good job so that the customer is satisfied. This could for example include economic considerations such as not to spend too much time which would increase the cost and decrease the profit. Interpersonal aspects also concern what the employer may impose on the workers (a good job, customer satisfaction, expediency). Also aesthetic aspects, such as “looking neat and tidy”, are part of what is regarded as a good job and what may make the customer satisfied.

Textual. The workplace specific resources for measuring provided efficiency and functionality. Resources can then have the role of facilitating the work; for example, the task is completed more quickly through the use of “rough” measuring units. When a measuring needs to be recorded, appropriate documents are included.

Institutional. Resources contribute to standardisation within the workplace, as well as between workplaces. Written forms can take this role as well as other resources for measuring. Notions concerning a “good job” also concern the institutional framing. The client is thus part of the institutional framing.

Concluding discussion

As stated previously we position this article within a social and critical paradigm. For our work, this quote by Valero and Zevenbergen (2004) is particularly relevant:

In mathematics education it is always possible to ask whose knowledge is being represented in society, schools and classrooms, and with what effects for the different participants in it. The recognition of the different and multiple positions that social actors can adopt in relation to and with the use of (school) mathematical knowledge is at the core of discussions of equity, social justice and democracy in mathematics education. (p. 2)

They continue by arguing that such social aspects are essential to an understanding of mathematics education practices in broader institutional contexts (see also Valero, 2004a). At the same time, such aspects form this broader understanding of the *social*. In terms of research on mathematics-containing activities in workplaces, our standpoint is that such an

understanding incorporates an interest in whose and what kind of knowing is represented in school mathematics, and also how this is connected to the broader social context. We know from earlier research (Fahrmeier, 1984; Lave, 1988; Masingila, Davidenko, & Prus-Wisniewska, 1996; Nunes Carraher, Carraher, & Schliemann, 1985; Nunes, et al., 1993; Scribner, 1985) that the mathematics that can be construed from workplace activities has connections to, but is not the same as, school mathematics. One way to put it is that workers' voices are missing in the school context, often also in prevocational studies.

The complexities of the workplace could be brought into school mathematics if we want to represent also the knowing of workers in different sectors. This is described by Steen (2003) in this way:

The contrast between these two perspectives—mathematics in school versus mathematics at work—is especially striking (Forman and Steen 1999). **Mathematics in the workplace makes sophisticated use of elementary mathematics rather than, as in the classroom, elementary use of sophisticated mathematics.** Work-related mathematics is rich in data, interspersed with conjecture, dependent on technology, and tied to useful applications. Work contexts often require multi-step solutions to open-ended problems, a high degree of accuracy, and proper regard for required tolerances. None of these features is found in typical classroom exercises. (p. 55; our emphasis).

What we have accomplished through utilising our analytical framework, based on a multimodal approach and a design theoretical approach, is to connect the people, the workers and their competence, to the workplace, and to the institutional framing. The three meta-functions have served the purpose of connecting the content (ideational) – the measuring, with relations between the people involved (interpersonal), with a special attention to the roles of resources (textual). The model by Selander (2008; see also Selander & Kress, 2010) helped us understand the institutional framing, and also the different kinds of communications that took place when we, on one hand, observed the work-processes, and, on the other hand, posed questions about it. What became clear to us in the analysis is what measuring accurately (Bishop, 1988) may mean in a workplace context, for example that precision for the loading process or healing process was accomplished *through* workplace-specific measuring units.

We would argue that our research is part of a development of research methods and analytical frameworks sensitive enough to do justice to the complexity and to the power of mathematical practices other than school-mathematics, for example, in workplaces. Included here is a view of the worker as self-governed and competent (Wedge, 2001) as well as an approach that there is much to learn from workplaces that can be brought into vocational education and training (VET) settings. This article is consequently an example of a study within what Wedge (2010b, p. 452) labels as sociomathematics: “a research field where problems concerning the relationships between people, mathematics and society are identified, formulated and studied.”

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References

- Adams, T. L., & Harrell, G. (2010). A study of estimation by professionals at work. *Journal of Mathematics & Culture*, 5(2). Retrieved from <http://nasgem.rpi.edu/files/4088>
- Bakker, A., Wijers, M., Jonker, V., & Akkerman, S. (2011). The use, nature and purposes of measurement in intermediate-level occupations. *ZDM Mathematics Education*, 43(5), 737-746. doi:10.1007/s11858-011-0328-3
- Baxter, M., Leddy, E., Richards, L., Tomlin, A., Wresniwiro, T., & Coben, D. (2006). *Measurement wasn't taught when they built the pyramids - was it?* Place of publication: The National Research and Development Centre for Adult Literacy and Numeracy (NRDC).
- Bernstein, B. (2000). *Pedagogy, symbolic control and identity: Theory, research and critique* (Rev. ed.), Lanham: Rowman and Littlefield.
- Bessot, A., & Ridgway, J. (Eds.). (2000). *Education for Mathematics in the Workplace* (Vol. 24). Dordrecht: Kluwer Academic Publishers.
- Bishop, A. J. (1988). *Mathematical enculturation. A cultural perspective on mathematics education*. Dordrecht: Kluwer Academic Publishers.
- Björklund Boistrup, L. (2010). Discourses of assessment actions in mathematics classrooms. In U. Gellert, E. Jablonka, & C. Morgan, (Eds.). *Proceedings of the sixth international mathematics and society conference 20th-25th March 2010, Berlin, Germany* (pp. 141-150).
- Björklund Boistrup, L., & Selander, S. (2009). Coordinating multimodal social semiotics and an institutional perspective in studying assessment actions in mathematics classrooms. In V. Durand-Guerrier, S. Soury-Lavergne & F. Arzarello (Eds.), *Proceedings of CERME 6, Sixth Conference of European Research in Mathematics Education* (pp. 1565-1574). Lyon, France: Institut national de recherche pédagogique.
- Cockcroft, W. H. (Chairman). (1982). *Mathematics counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools*. London: Her Majesty's Stationery Office.
- Delandshere, G. (2002). Assessment as inquiry. *Teachers College Record*, 104(7), 1461-1484.
- Douglas, M. (1986). *How Institutions Think*. Syracuse, N.Y.: Syracuse university press.
- Dowling, P. (1996). A sociological analysis of school mathematics texts. *Educational Studies in Mathematics*, 31(4), 389-415.
- Dowling, P. (1998). *The sociology of mathematics education: mathematical myths/pedagogic texts*. London: Falmer Press.
- Ellström, P. (1992). *Kompetens, utbildning och lärande i arbetslivet* [Competence, education and learning in working life]. Stockholm, Sweden: Norstedts Juridik.
- Ellström, P-E. (2010). Practice-based innovation: A learning perspective. *Journal of Workplace Learning* 22 (1/2).
- Engeström, Y. (2001). Expansive learning at work: Toward an activity-theoretical reconceptualization. *Journal of Education and Work*, 14(1), 133-156.
- Ernest, P., Greer B., & Sriraman, B. (Eds.). (2009). *Critical issues in mathematics education*. Charlotte, NC: Information Age Publishing.
- Fahrmeier, E. (1984). Taking inventory: Counting as problem solving. *Quarterly Newsletter of the Laboratory of Comparative Human Cognition*, 6(1-2), 6-10.
- Fitzgerald, A. (1976). School mathematics and the requirements of industry. *The Vocational Aspect of Education*, 28(70), 43-49. doi: 10.1080/10408347308000621
- FitzSimons, G. (2002). *What counts as mathematics?: technologies of power in adult and vocational education*. Boston: Kluwer Academic Publishers.
- Fitzsimons, G. (2013). Doing mathemtics in the workplace: A brief review of selected recent literature. *ALM Journal*, 8(1), 7-19. Retrieved from <http://www.ALM-online.net/images/ALM/journals/ALMij-volume8-1-april2013.pdf>.
- FitzSimons, G. E., Mlcek, S., Hull, O., & Wright, C. (2005). *Learning numeracy on the job: A case study of chemical handling and spraying*. Final Report. Adelaide: National Centre for Vocational Education Research. [Retrieved October 12, 2006, from <http://www.ncver.edu.au/publications/1609.html>]
- FitzSimons, G., & Wedege, T. (2007). Developing numeracy in the Workplace. *Nordic Studies in Mathematics Education*; 1, 12 49-66.
- Foucault, M. (2002). *Vetandets arkeologi* [The Archaeology of knowledge]. Lund, Sweden: Arkiv förlag.

- Gellert, U., & Jablonka, E. (2009). The demathematising effect of technology. Calling for critical competence. In P. Ernest, B. Greer & B. Sriraman (Eds.), *Critical issues in mathematics education* (pp. 19-24). Charlotte, NC: Information Age Publishing.
- Gustafsson, L., & Mouwitz, L. (2008). *Validation of adults' proficiency – fairness in focus*. Göteborg. National Center for Mathematics Education, University of Gothenburg.
- Halliday, M. A. K. (2004). Revised by Matthiessen, M. I. M. *An introduction to functional grammar*. London, UK: Oxford University Press Inc.
- Harris, M. (Ed.). (1991). *Schools, mathematics and work*. London: FALMer Press.
- Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for technomathematical literacies*. Abingdon, Oxon: Routledge.
- Hoyles, C., Noss, R., & Pozzi, S. (2001). Proportional reasoning in nursing practice. *Journal for Research in Mathematics Education*, 32(1), 4-27.
- Jablonka, E. (2003). Mathematical Literacy. In A. J. Bishop (Ed.), *Second international handbook of mathematics education* (pp. 75-102). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Johansson, M. C. (2013). Caring or counting? – An empirical paper problematizing different kinds of mathematics involved in a nursing aide's work. Paper presented at *Adults learning mathematics 20 'Critical moments in adult mathematics'*, Newport South Wales 1-4 July, 2013.
- Johansson, M. C. (In progress). Licentiate thesis.
- Kent, P., Bakker, A., Hoyles, C., & Noss, R. (2011). Measurement in the workplace: the case of process improvement in manufacturing industry. *ZDM*, 43(5), 747-758. doi: 10.1007/s11858-011-0359-9
- Keogh, J., Maguire, T., & O'Donoghue, J. (2013). Mathematics 'complexified' by the workplace: Routine mathematics constrained and mediated by workplace contexts. Paper presented at *Adults learning mathematics 20 'Critical moments in adult mathematics'*, Newport South Wales 1-4 July, 2013.
- Kress, G., Jewitt, C., Ogborn, J., & Tsatsarelis, C. (2001). *Multimodal teaching and learning: The rhetorics of the science classroom*. London, UK: Continuum.
- Lerman, S. (2006). Theories of mathematics education: Is plurality a problem? *ZDM. The International Journal on Mathematics Education*, 38(1), 8-13.
- Masingila, J., Davidenko, S., & Prus-Wisniowska, E. (1996). Mathematics learning and practice in and out of school: A framework for connecting these experiences. *Educational Studies in Mathematics*, 31(1-2), 175-200.
- Mellin-Olsen, S. (1987). *The politics of mathematics education*. London, UK: Kluwer Academic Publishers.
- Morgan, C. (2006). What does social semiotics have to offer mathematics education research? *Educational Studies in Mathematics*, 61(1), 219-245.
- Noss, R. (1998). New numeracies for a technological culture. *For the Learning of Mathematics*, 18(2), 2-12.
- Nunes Carraher, T., Carraher, D. W., & Schliemann, A. D. (1985). Mathematics in the streets and in schools. *British Journal of Developmental Psychology*, 3, 21-29.
- Nunes, T., Schliemann, A. D., & Carraher, D. W. (1993). *Street mathematics and school mathematics*. New York: Cambridge University Press.
- Popkewitz, T. (1988). Institutional issues in the study of school mathematics. Curriculum research. *Educational Studies in Mathematics Education*, 19(2), 221-249.
- Prediger, S., Bikner-Ahsbals, A., & Arzarello, F. (2008). Networking strategies and methods for connecting theoretical approaches: first steps towards a conceptual framework. *ZDM. The International Journal on Mathematics Education*, 40(2), 165-178.
- Salling Olesen, H. (2008) Workplace learning. In P. Jarvis (Ed.), *The Routledge international handbook of lifelong learning* (pp. 114-128). London: Routledge Mental Health.
- Scribner, S. (1985). Knowledge at Work. *Anthropology and Education Quarterly*, 16(3), 199-206.
- Selander, S. (2008). Designs for learning – A theoretical perspective. *Designs for Learning*, 1 (1).
- Selander, S., & Kress, G. (2010). *Design för lärande – Ett multimodalt perspektiv* [Designs for learning – A multimodal perspective]. Stockholm, Sweden: Norstedts Akademiska förlag.
- Skovsmose, O. (2005). *Travelling through education: Uncertainty, mathematics, responsibility*. Rotterdam, The Netherlands: Sense Publisher.
- Steen, L. A. (2003). Data, shapes, symbols: Achieving balance in school mathematics. In B. Madison & L. A. Steen (Eds.), *Quantitative literacy: Why numeracy matters for schools and colleges* (pp. 53-74). Princeton, NJ: National Council on Education and the Disciplines. Retrieved from <http://www.maa.org/sites/default/files/pdf/QL/WhyNumeracyMatters.pdf>
- Triantafyllou, C., & Potari, D. (2010). Mathematical practices in a technological workplace: the role of tools. *Educational Studies in Mathematics*. Retrieved from <http://link.springer.com/content/pdf/10.1007%2Fs10649-010-9237-6.pdf> doi:10.1007/s10649-010-9237-6.

- Valero, P. (2004a). Postmodernism as an attitude of critique to dominant mathematics education research. In M. Walshaw (Ed.), *Mathematics education within the postmodern* (pp. 35-53). Greenwich, CT: Information Age Publishing.
- Valero, P. (2004b). Socio-political perspectives on mathematics education. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 5-24). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Valero, P., & Zevenbergen, R. (2004). Introduction. Setting the scene of this book. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 1-4). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Van Leeuwen, T. (2005). *Introducing social semiotics*. London, UK: Routledge.
- Volmink, J. V. (1994). Mathematics by all. In S. Lerman (Ed.), *Cultural perspectives on the mathematics classroom* (pp. 51-67). Dordrecht, The Netherlands: Kluwer academic publishers.
- Wedeg, T. (2001). "Competence" as a construction in adult and mathematics education. In G. FitzSimons, J. O'Donoghue & D. Coben (Eds.), *Adult and life-long education in mathematics: Papers from WGA6, 9th International Congress on Mathematical Education (ICME9)* (pp.21-30). Sydney, Australia: Language Australia.
- Wedeg, T. (2003). Konstruktion af kompetence(begreber) [The construction of (concepts of) competence]. *Dansk Pædagogisk Tidsskrift* [Danish Pedagogical Magazine], 03, 64-75.
- Wedeg, T. (2010a). People's mathematics in working life: Why is it invisible? *ALM Journal*, 5(1), 89-97.
- Wedeg, T. (2010b). Sociomathematics: A subject field and a research field. In U. Gellert, E. Jablonka, & C. Morgan, (Eds.). *Proceedings of the sixth international mathematics and society conference 20th-25th March 2010, Berlin, Germany* (pp. 449-458).
- Wedeg, T. (2013). Workers' mathematical competences as a study object: Implications of general and subjective approaches. *Adults' mathematics: Working papers*, 2.
- Zevenbergen, R., & Zevenbergen, K. (2009). The numeracies of boatbuilding: New numeracies shaped by workplace technologies. *International Journal of Science and Mathematics Education*, 7(1), 183-206.

The French ‘Alternance’ Model: The Question of the Relationship Between Professional Skills and Academic Knowledge

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Abstract

In this paper I will first present the French ‘Alternance’ Model which is a way to conceptualise vocational education, in particular apprenticeship. ‘Alternance’ Model, as defined by Geay (1998), is a four dimensional model. I will explore the didactic dimension, centred on knowledge. I will discuss the notion of ‘real problem,’ and I will present pedagogical devices designed for apprentices. I will give examples of my work in mathematics education with young adults working in the trade sector: shop assistants, merchandisers, sales managers.

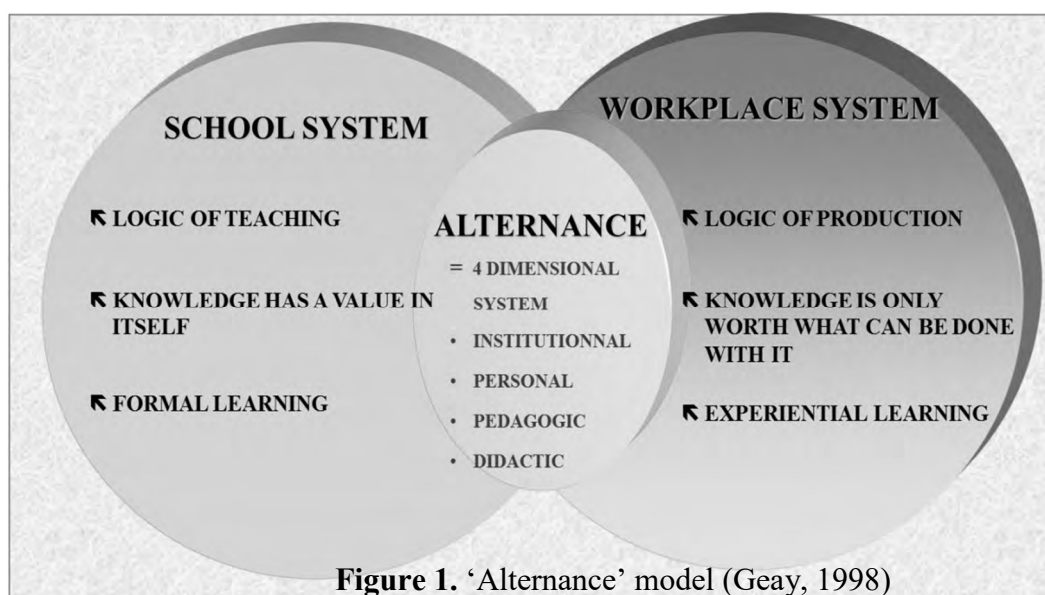
Key words: alternance, device, logic, workplace, system, objects, pragmatic concepts

French ‘Alternance’ Model

Geay (1998) defined ‘alternance’ as an interface system in between two systems, school and workplace (see figure 1). These two systems have two different logics, different ways of learning, and different ways to consider knowledge. His ‘Alternance’ Model has four dimensions: institutional (administrative organisation), personal (about learners’ identity), pedagogic (organization of the formal learning process), and didactic (link between disciplinary knowledge and experienced knowledge). The model works only if the four interlinked dimensions are activated.

This systemic approach is deeply rooted in French culture, because of the importance given to knowledge and also because the approach is centred on the learner. The learner is at the heart of the system and not just one element of it (as in Engeström’s model of expansive learning). However, it is also in opposition to French tradition as there is no hierarchy between different forms of knowledge. Many researchers, referring to different frameworks, share the conception of the learning process as a dialectical process between conceptualization in action and theoretical concepts and not as a simple vertical process directed toward mastery of theoretical knowledge.

I draw on a socio-constructivist framework based on Gerard Vergnaud’s theory of conceptual fields. For Vergnaud, the locus of learning is located in the individual, although he recognizes the importance of the socio-cultural environment. According to him, a concept is linked to a set of situations that give meaning to that concept. Operational invariants, such as theorem action and concepts-in-action, play an important role in the process of adapting to new situations. Vergnaud claims that we need to analyse professional activity and identify its conceptual components. According to him, knowledge has two forms: an operational/practical one and a predicative/theoretical one. He claims that alternance gives the opportunity for each form to support the development of the other.



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To explain this seminal notion of operational invariant, I will give an example from a research I conducted with shop-assistants (Hahn, 2000). I observed how they used percentages at school and in the workplace. I saw that, most of the time, to calculate a price without a 18.6% tax, they 'subtract' 18.6% instead of divide by 1.186. This is a very common and resistant mistake, an epistemological obstacle (Bachelard, 1970). Referring to Vergnaud's theory, apprentices used the theorem-in-action 'x% is inverse of +x%' associated with the concept-in-action that % is a unit.

'Real' Problems, Boundary Objects, and Pragmatic Concepts

We know that school and workplaces have different epistemologies (Noss et al, 2000) and it is not enough to recommend the use of 'real' problems or 'authentic' situations (Adda, 1976). We have to consider vertical and horizontal processes (Bakker, 2014).

It is often recommended to use 'real' problem in order to help learners to link what they learn at school with what they experienced in and out of school situations. But this is not enough, and it can even have negative effects as I will show in another example from my work with shop assistants. I observed that to calculate a 20% discount, they [the assistants] systematically multiply by 0.8 and then subtract the net price from the gross price, instead of just multiplying by 0.2. Of course, this method is not incorrect, but it is longer. I observed that this knowledge is rooted in local professional practices and reinforced by school practices. In fact, in the shop the assistants never calculate the amount of a discount; they always calculate the net price. At school, teachers ask them to

calculate the net price to fit to workplace practices. As a consequence, students no longer know how to calculate the amount of a discount directly (e.g., multiplying by 0.2 to calculate 20%). An excessive use of 'real' workplace problems at school may hinder the development of more general knowledge. It leads to the creation of didactical obstacles, due to school practices.

A way to overcome this difficulty could be to use boundary objects (Star & Griesemer, 1989; Akkerman & Bakker, 2011). Boundary objects are artefacts used in the workplace that also make sense at school. The use of boundary objects facilitates communication between different communities, in this case school and workplace. To help students to overcome the obstacle of additive percentage, I used different boundary objects. These boundary objects were instruction sent to shop owners by their professional unions when the tax rate decreased from 22% to 18.6%. For example, I used a poster sent to jewellery shops presenting the tax change as a discount: '2.3% on marked prices'. The jewellery shop apprentices were tasked with finding out what calculation was hidden behind this marketing trick and how to adapt it to other situations.

Another notion I integrated in my work is the notion of pragmatic concept. This notion was defined by Pierre Pastré (1998) who builds the field of 'professional didactics' drawing on Vergnaud's work, Russian psychology and French ergonomics. This notion is inspired by Vygotsky's everyday concept but it has a smaller range of validity as it is restricted to the professional field. Pragmatic concepts are forms of conceptualization that help to organize efficient action. They are linked to a class of situations and the relation between pragmatic concepts and scientific concepts is dialectic. Pastré claims that didactical situations based on pragmatic concepts enhance professional development. Here is an example of a pragmatic concept and how I used it:

Some years ago I was asked by an international soft drinks company to design a mathematics course for their merchandisers. I interviewed merchandisers and I studied their argumentation with department supervisors. I realised that merchandisers' strategies depended on how they conceptualise the situation. To sell a new product, a seasonal merchandiser first makes a diagnosis by evaluating the 'room'; then he adapts negotiation techniques. Department managers are usually reluctant to reference a new product because it is necessary to reorganize the shelves. If the merchandiser identifies a space big enough for the product by moving the others on the shelves, then he has good chances to convince the manager. Expertise in merchandising reflects awareness of this notion of 'room'. Evaluating the 'room' requires knowing about area and volume calculations and spatial representations, which was not expected. Note that in French we have two different words for 'shelves': 'étagères' in a general sense and 'linéaire' in retailing. 'Linéaire' is already a geometrical conceptualization of what shelves represents in retailing.

As this pragmatic conceptualization of room is central in the success of negotiation I designed a pedagogical situation based on it. Students were asked to prepare a negotiation strategy between a merchandiser and a department supervisor using the map of the department and the shelves, information on products, on competitors' sales etc.

Design

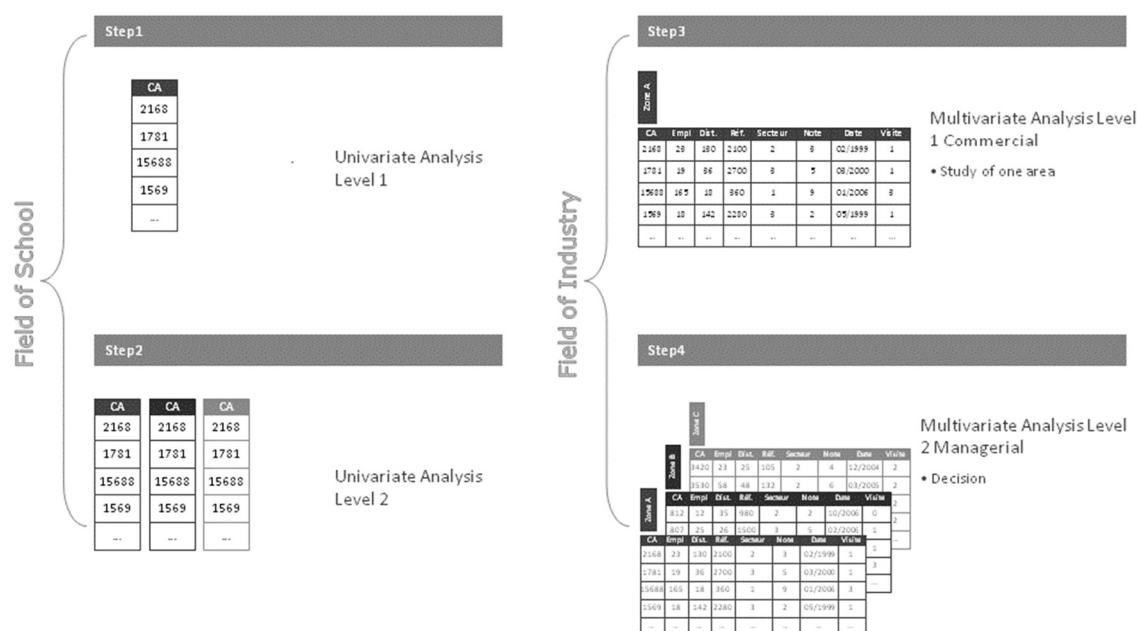
A design experiment is a specific pedagogical device which intends to address complex problems in education through the design of a learning environment interwoven with the testing of theory (Cobb et al, 2003, Bakker & Van Eerde, 2013). I built a design experiment in statistics to get some insight about how students link different types of conceptualizations and, at the same time, design a device that could be used to support the learning process.

The following is a short description of a design experiment I conducted with 36 postgraduate business students (Hahn, 2014). Most of them had previous work experience as salespersons and had applied for a master's degree to become sales managers. The problem on which the pedagogical device was built was an authentic situation based on a real firm and describing an event that really happened. Students had to choose between three sales areas in order to get the job of a sales manager. They were provided with an excel file of data collected on a sample of customers (firms) in each area. The pedagogical device involved four steps, including building a database.

To build the database I conducted a literature which included an a priori analysis (Artigue, 1988) in order to identify statistical obstacles. Through the literature review, it appeared that students seldom use numerical summaries (Konold & Pollatsek, 2002), and if they calculated summaries they make no use of common sense to answer statistical problems (Bakker, 2004). Students have a natural strategy to study extremes and to divide in subgroups (Hammerman et Rubin, 2004; Noss et al, 2000) and have difficulty to move from a local to a global point of view and to construct the concept of distribution (Makar & Confrey, 2005). For clarifications, it is important to consider 2 types of variation: within and between groups (Garfield & Ben-Zvi, 2005), as is shown in Figure 2.

I anticipated that through this device or model (see figure 2) students would have to move from a local to a global point of view and that, to choose the best area, they had to connect knowledge from both worlds and integrate statistics in decision making. The research results showed that the use of statistical summaries was limited, decreased as students move forward in the experiment, and was dependent on the context (the meaning of the variable) and not only on the distribution of numbers. In fact students were not solving the same problem as the procedure they used was primarily related to the personal experience they associated with the problem. Students' conceptions seem to be related to three forms of rationality: a technical rationality (application of techniques which are not put into perspective); a pragmatic rationality (use of intuitive strategies to meet a limited short-term objective); and a scientific/epistemic rationality (integration of theories to enlighten the problem).

The Device



1

Figure 2. The device

Although scientific rationality is the rationality that schools typically referred to, pragmatic rationality usually prevailed. It seems to me that each form of rationality is linked to a dominant identity: technical is linked to a student's identity, pragmatic to a salesman's identity, and scientific more to a manager's identity. Abreu (2000) claims that the resistance in using some specific knowledge can be explained by the fact that some identities are less valued than others. Indeed, the identity of the salesperson seemed to be more valued than the identity of a student. The challenge is to make students understand that both identities must converge if they want to gain access to the new identity of a manager. The few students who built a more managerial approach to the problem seem to question and link knowledge of different origins, trade and statistics, by emancipating themselves from the roles they had previously constructed.

Conclusion

In this paper, I described some pedagogical devices in mathematics and statistics designed to help learners to better link theoretical knowledge with professional knowledge. Apprenticeship gives the opportunity to design meaningful pedagogical devices, in order to make students confront different forms of rationality and help them to 'web' conceptualisations at different levels. This leads me back to the four interlinked dimensions of the 'Alternance' Model. The institutional organization of apprenticeship allowed me to design a pedagogical experiment that helps students to bridge Hahn, C. (2015). The French 'Alternance' model: The question of the relationship between professional skills and academic knowledge from different origins and supports the construction of their professional identity. Then the four interlinked dimensions were

activated and that is why the device had positive effects. If one believes, as I do, that mathematics education has an important role to play in Adult Education, then it is important to understand how mathematics is developed and recontextualized by learners (FitzSimons, 2014), how it contributes to the construction of the learner's identity, and how it allows the learner to understand the challenges and the changes of activities in different contexts. According to me, that is what the role of school should be.

References

- Abreu, G. de (2000). Relationships between macro and micro socio-cultural contexts: Implications for the study of interactions in the mathematics classroom. *Educational Studies in Mathematics*, 41, 1-29.
- Adda, J. (1976). Difficultés liées à la présentation des questions mathématiques. *Educational Studies in Mathematics*, 7, 3-22.
- Akkerman, S.F., & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research*, 81, 132-169.
- Artigue, M. (1988). Ingénierie didactique. *Recherches en Didactique des Mathématiques*, 9(3), 281-308.
- Bachelard, G. (1970). *La formation de l'esprit scientifique*. Paris: Vrin.
- Bakker, A. (2004). Reasoning about shape as a pattern in variability. *Statistic Education Research Journal*, 3(2), 64-83.
- Bakker, A., & Van Eerde, H.A.A. (2013). An introduction to design-based research with an example from statistics education. In A. Bikn-eArhsbahs, C. Knipping, & N. Presmeg (Eds.) *Doing qualitative research: methodology and methods in mathematics education*. Springer.
- Cobb, P., Confrey, J., diSessa, A., Lehrer, R. & Schauble, L. (2003). Design experiment in educational research, *Educational Researcher*, 32(1), 9-13.
- Fitzsimons, G. (2014). Commentary on vocational mathematics education: Where mathematics education confronts the realities of people's work, *Educational Studies in Mathematics*, 86(2), 291-305.
- Garfield, J. & Ben Zvi, D. (2005). A framework for teaching and assessing reasoning about variability. *Statistic Education Research Journal*, 4(1), 92-99.
- Geay A. (1998). *L'école de l'alternance*. L'Harmattan: Paris
- Hahn, C. (2000). Teaching mathematics to shop-assistant apprentices: Exploring content and didactical situations. In A. Bessot & J. Ridgway (Eds.) *Education for mathematics in the workplace* (pp. 159-166). Dordrecht, Netherlands: Kluwer Academic Publishers.
- Hahn, C. (2014). Linking academic knowledge and professional experience in using statistics: A design experiment for business school students. *Educational Studies in Mathematics*, 86(2), 239-251.
- Hammerman, J., & Rubin, A. (2004). Strategies for managing statisticaol mplexity with new software tools. *Statistics Education Research Journal*, 3(2), 17-41.
- Konold, C., & Pollatsek, A. (2002). Data analysis as the search for signal in noisy process. *Journal for Research in Mathematics Education*, 33, 259-289.
- Makar, K., & Cnofrey, J. (2005). 'Variation-talk': Articulating meanings in statistics. *Statistics Education Research Journal*, 4(1), 27-54.
- Noss R., Hoyles C., Pozzi, S. (2000). Working knowledge: mathematics in use. In A. Bessot & J. Ridgway (Eds.) *Education for mathematics in the workplace*. Dordrecht, Netherlands: Kluwer Academic Publishers.
- Pastré P. (1992). Didactique professionnelle. *Education Permanente* no. 111.
- Star, S. L., & Griesemer, J. R. (1989). Institutional ecology, 'translations' and boundary objects: Amateurs and professionals in Berkeley's Museum of Vertebrate Zoology, *Social Studies of Science*, 19, 387-420.
- Vergnaud, G. (1990). La théorie des champs conceptuels. *Recherches en Didactique des Mathématiques*, 10(2-3), 133-170.

Coordinating Learning Inside and Outside the Classroom in Vocational Education and Training (Vet)

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Abstract

For more than 100 years now teachers have been complaining that their new learners 'cannot calculate anymore (!)'. As it is very improbable that the situation has deteriorated from generation to generation over the last hundred years, we at the Swiss Federal Institute for Vocational Education and Training (SFIVET) think that there must be something wrong with the teachers' expectations. As a consequence we started the project 'Everyday mathematics at work'. The idea is to bring the two learning sites – school and company – closer together.

Key words: vocational, work-based, apprenticeship, training

Introduction

In Switzerland two thirds of adolescents start vocational education and training (VET) after lower- secondary education, i.e. after their compulsory nine years of schooling. VET is predominantly based on a dual system: practical, work-based training (apprenticeship) on three to four days a week at a host company is supplemented by theoretical classes (vocational subjects and subjects falling under Language, Communication and Society, LCS) on one to two days at a VET school. Vocational Subjects are usually not split up in separate 'subjects' but taught in a holistic manner. (For details see: <http://swisseducation.educa.ch/en/vocational-education-and-training-0>)

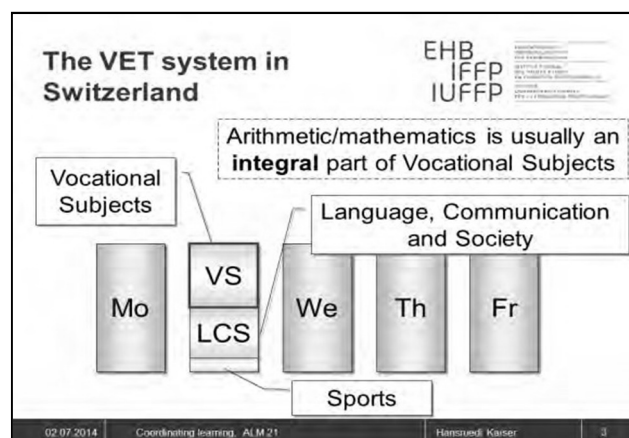


Figure 1: Four days at work, one day at school

The project ‘Everyday Mathematics on the Job’

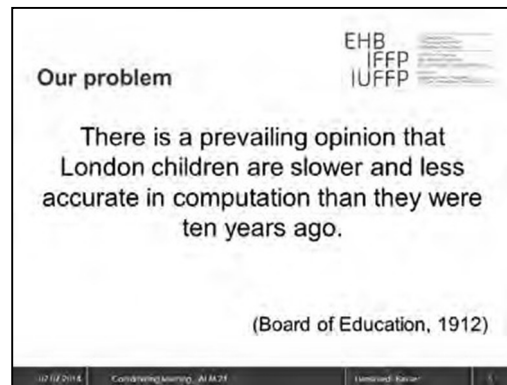


Figure 2: VET teachers complaining then (and now)

For more than 100 years now teachers have been complaining that their new learners ‘cannot calculate anymore (!)’. As it is very improbable that the situation has deteriorated from generation to generation over the last hundred years, we at the Swiss Federal Institute for Vocational Education and Training (SFIVET) think that there must be something wrong with the teachers’ expectations.

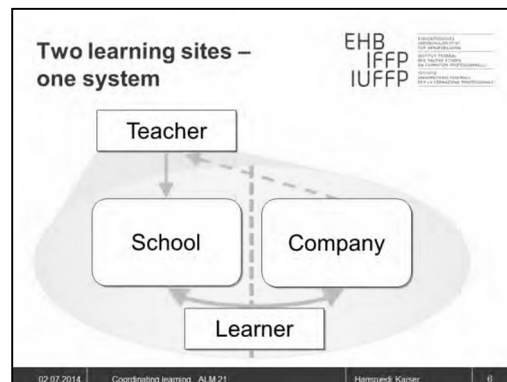


Figure 3: Helping learners by integrating school and work

As a consequence we started the project ‘Everyday mathematics at work’. The idea is to bring the two learning sites – school and company – closer together so that the learners do not have to ‘cross boundaries’ every time they go from school to work or vice versa, but experience the VET system as an integrated whole. One favorable precondition to this is that the teachers themselves are experienced professionals in the occupation they are teaching. They know both worlds very well and are in a good position to help the learners to integrate their learning at the different learning sites.

Background theory: Situated cognition

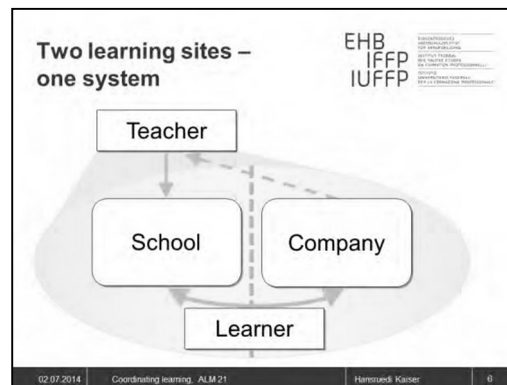


Figure 4: Stories about typical situations in everyday work life used as unifying language

To integrate the two learning sites the people at the two sites have to find a common language to talk about what the learners should learn. Abstract descriptions of competencies as found in regulatory frameworks are not very helpful, as they are insufficient to describe the ambiguous and open-ended challenges at the workplace (see e.g. Coben & Weeks, 2014; Lum, 2004). We found it more suitable to describe these challenges as typical situations of everyday life at the workplace (Kaiser, 2005a). To describe the situations we use stories which enable us to capture ‘soft’ aspects that otherwise are easily lost. So the unit of teaching is always an authentic situation and the challenges it poses.

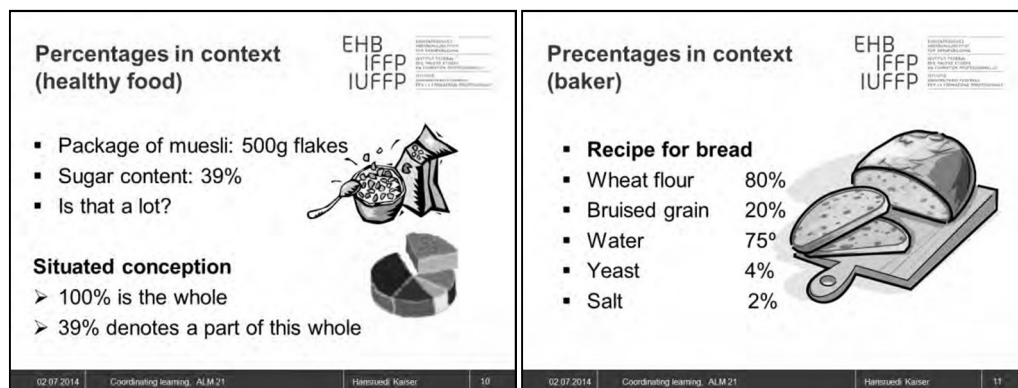


Figure 5: 100% being ‘the whole’ does not work when baking bread

This connects directly with the situatedness of knowledge. We believe that the complaints of the teachers about learners who ‘cannot calculate anymore’ are a direct consequence of them not seeing this situatedness. E.g. children usually learn to use percentages in a context where it is natural to think of 100% being ‘the whole’ (Figure 6; left side). If all went well they can handle all kinds of situations where this idea of ‘100% is the whole’ is applicable. However, when they enter an apprenticeship as baker they encounter professional bread recipes (Figure 6; right side). In this context, all ingredients are specified in percent of the amount of flour – which in no intuitive way is ‘a whole’! Intuitive ‘wholes’ would be the dough mixed out of all ingredients or the finished bread. Searching for an intuitive ‘whole’ the learners stumble and the teachers – rightly – complain that they ‘cannot do percentages’.

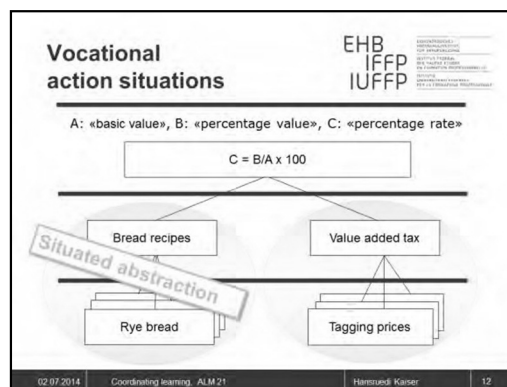


Figure 6: The two situations where bakers have to deal with the % sign

As a consequence we tell our teachers that they should not try to teach their learners ‘percentages’ (or ‘the rule of three’, etc.) but teach them to make bread. If the learners learn to use ‘baker percentages’ in the situation of ‘making bread’ (situated abstraction, Hoyles & Noss, 2004) and do not realize that, from a more abstract mathematical point of view, the ‘baker percentages’ are the same as the ‘part of a whole percentages’ they will still be good bakers. All the professions we have worked with so far do not know many different professional situations where the same mathematical concept is applicable. For example, the only other situation where bakers have to handle percentages is when they ‘calculate VAT’. It is therefore no problem to treat them as two different situations at school – and they are different: While calculating VAT the exact percentages to several decimal places is important; this is not the case with the percentages in a bread recipe.

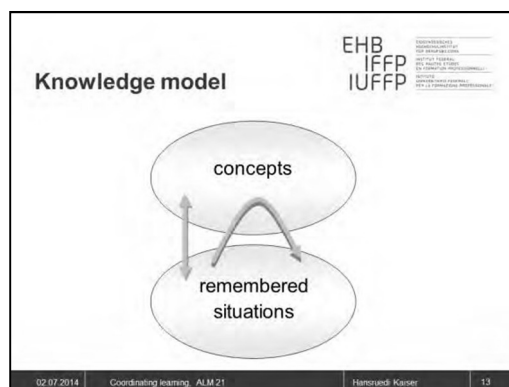


Figure 7: Two types of knowledge

To explain to our teachers why learners think as they think, we use a model which works with two different types of knowledge: Memories of self-experienced situations and learned concepts (Kaiser, 2005b; see Vergnaud, 1990 for a similar conception). Every time a learner encounters a new situation he or she is reminded of previous similar situations and tries to deal with the new situation in analogous ways as in these previous situations. As a consequence, the learners’ knowledge is portioned in packages of similar situations. Some learners manage to fuse the two packages ‘part of a whole percentages’ and ‘baker percentages’ to one package, but many do not.

Didactics: Learning how to use mathematics

The didactical model consists essentially of two rules:

1. Work from the concrete application to the abstract rules – and not the other way round.
2. Always stay in the context of situations that learners know from their workplace.

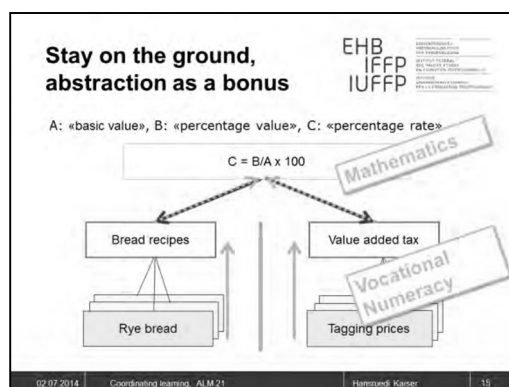


Figure 8: Rule A: Teach ‘to make bread’ and not ‘percentages’

Rule A was mentioned before: ‘Do not try to teach your learners ‘percentages’ but teach them to make bread’. This means: Do not start by recapitulating ‘percentages’ and then apply them to bread recipes. Start with bread recipes and explain, what the % sign means in this context, and make sure that the learners learn to handle the situation ‘make bread’ rather than ‘calculating percentages’. Be prepared when later switching to the VAT to start again at the bottom and to explain, how the % sign is used in this new context. Important is that the learners learn to handle each of the two situations professionally (cf. ‘contextual coherence’, FitzSimons, 2014). Not so important is that they see the ‘mathematical’ similarities between the two situations (‘conceptual coherence’, FitzSimons, 2014). Of course, if there is time and the learners are motivated, it is a good idea to discuss with them later on – after they feel confident with both situations – the similarities between the two situations and do some ‘mathematics’. This will help them later on to adapt to changes at the workplace or to continue with a program in higher education. But you will not be able to do that with all of your learners.

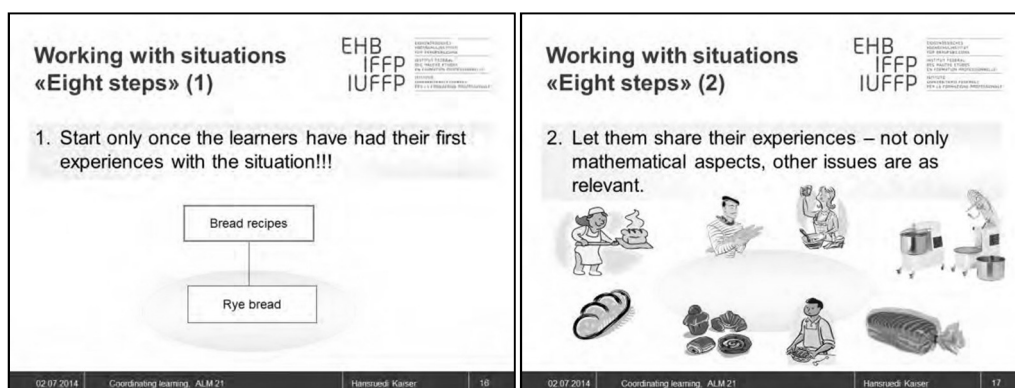


Figure 9: Bring the situation to the classroom (Steps 1 and 2)

For rule B we propose an eight step didactical model to our teachers. Step 1 in this model is actually more a stop-sign than a step. It just means: Do not try to teach the learners how to handle a situation they have not yet experienced at their workplace; it is a waste of time (see LaCroix, 2014)!

To work with a situation like ‘baking bread’ it is important that teachers and learners activate as many remembered situations as possible. This is the idea behind step 2. By listening to the learners’ stories the teacher also gains insight into how the learners perceive the situation and how this perception possibly differs from his professional perception.

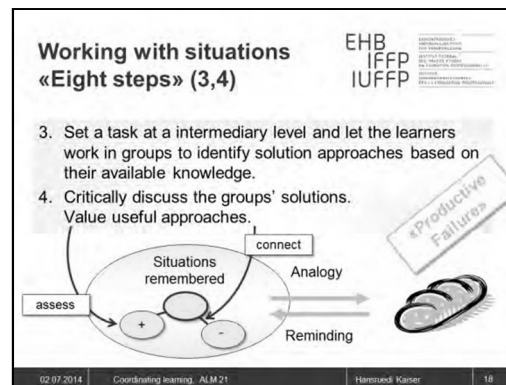


Figure 10: Find out and value what they already know (Steps 3 and 4)

Step 3 and 4: Kapur & Bielaczyc (2012) explored this way of working with learners’ prior knowledge under the heading of ‘productive failure’ and showed how effective it can be. The idea is to start with what the learners already know instead of complaining about what they do not know. Working on the task and discussing the solutions has two functions: 1) Connecting what will follow with the already existing experiences, 2) critically evaluating these experiences in the light of a professional way of handling the task. These old experiences will stay in the package of similar situations and will continue to influence what the learners do when baking bread. So it is important that the learners know which remembered situations are reliably good examples and which are examples of situations to avoid. This is possible when the task is simple and familiar enough to remind them of earlier experiences and at the same time demanding enough so that they encounter the limits of their prior knowledge (productive failure).

Ideally, a list of open questions is the result of step 4; questions on which the learners agree that they need an answer for. Sometimes there are no questions because the learners handled the situation already perfectly well. In this case the rest of the steps can (and should!) be skipped.

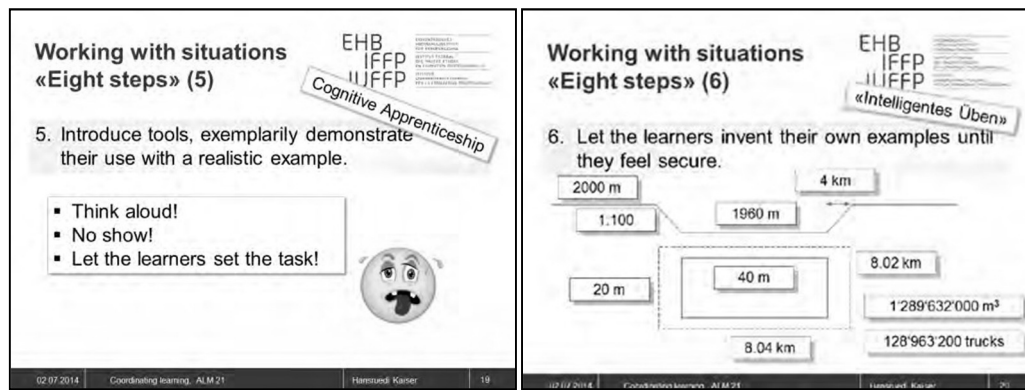


Figure 11: Model a professional solution and let them practice (Steps 5 and 6)

Step 5 provides the answers to the open questions from step 4 in form of a demonstration of how this type of situation is professionally handled. It corresponds to the modelling-step of the Cognitive Apprenticeship process (Collins et al., 1989; Weeks et al., 2013). We always tell our teachers that they should provide a real model and not a show. The learners should see and hear what a professional thinks and where even a professional has to think hard. As a rule we propose to not prepare a demonstration but to let the learners set the task and then try to solve it in front of them while thinking aloud.

Step 6 corresponds to the ‘coaching’, ‘scaffolding’ and ‘articulation’ parts of the Cognitive Apprenticeship process. Details about what is important in this step can be found in publications about Cognitive Apprenticeship. As an addition we propose our teachers not to work with a list of prepared examples but to let the learners invent their own examples (‘Intelligentes Üben’ [Intelligent practice], Leuders, 2009). There are several advantages to this: First, you do not have to prepare anything! Second, learners usually find teacher set tasks boring, but enjoy working on tasks prepared by their colleagues. And third, learner constructed tasks sometimes explore aspects of the situation a teacher would never think of. My favourite example is from the time when I worked with construction workers. The task was to calculate how many truckloads of dirt had to be removed while excavating a pit. They decided to make a deep pit (40 m), with walls not too steep (1:100) to reduce the risk of a collapse. That gave them an upper rim of the pit of 8 by 8 kilometres and about 129 million truckloads of dirt. We laughed a lot but at the end several of the construction workers said that the example helped them a lot to understand what a slope of 1:100 or 3:4 really means. With the usual teacher set examples with ‘realistic’ slopes of 2:1 und 3:2 there would not have been enough variation to get a feeling for the differences.

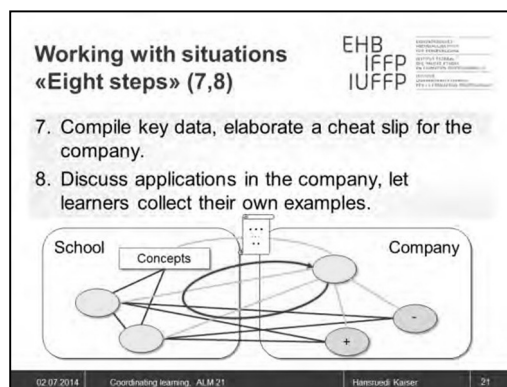
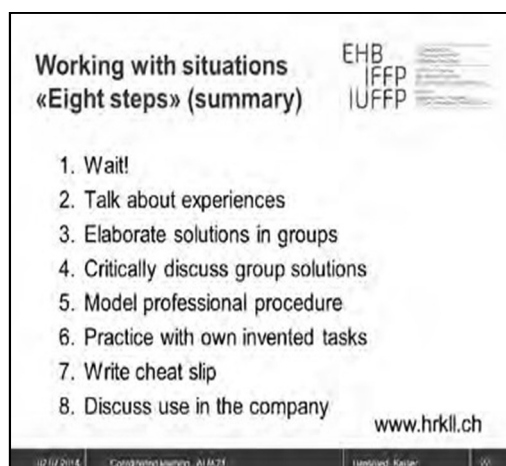


Figure 12: Help them to transfer to the workplace (Steps 7 and 8)

The function of the last two steps is to bring the learning process from the classroom back to the workplace. Step 7 prepares that move. The idea is that the learners construct an external memory that will help them to remember essential details of what they learned in school, once they are back at work. Step 8 has two parts. Part one is a discussion where the teacher and the learners try to anticipate what will happen when the learners begin to use at the workplace what they just learned in school. Part two takes place a week (or more) later. The learners come back to the classroom, tell what has happened, what did work and what did not, and where the problems were when they tried to apply the concepts and techniques learned at school. Solutions for these problems are discussed together and ideally at the end – after several weeks – every learner can add at least one positive example to his or her memory of remembered situations.

If all goes well, what happens by following the ‘Eight Steps’ is: The learners start with some remembered situations from the workplace (the brown circles in Figure 12). In steps 3 and 4 they learn in which instances these experiences have already been helpful to solve a new task and in which they have not (the plus and minus signs within the brown circles). Then in step 5 and 6 they add a few new situations to their memory by watching the teacher model and by working on self-constructed tasks. These memories are connected to the old situations and to some theoretical concepts (blue lines). Before going back to work they write a cheat slip (a kind of boundary object; Hoyles & Noss, 2004) which is in their memory also connected to the school situations (brown arc on top). Back at work they encounter a new situation (yellow). This new situation will remind them of the old workplace situations, which will remind them of the new school situations, which will remind them of the newly learned



concepts and the cheat slip. Based on all the remembered situations, the new concepts, and the cheat slip, they will try to handle the new workplace situation. They will end up with a new (hopefully positive) memory of a workplace situation which is not only connected to memories of old workplace situations but also to memories of school situations (red arc).

Figure 13: The ‘Eight Steps’

I presented the ‘Eight Steps’ here in the way we tell our teacher trainers what they should tell the teachers about what the teachers should do in their classrooms so that in the end the learners learn something useful for their work at the workplace.

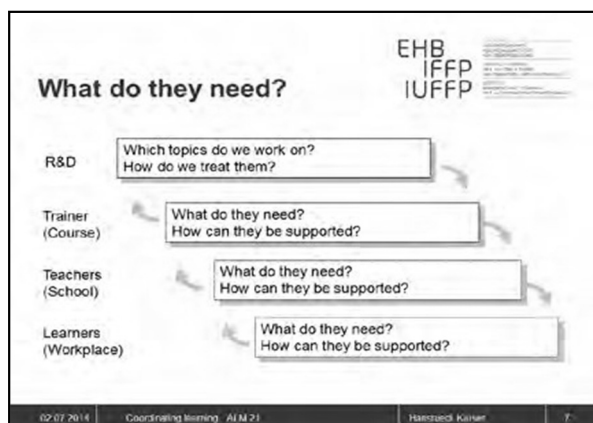


Figure 14: The transmission pipeline

Conclusion

There are many details to each of the eight steps and it is not very likely that each and every one survives the transmission pipeline (see Figure 14). When we watch teachers we see many ‘mutations’ to our ideas – even ‘lethal’ ones (Brown & Campione, 1996). But one advice seems to survive: ‘Do not prepare ‘word problems’; work with real situations the learners tell you about’. Already, this is great, because once the teachers start to do this, they have to do steps 1 & 2. They will then realize that the learners know more than they always thought (steps 3 & 4) and they will have to tune their ‘model’ (step 5) to what is really going on at the workplace. This will help them realizing that applying this ‘model’ at the workplace (step 8) is never straightforward but a major step. All this happens because once they allow the learners to talk about what is going on at the workplace, the learners will insist on making connections between school and work. As two teachers told us: ‘The learners start to feel co-responsible for what is going on in the classroom. They want to show us how it is really done at the workplace. And they become co-teachers explaining and showing things themselves to each other.’ (For more information about the experiences of the two teachers, see Califano & Caloro, 2013.)

References

- Board of Education (1912) quoted from: Anderson, J. (1999). Being Mathematically Educated in the 21st Century. In C. Hoyles, C. Morgan & G. Woodhouse (Eds.), *Rethinking the Mathematics Curriculum* (pp. 8-21). London: Falmer Press.
- Brown, A. L., & Campione, J.C. (1996). Psychological theory and the design of innovative learning environments: On procedures, principles, and systems. In L. Schauble & R. Glaser (Eds.), *Innovations in learning: New environments for education* (pp. 289–325). Mahwah, NJ: Erlbaum
- Califano, I., & Caloro, A. (2013). *Fachrechnen vom Kopf auf die Füße gestellt*. Arbeitsjournal, Pädagogische Hochschule Zürich, Zürich. Retrieved from http://www.hrkll.ch/vom_Kopf_auf_die_Fuesse/Beispiele/Gem_Arbeitsjournal%202%20_Adriano%20_Ivan.pdf
- Coben, D., & Weeks, K. (2014). Meeting the mathematical demands of the safety-critical workplace: medication dosage calculation problem-solving for nursing. *Educational Studies in Mathematics*, 1-18. doi: 10.1007/s10649-014-9537-3
- Collins, A., Brown, J. S., & Newman, S. E. (1989). Cognitive Apprenticeship: Teaching the crafts of reading, writing, and mathematics. In L. B. Resnick (Ed.), *Knowing, learning and instruction* (pp. 453-494). Hillsdale, N.J.: Lawrence Erlbaum.
- FitzSimons, G. E. (2014). Commentary on vocational mathematics education: where mathematics education confronts the realities of people's work *Educational Studies in Mathematics*, 86(2), 291-305.
- Hoyles, C., & Noss, R. (2004). *Situated abstraction: mathematical understandings at the boundary*. Paper presented at the ICME-10, Copenhagen.
- Kapur, M., & Bielaczyc, K. (2012). Designing for Productive Failure. *Journal of the Learning Sciences*, 21(1), 45-83.
- Kaiser, H. (2005a). *Wirksame Ausbildungen entwerfen - Das Modell der Konkreten Kompetenzen*. Bern: h.e.p. verlag.
- Kaiser, H. (2005b). *Wirksames Wissen aufbauen - ein integrierendes Modell des Lernens*. Bern: h.e.p. verlag.
- Kaiser, H. (2008). *Berufliche Handlungssituationen machen Schule*. Winterthur: Edition Swissmem.
- Kaiser, H. (2011). *Vorbereiten auf das Prozentrechnen im Beruf*. Praxis der Mathematik in der Schule, 53(41), 37-44.
- Kaiser, H. (2013). *Ansätze für eine berufsbildungsspezifische Didaktik des Fachrechnens*. bwp@ Berufs- und Wirtschaftspädagogik – online, Ausgabe, 24, 1-21. www.bwpat.de
- LaCroix, L. (2014). Learning to see pipes mathematically: pre-apprentices' mathematical activity in pipe trades training. *Educational Studies in Mathematics*, 1-20. doi: 10.1007/s10649-014-9534-6
- Leuders, T. (2009). Intelligent üben und Mathematik erleben. In L. Hefendehl-Hebeker, T. Leuders & H.-G. Weigand (Eds.), *Mathemagische Momente* (pp. 130-143). Berlin: Cornelsen.
- Lum, G. (2004). On the non-discursive nature of competence. *Educational Philosophy and Theory*, 36(5), 486-497.
- Vergnaud, G. (1990). Epistemology and Psychology of Mathematics Education. In P. Nesher & J. Kilpatrick (Eds.), *Mathematics and Cognition. A Research Synthesis by the International Group for the Psychology of Mathematics Education* (pp. 14-80). Cambridge MA: Cambridge University Press.
- Weeks, K. W., Hutton, B. M., Coben, D., Clochesy, J. M., & Pontin, D. (2013). Safety in numbers 3: Authenticity, Building knowledge & skills and Competency development & assessment: The ABC of safe medication dosage calculation problem-solving pedagogy. *Nurse Education in Practice*, 13(2), e33-e42. Retrieved from [http://www.nurseeducationinpractice.com/article/S1471-5953\(12\)00199-0/pdf](http://www.nurseeducationinpractice.com/article/S1471-5953(12)00199-0/pdf)

Connecting Mathematics Teaching with Vocational Learning³⁰

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Abstract

For many vocational students in England, mathematics is now a compulsory part of their programme, yet the inclusion of an academic subject within a vocational course presents challenges. In this paper, an analysis of a series of case studies of vocational student groups in Further Education colleges in England shows how contrasting practices in 'functional mathematics' and vocational classes reinforce perceptions that mathematics is an isolated and irrelevant subject. Some mathematics teachers made contextual connections by embedding mathematical problems in vocationally-related scenarios but distinctive socio-cultural features of vocational learning situations were often absent from mathematics classes. Addressing this disconnection requires a pedagogical approach and classroom culture that links mathematics learning with vocational values. The findings suggest that adopting mathematics classroom practices that reflect the surrounding vocational culture creates greater coherence for students and has positive effects on their engagement with mathematics learning.

Key words: mathematics, functional mathematics, vocational education

Background to the study

The separation of vocational and academic pathways in post-16 education is a result of long-standing divisions that remain unresolved within the English education system (Young, 1998). Entry to the academic pathway is largely controlled by success in GCSE examinations at age 16 and those with low GCSE grade profiles often transfer to vocational pathways in separate institutions such as Further Education colleges. The divisions between the academic

³⁰ This article is a peer reviewed contribution which appeared first in the ALM Special Edition Journal, Volume 10(1) – August 2015. Copyright © 2015 by the authors. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution International 4.0 License (CC-BY 4.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are properly cited.

and vocational pathways are not only institutional but there are distinct differences in the curricula, qualification types and forms of knowledge associated with each strand. Students have constrained choices in post-16 education within a highly-stratified system (Pring et al., 2009) that tends to prioritise the academic over the vocational.

Within vocational education, both the mathematical skill levels of students and the qualifications undertaken have attracted criticism (Wolf, 2011). Historically, many low-attaining 16-year-olds have taken no further mathematics qualification by age 18 (DfE, 2014). Recent policy changes, however, now require these students to work towards re-sitting the GCSE mathematics examination until they achieve a grade C. When coupled with the recent extension of compulsory education to age 18 years in England, this means that many more students on vocational pathways now learn mathematics as a compulsory part of their study programme. This GCSE mathematics curriculum is traditional and academic in nature and so does not sit easily with their vocational learning.

The research reported herein was conducted prior to these national policy changes, with students who were taking a Functional Mathematics qualification rather than re-sitting the GCSE examination. Functional Mathematics focuses on problem solving and applications in ‘real life’ scenarios and, for most students in the study, the subject was compulsory due to college policies, although it was not a government requirement at the time. This paper examines the contrasts between mathematics teaching and vocational learning that emerged from a wider study of the students’ learning experiences of mathematics and was primarily concerned with students aged 16-18 years. Before examining the research findings, some of the relevant historical academic-vocational tensions are discussed.

Divisions of knowledge, curriculum and pedagogy

The institutional divisions within the English education system can be traced back to the separate establishment of schools, work-related training and adult education. These educational traditions have continued without a coherent overarching policy for education as a whole (Maclure, 1991; Young, 1998). The academic and vocational education traditions that have grown from these roots have different purposes, curricula and qualifications but also reflect longstanding societal hierarchies (Hyland, 1999). The “links between the stratification of knowledge in the curriculum and patterns of social inequality and distribution of power in the wider society” (Young & Spours, 1998, p. 51) are evidenced in the privileging of academic over vocational pathways.

Low-achieving students often undertake vocational qualifications after the age of 16 although these qualifications are seldom considered in schools (Hodgson & Spours, 2008) where the focus remains on academic GCSEs, both prior to age 16 and after the parting of the academic-vocational ways. Vocational training in Further Education colleges has the twin goals of developing practical competencies and acquiring relevant technical knowledge in order to prepare individuals for employment. In contrast, academic qualifications in post-16 pathways prepare students for higher education and GCSE mathematics continues to act as a highly-valued ‘gate-keeper’. Despite some attempts to bridge the divide by increasing the academic rigour of vocational qualifications or bringing vocational education into schools, these initiatives have historically had limited success (Hyland, 1999).

The teaching of academic and vocational subjects draws upon contrasting traditions (Lucas, 2004). For vocational education one of the major influences has been the close association with the apprenticeship model of learning, in which the teacher, as an occupational expert, demonstrates skills for students to replicate until they achieve competence in a ‘community of practice’ (Lave & Wenger, 1991; Wenger, 1999). Teachers may take a range of roles

within vocational workshops and classrooms but practical activity is particularly important in a learning process that is essentially social and collective (Unwin, 2009); the emphasis is on developing competency within a community rather than acquiring knowledge (Hyland, 1999)

The academic strand within Further Education reflects a more classical, liberal approach to education in contrast to the practical usefulness valued by vocational areas. Robson (2006) argues that pedagogy needs to reflect the disciplinary context but this causes an uneasy relationship when a subject such as mathematics is taught as part of a vocational programme. Learning mathematics for vocational purposes focuses activity on a particular context and practical need but this utilitarian view (Ernest, 2004) is in tension with the broader appreciation of mathematics and abstract knowledge valued in academic pathways.

Vocational departments in Further Education colleges associate strongly with their particular occupational values (Robson, 1998). The tendency for students to adopt these values (Colley, James, Diment, & Tedder, 2003) suggests that students primarily focus on their vocational goals, resulting in perceptions that subjects with no clear vocational purpose are peripheral. Such values are key components of departmental culture but are also important influences in the teaching of mathematics (Bishop, 2001; FitzSimons, 1999). Against this background of historical traditions, our interest here is in the differences between students' experiences of mathematics and vocational learning, including the pedagogies and values enacted in these lessons.

Research questions and methods

The research questions of interest in this paper are:

- In what ways are students' experiences of learning in vocational sessions and Functional Mathematics classrooms related?
- How does this affect their learning of mathematics?

To answer these questions we compare teaching and learning approaches in mathematics and vocational sessions, using lesson observation data from a wider study of vocational students' experience of functional mathematics in Further Education.

The research involved a series of nested case studies within vocational areas in three Further Education colleges, from which cross-case and within-case comparisons could be made. Seventeen different student groups were involved from the vocational areas of Construction, Hair and Beauty, and Public Services and each student group formed a separate case study. The research was exploratory as well as explanatory and used multiple methods, both qualitative and quantitative, to provide triangulation between sources and methods. Drawing on ideas involved in grounded theory, an iterative process of analysis was used that involved the coding of qualitative data and constant comparison to identify emerging themes.

In addition to the lesson observations of the same student groups in Functional Mathematics and vocational sessions, data was obtained from student focus group discussions, interviews with Functional Mathematics teachers, interviews with vocational teachers, staff questionnaires and individual student card-sorting activities. In the card-sorting activities students either ranked statements, or placed statements on a Likert scale, to describe their experiences of school and college. In the following section we present some of the relevant results from these activities as background before examining the lesson observation data.

Research findings

When students ranked statements about their reasons for coming to college, the dominant reasons that emerged from the analysis were 'I was interested in the course' and 'I wanted to

improve my education'. Focus group discussions provided further evidence that most students were interested in their vocational courses and valued the opportunity to choose the direction of their education, even though these choices were somewhat constrained by their GCSE profiles.

Secondly, students placed statements regarding their experiences of college on a Likert scale and discussed these in focus groups. Most students depicted college in positive terms (See Table 1 in Appendix I) referring to features such as being treated in a more adult manner, experiencing greater freedom, having more agency and taking more responsibility for their own learning. These results suggest that values relating to adulthood and employment were important to students and welcomed their presence in the college culture. In contrast, many students referred to their experiences of mathematics in school in negative terms (Table 2). Focus group discussions provided further evidence that most students approached college with a view that mathematics was a remote and irrelevant academic subject, one associated with previous failure and disaffection.

Within this context, where many students were positive about their general experience of college but showed an initial negative disposition towards mathematics, we compare their experiences in vocational and mathematics sessions. The differences will be set out using two short summaries of observed sessions. These exemplify the high contrast between vocational sessions and the traditional features of mathematics lessons that were evident in many of the seventeen case studies. After summarising the key features, the approaches used by some functional mathematics teachers to connect the two learning situations will be considered.

Observation A: Beauty Therapy students in the training salon

The students were giving facial treatments to clients. This involved individual skin consultations and one-to-one practical work. One student, Nina, was demonstrating the treatments on a “doll” (artificial head) to students who had missed the previous session. Nina explained the stages of the facial and how each had to be completed properly but within a timescale of about 30 minutes since extra time would lose money for the business. Another student, Gemma, was acting as the salon manager: replenishing products, keeping records of the treatment times and generally making sure the salon was running smoothly. Quiet, relaxing music was playing as each student worked individually on their client. Several times during the session the students were reminded by the teacher to talk solely to their client and not chat to other students. All the students were wearing clean uniforms and seemed to have taken considerable care over their personal appearance. Students were expected to maintain their own uniforms, have their hair tied back and keep jewelry to a minimum. Apart from moderating the atmosphere, the vocational teacher watched and advised, acting as a guide and source of further information when necessary.

Observation B: Functional mathematics with Public Services students

The session took place away from the Public Services vocational area. Space was tight and although the students could all be seated at tables there was little room for the teacher, David, to move between them. This had an impact on the lesson since it was difficult for him to check work, give feedback and support individual students. After a formal teacher-centred introduction and some worked examples on the board, the main activity was to complete a series of worksheets about areas and perimeters. These were given out one at a time so that the completion of any worksheet was quickly followed by the provision of another. David tried to circulate to mark work and encourage students to participate but it was difficult to get students to engage

with the work and frequent reminders were needed to keep them on task. He worked hard to keep distractions under control by reminding students to be quiet and get on with their work. These attempts to impose a working environment dominated the session and, despite being calm and persuasive, David's strategy seemed largely ineffective. Towards the end of the lesson students who had completed the work were allowed to go early whilst the others were retained and urged to continue until the official end of the lesson.

In the vocational session students were expected to adopt professional standards of behaviour and take significant responsibilities such as making decisions about treatments, supervising other students and providing customer care. In contrast, the Functional Mathematics lesson was a tightly structured, teacher-controlled session, closely resembling a typical school mathematics classroom. Students had little opportunity to make decisions about the learning process or take responsibility for their own progress as the whole process was largely controlled by the teacher.

Within the training salon there were clear rules regarding personal appearance and professional conduct but there was also considerable freedom. Students were expected to focus on their client during the session but walking around to collect equipment or products was part of the normal routine. Unprofessional chatter with their peers was prohibited but consulting with other students for support or advice was an accepted feature of working practice. In the functional mathematics classroom, space was constrained and students were expected to remain seated throughout the session. This created a very different environment and influenced the way in which learning took place.

David's approach to teaching was topic-based and the lesson involved an explanation of the mathematical content before demonstrating the processes through worked examples on the board. This was followed by student work on further examples that they were expected to complete quietly and independently. For David, mathematics should be learned in an organised, orderly classroom with clear rules enforced by the teacher. In contrast, the teacher's role in the salon was mainly to observe and advise. Students learned from one other as well as from their teacher in this collaborative and supportive environment.

There were further contrasts in the type of tasks used. In the vocational session practical skills and theory were integrated into tasks. For example, relevant theory about skin types needed to be recalled and used during consultations with clients. Tasks in the salon would generally take some time to complete and there was some flexibility about the time taken for each component as long as the overall treatment was completed within a reasonable timescale. Learning in David's classroom mainly involved short written tasks with the expectation that students would remain 'on task' and completion would be followed immediately by additional written work.

These two approaches to teaching and learning seemed to be based on contrasting values and assumptions regarding the role of the teacher, the environment and the processes that would be most effective for learning. Relationships between the teachers and students in these two examples were very different, as were the social structures and classroom cultures in which roles and relationships were embedded.

Comparisons with other observations of vocational sessions in salons, workshops and classrooms, showed that this Beauty Therapy session was very typical. A cross-case analysis led to the identification of four main areas in which there were common characteristics:

- *Responsibility, agency and freedom.* Students worked within loose frameworks of rules that related to health and safety requirements or professional standards but had freedom to

make individual decisions. They were expected to take responsibility for their learning and were placed in positions of responsibility for clients or other students. There was freedom of movement around the vocational salons, workshops and classrooms.

- *Vocationally-related values and expectations.* Adult and work-related values, dispositions and behaviours were encouraged and evident in most sessions.
- *Student-focused learning through guided activity.* Learning processes centred on developing practical competencies through replication of skills demonstrated by respected vocational experts. Their role was to facilitate learning, with students acting as apprentices in a community where informal peer learning was often evident.
- *Integration of knowledge and skills into substantial tasks.* Practical skills were highly valued but knowledge from theory sessions was often intertwined into tasks. Tasks were usually substantial with multiple elements and time-scales stretching over days or weeks. Students worked at their own pace, making individual decisions about the order of the sub-elements and the methods to use.

These four areas contrasted with the formal, traditional approach to teaching mathematics in David's lesson where the following key features were identified:

- *Teacher authority and control.* The rules in the classroom reflected the values and priorities of the teacher-authority who expected students to comply. The teacher directed and controlled the learning process. Students had little agency in their work. They were expected to remain seated throughout the session, to work quietly, individually and follow directions.
- *Academic values and expectations.* The students were learning a subject as a series of disconnected topics, through a process of knowledge transfer rather than developing a set of skills.
- *Teacher-led activity.* The lesson was planned and closely directed by the teacher. Mathematical knowledge was delivered to students who did not aspire to be mathematicians and had little sense of how this learning might be useful.
- *A focus on written work.* There was a reliance on worksheets and written solutions to questions. The tasks were usually short and students were expected to remain 'on task' throughout the lesson.

Not all of the mathematics lessons were, however, like David's. We now consider those cases in which the Functional Mathematics teachers adapted to the vocational environment with lessons that were better connected to the students' vocational learning experience. The key features of these lessons are, again, presented using a short lesson observation as an exemplar, followed by a summary of the common characteristics of similar lessons from the cross-case analysis.

Observation C: Functional mathematics with Hairdressing students

The session took place in a separate building, some distance from the main Hairdressing area. As the students arrived the teacher, Richard, greeted them individually and engaged in relaxed conversations about what they had been doing both inside and outside college since the last class. His introduction to the lesson involved a class discussion about using units of time. Students readily talked about their difficulties, both asking and responding to questions until they were satisfied that they understood the concepts and processes involved. The main task in the lesson was to draw up an appointment schedule for a hair salon from a list of requests for appointments involving different hair treatments. This required students to use vocational knowledge about the time needed for each treatment and considerations

about appropriate business decisions, in conjunction with mathematics. The students produced individual schedules, using different methods and formats, but discussed their strategies and decisions freely with each other. The teacher supported and guided by asking students individually about their methods, assumptions and decisions. Finally, the teacher checked their progress with a longer-term integrated homework assignment in which students were using vocational knowledge, English and mathematics to produce a business plan for a new hairdressing salon.

Although the physical separation of the mathematics classroom from the students' vocational base was similar to the situation of the Public Services lesson, the key pedagogical features contrasted with those observed in David's lesson and were more closely aligned to the vocational session. Similar features were evidenced in a number of Functional Mathematics sessions and the cross-case analysis suggested the following key features of a more 'connected' functional mathematics classroom:

1. Teachers adopted pedagogies that made connections through context, classroom discourse and programme synchronization:
 - Using vocational situations as the context for mathematical problems. This was effective when the details of these scenarios were accurate and resembled situations that students had actually experienced;
 - Encouraging an integrated discourse about mathematics in students' lives by using informal conversations and interests as a basis for improvising discussions about applications of mathematics;
 - Synchronizing the Functional Mathematics scheme of work with the vocational programme to increase perceptions of relevance.
2. Teachers developed classroom cultures that were more in keeping with values of the surrounding vocational culture by:
 - Creating flatter social structures than those in traditional school mathematics classrooms;
 - Adopting a supportive, facilitating role;
 - Developing equitable relationships with students;
 - Using peer learning as a key learning strategy.

In cases where these features were present, our cross-case analysis suggested that students responded more positively to learning mathematics than in classrooms where the pedagogy and culture were more traditional. These features seemed to reduce the sense of disconnection between the mathematics classroom and the vocational programme and as a result their engagement with mathematics improved.

Discussion

For many of the students in this study, learning mathematics was perceived as separate from their vocational learning. However, when learning mathematics was connected to students' vocational development, values and culture then the subject generally became more relevant, meaningful and coherent. Although students retained a narrow focus on their vocational area (Hodgson & Spours, 2008) and only identified a limited utilitarian purpose for mathematics (Ernest, 2004), their acceptance of Functional Mathematics as a vocationally-relevant subject represented a shift in perspective that had a positive effect on their engagement with mathematics.

The students in this study were in a transition from school education to the workplace and

were experiencing the tensions between formal, abstract academic learning and the development of vocational skills to achieve professional competence. As FitzSimons (1999) explains, mathematics in the workplace becomes a tool, in contrast to being the object of activity in mathematics classrooms. The transition from school to the workplace therefore involves changing students' perceptions of mathematics from object to tool, but this is a gradual process and not straightforward. In the interim period of being a 'trainee' in college students are caught between these two positions.

The mathematics learning of vocational students is situated in a complex socio-cultural environment, influenced by contrasting educational and vocational values and traditions. Although historical, social and cultural influences affect values generally in mathematics classrooms (Bishop et al., 1999), the co-existence of Functional Mathematics lessons within vocational programmes require students to change between cultures with typically dissonant values, unless cultural divisions can be bridged and values harmonized. In practice, students tend to adopt the values of their vocational area, as indicated by previous research (Colley et al., 2003) and the alternative value system, that frames much mathematics teaching, generates tensions. Some reconciliation of these different cultures is necessary to enable students to see learning mathematics as an integral and meaningful component of their vocational training.

Values relating to employment and adulthood were dominant in the general college culture and were also important to students. In some cases Functional Mathematics teachers created social structures that facilitated a more open and equitable classroom culture and this was better aligned to these values. Others embraced specific vocational values, such as teamwork for Public Services, in their teaching approaches. These adjustments to classroom culture provided a more coherent learning experience and helped stimulate student engagement.

In the observed vocational sessions, the role of the teacher was one of an occupational expert in a learning community similar to a 'community of practice' (Lave & Wenger, 1991; Wenger, 1999). In this social arrangement students were learning from the teacher's expertise and from one other by developing practical competencies coupled with technical knowledge. Functional Mathematics lessons involved a different learning process as students were neither aspiring to be mathematicians nor intending to be teachers of mathematics and therefore a 'community of practice' model was inappropriate.

The analysis suggests that the practices of a *connected* mathematics classroom in colleges can enable students to bridge some of these divisions by presenting mathematics as a subject that is not confined to the domain of academic knowledge but can also constitute vocationally-related skills. The pedagogy of the *connected* classroom in this study reflects some of the principles of embedding from previous research (Eldred, 2005; Roberts et al., 2005) but it also highlights the importance of shared values and compatible cultures in mathematics classrooms for vocational students. Bridges between different practices, of vocational learning and mathematics teaching, were constructed using key points in these separate discourses to make connections (Evans, 1999). These connections enabled a form of 'boundary crossing' that reconciled some of the conflicts for students between their vocational training and their learning of mathematics. Although the learning processes for mathematics and vocational skills retains some fundamental differences, these approaches suggest ways in which greater coherence and better engagement can be brought into the student experience of learning mathematics in vocational education.

Conclusions

The academic-vocational divisions and tensions of the English education system were evident

in the student experience through contrasts in the pedagogy and purpose of learning in vocational and mathematics sessions but there were also important differences in the social structures, culture and values within the two separate learning environments. For students in transition from school to the workplace, the vocational training phase is characterised by changing values and shifting perspectives as students become more orientated towards employment. Bridging the divisions and providing a coherent, meaningful experience of mathematics learning for vocational students requires an understanding of this transition, a non-traditional approach to mathematics teaching and a classroom culture that reflects the values of the surrounding environment that are important to students. The effects of these features within the classroom were significant for students in the study and suggest aspects of teaching in a vocational environment that need to be considered seriously alongside general and subject-specific pedagogy.

In the light of recent policy changes in England it seems that the move towards the more knowledge-based, academic GCSE mathematics qualification rather than a 'functional' curriculum is likely to create a greater distance between mathematics and vocational learning for students. Further research is needed to ascertain the actual effects of these policy changes on the dispositions and attainment of students who are required to re-sit GCSE mathematics courses but there are clear indications in this study that addressing the cultural divisions between mathematics and vocational learning is an important factor in creating a meaningful and successful experience for students. These findings have implications for the training of mathematics teachers for Further Education. They also raise questions for policy-makers for whom the achievement of an academic minimum standard in mathematics is privileged over engaging students in a meaningful experience that prepares them for the workplace.

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References

- Bishop, A. J. (2001). What values do you teach when you teach mathematics. In P. Gates (Ed.), *Issues in mathematics teaching* (pp. 93-104). London: Routledge Falmer
- Colley, H., James, D., Diment, K., & Tedder, M. (2003). Learning as becoming in vocational education and training: class, gender and the role of vocational habitus. *Journal of Vocational Education and Training*, 55(4), 471-498.
- DfE. (2014). GCSE and equivalent results in England 2012-2013 revised. Retrieved from <http://www.gov.uk/government/publications/gcse-and-equivalent-results-in-england-2012-to-2013-revised>.
- Eldred, J. (2005). *Developing embedded literacy, language and numeracy: supporting achievement*. Leicester: NIACE.
- Ernest, P. (2004). Relevance versus Utility. In B. Clarke, D. M. Clarke, G. Emanuaelson, B. Johansson, D. Lambin, F. Lester, A. Wallby & K. Wallby (Eds.), *International Perspectives on Learning and Teaching Mathematics* (pp. 313-327). Goteborg: National Center for Mathematics Education
- Evans, J. (1999). Building Bridges: Reflections on the problem of transfer of learning in mathematics. *Educational Studies in Mathematics* 39(1-3), 23-44.
- FitzSimons, G. E. (1999). *Values, vocational education and mathematics: Linking research with practice*. Paper presented at the 7th Annual International Conference on Post-Compulsory Education and Training, Brisbane, Australia.
- Hodgson, A., & Spours, K. (2008). *Education and training 14-19: curriculum, qualifications and organization*. London: Sage.
- Hyland, T. (1999). *Vocational studies, lifelong learning and social values: investigating education, training and NVQs under the new deal*. Aldershot: Ashgate.
- Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.

- Lucas, N. (2004). *Teaching in further education: New perspectives for a changing context*. London: Institute of Education.
- Maclure, S. (1991). *Missing Links: The Challenges to Further Education*. London: Policy Studies Institute.
- Pring, R., Hayward, G., Hodgson, A., Johnson, J., Keep, E., Oancea, A., ... Wilde, S. (2009). *Education for All: The Future of Education and Training for 14-19 Year-Olds*. Abingdon: Routledge.
- Roberts, C., Gidley, N., Eldred, J., Brittan, J., Grief, S., Cooper, B., ... Castellino, C. (2005). *Embedded teaching and learning of adult literacy, numeracy and ESOL: Seven case studies*. Leicester: NIACE.
- Robson, J. (1998). A profession in crisis: status, culture and identity in the further education college. *Journal of Vocational Education and Training*, 50(4), 585-607.
- Robson, J. (2006). *Teacher professionalism in further and higher education: challenges to culture and practice*. Abingdon: Routledge.
- Unwin, L. (2009). *Sensuality, sustainability and social justice: vocational education in changing times*. London: Institute of Education.
- Wenger, E. (1999). *Communities of Practice: Learning, Meaning and Identity*. Cambridge: Cambridge University Press.
- Wolf, A. (2011). *Review of vocational education* (DfEE Ed.). London: Department for Education.
- Young, M. (1998). *The Curriculum of the Future: from the 'new sociology of education' to a critical theory of learning*. London: Falmer.
- Young, M., & Spours, K. (1998). 14- 19 Education: Legacy, opportunities and challenges. *Oxford Review of Education*, 24(1), 83-97.

Appendix I

Table 1. Student Views on What College is Like

QUESTION B: What is college like? (compared to school)					
STATEMENT	SA	A	N	D	SD
There is more freedom than there is in school	45	42	9	6	1
It has been easy to make friends	32	48	23	0	0
I get on with the staff in college	25	58	16	4	0
You are treated better at college than school	26	50	21	6	0
College work is easier than school	8	32	38	21	4
The staff treat you like adults	23	51	15	10	2
My course is interesting	47	44	8	2	1
I like the subjects I do	32	57	10	4	0

Table 2. Student Experiences of Mathematics in School

QUESTION D: When you did Math at school how did you feel?					
STATEMENT	AA	S	H	O	AN
I worked hard	13	34	31	14	9
It was difficult	9	29	28	28	8
I got distracted	22	31	16	29	4
I liked Math	9	20	14	20	38
I felt stressed	15	20	19	29	19
I was bored	21	20	26	27	7
I liked the teacher	11	23	19	13	36
I felt confident	4	22	21	38	17
It was interesting	2	14	18	34	34
I understood it	6	30	27	24	15
It was confusing	14	21	23	38	6
I could have done better	34	30	17	19	0

Professional Development

Making meaning in maths

Adult Numeracy Teaching: a course for teachers

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Background: who needs training?

Adult numeracy is a small educational field with little published research, and only a small number of teaching materials and professional development and training packages available. Since the late 1980s Australia has become quite active in the adult numeracy area and now has developed quite a range of adult numeracy curriculum, teaching resources and professional development options for teachers.

One of our priorities in Australia has been to develop a range of training packages for tutors and teachers. This has been seen as important partly because the majority of teachers were in fact adult literacy teachers with little knowledge or experience in teaching numeracy or basic mathematics. As well, many of these teachers were anxious about mathematics themselves. On the other hand, it was felt that traditional maths teachers might not have had a chance to learn how to effectively and sensitively teach adult students who have a history of failing traditional school mathematics. Therefore, if there was to be successful adult numeracy teaching these two groups of teachers needed to be trained and supported.

Although a tradition had grown up of offering adult numeracy sessions at literacy conference or even running some adult numeracy workshops, there was very little opportunity during the developmental period of the 1980s for teachers to undertake any substantial training in adult numeracy. The first such program to be developed was *Breaking the Maths Barrier* (Marr and Helme, 1991), which was the result of national activities during International Literacy Year, 1990. *Breaking the Maths Barrier* has been instrumental in training adult numeracy teachers across Australia. It is a flexible package and provides a range of innovative, hands-on and practical activities that can be presented in a variety of ways.

However it was felt that there was a growing need for a comprehensive training package that was more theoretically based and which would provide opportunities for participants to reflect seriously on their current practice, beliefs and theories of adult numeracy. This led to the development of the course, *Adult Numeracy Teaching: Making Meaning in Mathematics* (ANT), an 84- hour professional development course designed as a continuation and further development of existing professional development packages, such as *Breaking the Maths*

Barrier leading in turn to postgraduates study. ANT is therefore targeted at experienced adult literacy, language and numeracy teachers.

ANT was an initiative of the National Staff Development Committee for Vocational Education and Training which works under the Australian National Training Authority. In 1993 they advertised a project, Teaching Mathematics in Adult Literacy and Basic Education, which was awarded jointly to the Centre for Language and Literacy at the University of Technology, Sydney (UTS) and the Adult Basic Education Resource and Information Service (ARIS) at the Victorian Office of the National Languages and Literacy Institute of Australia (NLLIA). The project team was supported by several experienced adult numeracy teachers and trainers from NSW and Victoria.

From the beginning, there was interstate consultation; we undertook a survey with state and territory teachers and educators to help identify needs, and we completed a literature search. The project team worked largely as a cooperative group, calling on the experience and expertise of a wide and enthusiastic group of consultants. By early 1994, we had endorsed and refined a model for the course content, and considered in detail the aims and learning outcomes, and the content issues and areas. The detailed written course was piloted in two states, NSW and Victoria, later in 1994 and in 1995. A Train-the-Trainers program was held in early 1995 for representatives from all states and territories, and the course was officially launched in September that year.

Numeracy: not less than maths but more

The question the project team began with was:

What should numeracy teachers know and be able to do after this course?

The answer proposed after some discussion was:

After this course a teacher

- 1) *should have a critical appreciation of the place of mathematics in society, and*
- 2) *should be able to initiate appropriate learning activities by identifying the numeracy needs of students and responding from a variety of approaches to teaching and a range of appropriate mathematical resources and knowledge.*

These concerns led us to a structure that tries to weave together three strands: knowing about maths; learning (and teaching) maths; and doing maths. This last strand is central to the course: clearly to teach numeracy you must know how to do mathematics.

The development throughout the ANT course is based on the assumption that the Adult Basic Education students of the teacher participants have usually failed in a transmission-type maths education and need alternative teaching approaches. Through engaging the participants in discussion and a wide range of specific mathematical activities, we aim to get the presenters and participants to compare the assumptions behind a transmission type pedagogy, with those informing a more constructivist approach, and the critical standpoint which is gradually incorporated. What do we mean by these approaches?

Most of us cut our mathematical teeth on the well-tried *transmission* or *positivist* model of mathematics teaching. In this model, knowledge is taken as external and objective and facts

are neutral. Instruction emphasizes rote-learning and mastery of atomized content, and there is little concern with maths as a way of seeing and thinking or with the uses of maths in understanding the world. Such an ideology grew up at a time when mass education was a new venture, classes were large and instrumental outcomes predominant. The quantified world of capitalism needed clerks and bookkeepers. Schools produced them.

This ideology still permeates a lot of mathematics classrooms, but it is contested, especially in primary and adult basic education, by a second model – the *constructivist* ideology – which centres on the human dimensions of knowledge. Changes in mathematics education in the ‘sixties were, in part, a response to the success of the Russian Sputnik, and an effort to produce more creative mathematical thinkers. One of the core changes was a shift to include process as well as product, relational thinking as well as instrumental. In the constructivist model, knowledge – which we construct for ourselves – is seen as being viable rather than ‘true’, and the role of the teacher is a facilitator and provoker of this learning. Meaning is an important focus of our teaching; and as can be seen in many adult classes committed to overcoming maths anxiety, the teaching is student paced, with an emphasis on problem-solving, process and cognitive conflict. This ideology has a greater appreciation of the nature of mathematics and of ability, but, in spite of its focus on maths as a human construction, it has little appreciation of how we can decode the varying mathematical worlds that learners bring to the classroom from their different social backgrounds. And it does not even begin to understand how received mathematical knowledge is shaped by social assumptions that derive from class, gender, race and ethnicity.

It is the sense of the interconnectedness of power and mathematical practice that extends the constructivist ideology into what we have been calling *critical constructivism*. This model owes much of Freire’s pedagogy of the oppressed. Numeracy – like literacy – can be a double-edged sword: some programs use methods and materials that domesticate rather than liberate. Numeracy that emancipates should be based on methods and materials that increase autonomy and social understanding. In this model, knowledge, as well as being assumed to be ‘viable’ and constructed by the learner, is also situated and political, and learning and teaching involved questioning power relations both within and outside the classroom. A critical constructivist approach to numeracy starts from the experiences and perspectives of the learners and the local community, learning and evolving mathematics that is relevant to their needs – helping students to become strong within their own culture, and to learn at the same time how to critically appropriate knowledge from a wider range of experiences. These three (or more accurately, two and a half) modes are summarized in Table 1.

Table 1: Two (and a half) contrasting views of knowledge and their implications for learning and teaching.

	Positivism	Constructivism	Critical Constructivism
Knowledge as	<ul style="list-style-type: none"> - External reality - Objective - Above human - True - God-given - Unproblematic 	<ul style="list-style-type: none"> - Internal reality - Subjective - Human activity - Viable - Socially negotiated - Problematic 	As for constructivism + <ul style="list-style-type: none"> - Situated - Political
Learning as	<ul style="list-style-type: none"> - Reception of information - Absorption of facts - Reproductions 	<ul style="list-style-type: none"> - Constructing - Dealing with perturbation - Reconstructing 	As for constructivism + <ul style="list-style-type: none"> - Purposeful - Critically reflective - Challenging power relations
Teaching as	<ul style="list-style-type: none"> - Transmission - Expert - Concern for product 	<ul style="list-style-type: none"> - Questioning, provoking - Collaborating, facilitating - Concern for process 	As for constructivism + <ul style="list-style-type: none"> - Awareness of power relations - Negotiating for power relations

For example: theories in practice

The best way to illustrate how these different theories might influence our teaching, is to reflect on how a particular topic might be approached from each perspective. We will arbitrarily use the concept of an ‘average’ or ‘arithmetic mean’ in this case, although countless other topics could well be used.

How would such a concept be taught from a positivist point of view: using a transmission style of teaching? We do not have to look very far to answer such a question. The arithmetic mean, or average is one of the lynch pins of Quality Control measures in many manufacturing

industries. We have observed typical training programs in which this concept is introduced via a rule-based or algorithmic approach:

$$\frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

The trainer puts the overhead on the screen then goes over the steps once or twice to ensure that everyone ‘understands’ what to do. In most cases this technique seems to be effective. Most members of the group will repeat the operation several times in the training session and it is ‘done’. They have learned averages! Oh but what has Tony done? He appears to have made a little mistake. He has calculated the mean of 215, 235, 270, 195, 276, 214, 200, 190, 232, and 205 and is waiting to see if his answer of 2052 is correct. Oh I see, he has forgotten to divide by ten. We can fix that up easily, just remind him about that last step. Give him some more practice.....and so it goes. And so the transmission-style teacher wonders: how can I get Tony to remember that last step?

What might a constructivist wonder? It would be something more like: how can I get him to engage with the meaning? After all, we might, if we reflect on it, realise that Tony has absolutely no feel for what ‘answer’ to expect. He may not even realise what an average is supposed to be. No-one has talked about the idea of an average, or related this calculation to the common meaning of the word as used in everyday parlance. The students have not had a chance to guess at the expected outcome, or see it as a sort of central indication of the quantities in front of them before starting the calculations – it could anything really.

In a more constructivist approach to the topic, all of these things would occur. In such a training session we might talk about our average age – guessing it as we look around the room. We might conjecture how it would change if the oldest person was not there, or if a new young worker was recruited. We might then move on to ideas of measurement and averages by comparing heights. One favourite activity in this vein, is to give small groups of three or five people a streamer and ask them to: *Use this streamer to display the average height of your group (no rulers or tape measures to be used).*

As a reader of this task, you may have already decided how it should be done and may give it to a class with the expectation that they will do it just that way. Surprisingly, every time we have tried it, each group has a different plan of attack. This variety, and the different ways of ‘seeing’ averages means that there is a great deal to discuss when groups share their techniques with one another. The sharing out of the extra bits of streamer, or giving more to those who have less until all are equal is a common way of looking at it, as is the amalgamation of all the streamer pieces, followed by equal division amongst the three or five. (3 or 5 rather than 4 means that the group has to be more conscious of the equal division process.). Such activities facilitate reflection on the meaning of the term ‘average’ and provide opportunity for appreciation for the steps within the mathematical process. It is a step along the constructivist pathway, and some distance from the rule-based transmission approach.

What then of the next perspective, which we are calling critical constructivism? This approach should take us further than the mathematical principles, and into some analysis of the use of ‘averages’ in the real world arena where it impacts on the lives of our students. A good activity for beginning this dialogue is one of a series created for *Numeracy on the Line* (Marr, Anderson and Tout, 1994). In this activity, the participants are given small wage slips reflecting an invented position in a hypothetical factory: several workers at ‘Production worker level 1’ several but fewer at ‘Production worker level 2’ and again several but fewer

at 'Production worker level 3'. One participant is given the 'Production Manager's' slip – naturally at a considerably higher salary than the production workers. Participants then compare their wage slips, predict what the average wage will be and discuss ways of calculate it. Once the calculations are completed and compared, the distortion created by the higher figure of the Production Manager's salary can be discussed. Asking the participants to form a line from lowest to highest salary and reflecting on the relative position of the 'average' in the line up – higher than all of the workers and closer to the Manager – provides a graphic illustration of the non-representative nature of the average and a chance to ask 'What would be a more useful figure to inform people about your wages?' From this discussion the alternative averages of median and mode can be introduced and the way is opened for subsequent analysis of 'average' statements by the media and political interests.

The organisation of the course

The course itself consists of four sections. Each section includes opportunities for mathematical activities, group discussion, journal reflections and curriculum investigations.

Section A: Exploring practice

- introduces issues and flags the need for theories
- journeys through a number of crucial mathematical areas including place value, basic operations and algebra, raising issues of meaning making, language, maths in practice and cultural contexts.

Section B: Maths as a human construction

- focuses on learning some maths, largely to do with space and shape, using a constructivist approach
- reflects upon this learning experience in journal
- looks at role of teacher in all this: how have we been taught, how do we teach, how do we want to teach?
- makes cultural and historical connections
- concludes with a substantial mathematical investigation by participants

Section C: Maths as a critical tool

- involves learning some more maths, including statistical and other everyday applications, within a critical framework
- looks upon this learning experience in journal
- concludes with participants doing a substantial project excavating maths from everyday sources and developing appropriate teaching or curriculum materials

Section D: Naming theories: implications for practice

- extracts from previous discussion the three theories of transmission, constructivism, and critical constructivism
- applies these theories to teaching, planning a curriculum, and assessment
- comes to the conclusion that
numeracy is not less than maths, but more

In conclusion, the writers of ANT hope that participants in the course will enjoy it, finding in it a challenge for themselves and the possibility of helping their students to move from an understanding of:

numeracy as doing basic maths

to

numeracy as being at-home with mathematics

to

numeracy as critical mathematics

In the last year or so, the course has been run about ten times, and in most states, and it has generated much enthusiasm and many questions. It is acting as one way of widening awareness and debate about adult numeracy. And where will we as a field go next? What will be the new directions? Where will we be in five years time?

Bibliography

Freire, Paulo (1970). Pedagogy of the oppressed. New York: Seabury.

*Johnston, Betty and Dave Tout with Liz Agars, Beth Marr & Keiko Yasukawa (1995) Adult Numeracy Teaching: Making Meaning in Mathematics. National Staff Development Committee for Vocational Education and Training, Melbourne.

*Marr, Beth, Chris Anderson & Dave Tout (1994). Numeracy on the Line: Language-based Numeracy activities for Adults. National Automotive Industry Training Board, Vic.

*Marr, Beth and Sue Helme (1991) Breaking the Maths Barrier. Department of Employment, Education and Training, Canberra

*available from Adult Basic Education Resource and Information Service (ARIS), Language Australia, Level 9, 300 Flinders ST, Melbourne, Vic 3000, Australia

Adult Numeracy Tutor Training in England – past, present and future

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Abstract

This session involved participants in exploring the past, present and future of adult numeracy tutor training in England in the light of changes in the organization and accreditation of training of adult educators which are likely to have an impact on adult numeracy practitioners. We looked at examples of practice in England and Australia and FENTO (Further Education National Training Organisation) for teaching and learning standards. Adult numeracy tutor training has always been the ‘poor relation’ – is a brighter future about to dawn?

Introduction

Numeracy tutor training in England has always been the ‘poor relation’ – of tutor training for adult literacy and, in common with other areas of post-compulsory education, of teacher education for the school sector; internationally, the present system compares poorly with that in Australia (Coben & Chanda, 2000). But with the publication of the UK government’s recent major report on adult literacy and numeracy education, the Moser Report (DfEE, 1999a), the development of the FENTO(1999) standards for the training of teachers for the post-16 sector and the dawning of the UNESCO-sponsored Mathematics Year 2000, are the poor relation’s fortunes about to change for the better? In this paper we shall consider this question in the light of a review of the present situation, including an outline of the Moser recommendations on teacher training, followed by a look at the background to the current situation. We shall then look briefly at examples of numeracy tutor training in Australia and England.

The present situation

The present situation is one of transition. An accreditation framework for adult numeracy teachers exists, and is, described briefly below (for more detail, see Coben & Chanda, 2000). The framework was developed in isolation from developments in adult numeracy education and adult numeracy teacher education outside the UK and in isolation from parallel developments in Initial Teacher Education and in mathematics education in schools. It suffers from the incoherence that is a feature of teacher education in the post-compulsory further education (FE) sector, where teachers are not yet required to hold a teaching qualification and “there is no coherent pathway or framework for their ongoing professional development”

(Ecclestone, 1996:146). Against this background, basic skills teaching is described in the Moser Report (DfEE, 1999a), as “marginalized, remaining something of a Cinderella service” (DfEE, 1999a:Summary and Recommendations, 1.2).

But Cinderella may be about to go to the ball: the Moser Report proposes a National Strategy, comprising:

- National targets
 - An entitlement to learn
 - Guidance, assessment and publicity
 - Better opportunities for learning
 - Quality
 - A new curriculum
 - A new system of qualifications
 - Teacher training and improved inspection
 - The benefits of new technology
 - Planning of delivery
- (DfEE, 1999a, Summary and Recommendations, 1.12)

The recommendations on teacher training are summarized in the Report as follows:

1.38 Without enough teachers there is little hope of achieving the proposed targets. At present, too many teachers teach part-time, and some are inadequately prepared. To achieve our aims, many more teachers will need to be trained to teach for the new curriculum. We shall require over 15,000 full-time equivalent teachers in this area, compared with under 4,000 at present. Teacher training programmes will have to be commensurate. And a new qualification for teachers should be developed jointly by QCA [the Qualifications and Curriculum Authority¹], the Further Education National Training Organisation (FENTO) the BSA [The Basic Skills Agency²] and others.

(DfEE 1999a, Summary and Recommendations, 1.38)

These recommendations are set out in detail in the main Report:

- (i.) All new staff and volunteers should undertake recognized initial training in teaching literacy and numeracy to adults
 - (ii.) The BSA and the new Further Education National Training Organisation (FENTO) and others should work together to produce new qualifications for teaching basic skills to adults
 - (iii.) By 2005 all teachers of basic skills should have this qualification or an equivalent
 - (iv.) Diploma courses in teaching basic skills to adults should be established in University Education Departments.
 - (v.) Intensive courses for teachers to become familiar with the new curriculum should be mounted.
- (DfEE, 1999a, Recommendation 14)

These recommendations are timely, since a national framework for teacher training in the FE sector is currently emerging³. Changes in the organization and accreditation of training of adult educators will have an impact on adult numeracy practitioners, although the exact nature of the changes is not yet clear. The UK government's Department for Education and Employment recently announced that FENTO and the BSA have begun developing a new initial training framework and qualification for new entrants to basic skills teaching, which will be closely aligned to the new training framework being developed for FE teachers in general. In addition, a tool kit for basic skills teachers will be produced which will include details of the new standards and curriculum for literacy and numeracy as well as guides to effective pedagogy and approaches to basic skills education. Furthermore, an intensive training programme is to be introduced for all existing basic skills teachers. (DfEE, 1999b).

So the present sorry situation in numeracy teacher education is about to change. But will it change for the better? It remains to be seen what form the new framework, with its associated training and tool kit will take, but, like the outgoing accreditation framework for numeracy teachers, described below, it has developed largely without benefit of research and underpinning theory. There has been little involvement of universities, which are, after all, institutions where educational research is undertaken. More study and research – both theoretical and empirical – is required into all aspects of adult numeracy and adult numeracy teacher development. Universities also provide postgraduate training, but postgraduate courses specifically geared to helping adult numeracy teachers develop their ideas and their skills are rare and research students in the field of adult mathematics education still more rare. Links urgently need to be forged between the 'licensing' and professional development of practitioners in the new accreditations framework, encouraging both practical teaching skills and academic study and research leading to higher academic qualifications. It is to be hoped that the emerging framework will facilitate this. Moser Recommendation 14 (iv) states that "Diploma courses in teaching basic skills to adults should be established in University Education Departments" but it is not clear what sort of qualifications are envisaged.

Teacher education for adult numeracy: the background to the current situation

Before 1990, a limited amount of training for adult numeracy tutors in England and Wales was funded by Local Education Authorities (LEAs) and by ALBSU through its Regional Training Grants programme. Most was non-accredited but some practitioners completed the RSA⁴ Diploma in Teaching and Learning in Adults Basic Education and in some areas customized training was developed through the RSA's Customer-Specific Accreditation Scheme. Provision was patchy, with less training provided or requested for numeracy than for literacy teaching. The content of training courses reflected the practice-based, a theoretical focus of numeracy teaching, imbued with notions of good practice which aspired to: start from where the student is; enable students to work at an individual pace; base work in contexts relevant to the adults concerned; base content on what the students need or want to know in the short term; an absence of examinations.

ALBSU's national consultation on the form and funding of its Regional Training programme in 1983 found that there was consensus on the need to improve coherence in teacher development in this area (ALBSU, 1988). Accordingly, in 1985 ALBSU launched its Basic Skills Accreditation Initiative (BSAI), sponsoring three pilot projects exploring approaches to accreditation and describing occupational standards for work in adult basic education (ABE). The intention was: to confirm and develop existing good practice rather

than to prescribe ideal practice; to acknowledge the varied starting points of experienced ABE tutors; to develop an accreditation framework rather than an examination-based course; and to facilitate different routes to accreditation (for example, through the accreditation of prior experiential learning).

The emerging accreditation framework was intended to be less demanding than the RSA Diploma in Teaching and Learning in Adult Basic Education. It was intended initially for experience practitioners (many of whom had by this time attended a considerable number of non-accredited training events) and later for those who had successfully completed a period of initial training recognized by ALBSU. It was hoped that the accreditation framework would harmonise with the findings of the pilot projects and that adult basic education provision would become publicly accountable through the ability of tutors practitioners to design and deliver appropriate curricula and through the introduction of measurable outcomes for students, such as examination passes.

In 1990, ALBSU launched the first version of its accreditation scheme, validated by the City and Guilds of London Institute. Accreditation was available to practicing tutors through the in-service Certificate for Teaching Basic Communication Skills to Adults (CG9281), and the pre-service Initial Certificate in Teaching of Basic Skills to Adults (CG9282 Literacy and CG9283 Numeracy, hereinafter called the Initial Certificate) for potential tutors and newly-appointed volunteers.

The Certificate for Teaching Basic Communication Skills to Adults (hereinafter called the CG 9281 Certificate) was not directly linked to training. Instead, it offered the opportunity for the accreditation of candidates' prior experience, supported by mentors, who facilitated the process of evidence-gathering and assessment of competence. The new framework was widely seen by funders as cheaper to implement than the provision of training courses.

Mentors, whose training was initially funded by ALBSU, were not required to be experienced or qualified in their candidate's field. In practice many found it difficult to support candidates specializing in areas outside their area of expertise and since more literacy and numeracy tutors were available to become mentors, this meant that numeracy tutors were at a disadvantage. Many numeracy tutors had difficulty in identifying a suitable mentor in their institution, or even in their region. Instead of the effective and economical system that had been anticipated, organisations found it difficult to manage and operate. By the end of the first year, fewer than 20 numeracy teachers nationally had taken up the scheme. ALBSU's internal review found the system to be unwieldy, open to differences in interpretation and difficult to steer towards successful completion.

Accordingly, in tandem with the development of the national Training and Development Lead Body (TDLB⁵) Standards for Training and Development, the CG8281 Certificate was revisited in 1992 to match more closely the competence-based National Vocational Qualification (NVQ⁶) format. The result was the CG9285 Certificate in Teaching Basic Skills to Adults (CG9285-020 Literacy and CG9285-021 Numeracy, hereinafter called the CG9285 Certificate). The revised CG9285 Certificate was more detailed and prescriptive in terms of performance criteria and evidence requirements, but serious implementation problems remained. In November 1995 it was itself revised to correspond to the standards of the Further and Adult Education Teachers' Certificate (CG7306), for which it represents the element of contextualization of teaching in either numeracy, literacy or English to Speakers of Other Languages (ESOL).

Meanwhile, at the pre-service level, the Initial Certificate was intended to introduce national standardization of volunteer training programmes and subsequently to become the entry qualification for literacy and numeracy work in Post-Compulsory Education and Training (PCET). The aim was to provide new tutors with a tool kit for survival in adult education generally. Consideration of numeracy was secondary, conceived as the contextualization of generic competence.

The Initial Certificate lacks cohesion and rigor and, with a taught element of only 16 hours, fails to provide teachers with the detailed knowledge and skills base they need. The practical aspect of the course relates to needs analysis, programme design and delivery to one student, despite the fact that numeracy tutors are much more likely to find themselves working with groups in further education colleges on a variety of programmes which include numeracy. The course introduces the trainee to some kinds of adult numeracy provision, but does not prescribe the study of pedagogy or the analytical development of teaching strategies with reference to adults learning numeracy, as is clear from perusal of the support materials for intending numeracy tutors (Brittan, 1993).

However, assuming that a numeracy practitioner, armed with CG9283 Initial Certificate, were to gain employment as a teacher of numeracy to adults, there is, in the current climate of cutbacks and preoccupation with management and assessment qualifications, only a slim chance of being trained on the job as a teacher of numeracy to adults. There may not be a numeracy colleague or mentor available, there is even less likely to be a numeracy-specific skills development programme. Part-time tutors may not have access to any training that is available and will probably be expected to undertake professional development in their own time.

After six months on the job, the CG9285 Certificate in Teaching Basic Skills is suggested as next-step professional development for the individual member of staff, a step which also helps the organisation to satisfy the staff training profile of The Basic Skills Agency's Quality Kitemark. The problem is that this is a competence-based accreditation scheme which assumes the underpinning knowledge and understanding have already been acquired and can be demonstrated in practice. As such it is vulnerable to the criticisms made of other competence-based assessment and accreditation schemes (principally NVQs/SVQs) that they are: based on a behaviourist model, ruthlessly applied; 'jargon-heavy' and 'content-light'; bureaucratic and time-consuming in operation, and hence expensive; variable in assessment standards; inadequate as a tool for the assessment of knowledge and theory.

There is no requirement for training to be available or undertaken to ensure that candidates have access to underpinning knowledge and understanding. In the real world of implementation of such schemes the requirement to demonstrate knowledge is all too easily glossed over. Through the achievement of CG9285 Certificate, practitioners gain experience of portfolio preparation and (*de facto*, but not *de jure*) a licence to continue to practice. But they have not been exposed to the richness of relevant knowledge, neither do they have the opportunity to reflect on their own practice with a view to growing and developing as professionals.

Although ALBSU's financial support for those endorsing the accreditation scheme provided a kick-start, by the mid-1990s the low take-up at both pre-service (Initial Certificate) and in-service (Certificate) level pointed to a lack of confidence in the potential of the scheme to meet either the training needs of newly-appointed staff or to act as a route to employment. Certification in numeracy teaching was revealed as unpopular by comparison with literacy. Given that the majority of attendances on Initial Certificate courses and Certificate accreditations are initiated by funders, rather than on the basis of demand from

individual practitioners, this would suggest that even those responsible for planning and implementing staff development have a lesser interest in certification for numeracy than for literacy.

Research by Joy Joseph (1997) amongst UK members of Adults Learning Mathematics – A Research Forum (ALM) confirms this picture of the relative unpopularity of the qualifications in numeracy teaching by comparison with literacy teaching amongst paid staff, and of the relative unpopularity of the CG9285 Certificate, by comparison with the Initial Certificate. Joseph found more volunteers achieving the Initial Certificate in numeracy than in literacy, in contrast to the position amongst paid staff. She found a fairly evenly mixed reaction to the Initial Certificate. However, the majority of respondents condemned the CG9285 Certificate, on candidate describing it as a nightmare (Joseph, 1997:181).

Joseph concluded that accreditation is now driving rather than following the curriculum for both tutors and students that the pressures of the current climate militate against good practice in adult numeracy work. She suggested that all trainee adult basic education tutors should be encouraged to follow training courses in both literacy and numeracy, since so many students have difficulty in both areas. Most importantly, the decline in numeracy teacher development amongst paid staff, particularly at the higher levels, must be reversed. As she says “Low numbers in training lead to poorer quality training, and a downward spiral could easily develop” (Joseph, 1997:182).

Examples of numeracy tutor training in England

In the discussion at ALM-6, Joy Joseph pointed out that the evolution of training for numeracy tutors seems to have been dependent on a number of factors, such as local demand, the presence in any area of a ‘pro-numeracy’ budget holder, and a trainer with the necessary expertise to develop it. Examples were shown from the area around Bristol in the west of England, where a local certificate was developed in the early 1990s. It was subsequently abandoned following the introduction of CG9285 Certificate (although much of the content was retained to support the underpinning knowledge for this) and is now being redeveloped for accreditation through the Open College Network (OCN⁷). Given that a new national scheme of certification for Basic Skills tutors is under review following the publication of the Moser Report, it could be questioned whether a development of this kind is appropriate at this stage. The rationale for proceeding has three bases:

- (i.) A replacement for the CG9285 may not be ready as soon as projected, and there are tutors who need training now;
- (ii.) As the scheme is being written with FENTO Standards and the Moser Report in mind, it is likely that the actual content will be of use as and when the new scheme becomes available;
- (iii.) By opting for OCN accreditation rather than a content-only course, there should be clear ways of mapping competences on to any new scheme, thus making the Accreditation of Prior Learning (APEL) easier.

The new City of Bristol Certificate is being designed to offer alternative levels of credit, to accommodate tutors with differing levels of experience, needs and motivation. General descriptors for these levels are (following from a foundation of competencies, knowledge and understanding in predictable and structured contexts):

The acquisition of a broader range of competences; knowledge and understanding which demonstrates the extension of previous abilities in less predictable and structured contexts and prepare the learner to progress to further achievement;

The acquisition of a more complex range of competences, knowledge and understanding in contexts which develop autonomous, critical and analytical abilities that prepare the learner to progress to further independent achievement.

An example of a specified 'Learning Outcome' for the Numeracy module might be 'recognise the importance of attitudes and emotions in the development of number skills'. Assessment Criteria for Level 2 lay down that the learner must be able to 'state the barriers to learning mathematics experienced by some adults'. At Level 3 this becomes 'state the barriers to learning mathematics experienced by some adults, including the cultural expectations of particular groups in society'. Similarly, 'devise strategies for developing positive attitudes to learning mathematics' (Level 2), becomes 'devise strategies and resources with particular reference to a group identified above'. In other cases, references to theories and/or research findings are asked for at the higher level, or for evidence of practice within a group setting rather than with individual students.

The main concern for the success of this module is numbers. First soundings suggest that literacy and IT will be popular, with very low numbers taking up numeracy, at least internally. This is a perennial problem with a minority subject. It is hoped that the impact of the Moser recommendations and Mathematics Year 2000 might begin to put numeracy more firmly on the map.

Colleagues attending the workshop from other areas of England also had experience of locally designed and accredited courses. For example, in Dorset, Isobel Peters reported that Audit Basic Education is still largely delivered outside the Further Education College system, although local government re-organisation has complicated countywide delivery to some extent. However numeracy tutor training has been successfully delivered across the county. Wendy Gornall reported on the work of the Birmingham Maths/Numeracy Research Group (1999, 1997), which consists of mathematics/numeracy tutors and managers from across the city. On tutor-training, the Group's 1997 Report recommended the encouragement of multi-method approaches to learning, providing hands-on practical activities to address the curriculum needs identified by the Research Group. Since then, various developments had taken place, including a 10-hour module promoting innovative teaching and introducing participants to fun and interesting ways of acquiring mathematics skills. Resource bases of teaching materials had been established and duplicates of these were used for tutor training. Termly numeracy support meetings offer tutors the chance to reflect on their own practice, to meet and discuss issues with colleagues, to address particular teaching needs and to hear outside speakers. A second working group has now met to examine accreditation needs across the curriculum (Maths/Numeracy Research Group, 1999:5). All in all, Birmingham appeared to offer exciting opportunities for teacher development which many of those in rural areas or cities with a smaller number of numeracy tutors would envy.

Adult numeracy teacher development in Australia

By contrast to their colleagues in England, Australian adult numeracy educators already have available to them a comprehensive set of staff development tools. 1995 saw the publication of *Adult Numeracy Teaching – Making Meaning in Mathematics* (National Staff Development Committee for Vocational Education and Training and the Commonwealth of Australia, 1995), also known as (and cited here as) ANT. ANT is described by three members of the team which produced it, Betty Johnston, Beth Marr and Dave Tout, as

an 84-hour professional development course designed as a continuation and further development of existing professional development packages, such as *Breaking the Maths Barrier* (Marr & Helme, 1991) leading in turn to postgraduate study (Johnston, Marr & Tour, 1997:166).

The primary purpose of ANT, as described in the ‘Information for Presenters’ (ANT, 1995:viii), is

to blend theory and practice about teaching and learning adult numeracy within a context of doing and investigating some mathematics, whilst developing a critical appreciation of mathematics in society.

The project team began with the question: “What should numeracy teachers be able to do after this course?”. Their answer was:

After this course a teacher

1) should have a critical appreciation of mathematics in society, and should be able to initiate appropriate learning activities by identifying the numeracy needs of students and responding from a variety of approaches to teaching and a range of appropriate mathematical resources and knowledge. (ANT, 1995:x)

The structure of ANT accordingly tries to weave together three strands: “knowing about maths; learning (and teaching) maths; and doing maths. This last strand is central to the course: clearly to teach numeracy you must know how to do mathematics” (ANT, 1995:x). The ANT team thus tackle head on the question of the teacher’s own grasp of mathematics, a question which parallel British initiatives have signally failed to address. However, we await implementation of the Moser Recommendations with interest.

In their conference paper for the third ALM conference, Johnston, Marr and Tout (1997) point out that ANT was an initiative of the National Staff Development Committee for Vocational Education and Training, which works under the Australian National Training Authority. The Committee awarded the project jointly to the Centre for Language and Literacy at the University of Technology, Sydney (UTS) and the Adult Basic Education Resource and Information Services (ARIS) at the National Languages and Literacy Institute of Australia (NLLIA) (Johnston, Marr & Tout, 1997:166). The ANT project thus embodied the link between research and practice, between a university and a training and staff development organization that is greatly needed in the UK. It did so on the basis of interstate consultation with educators and a search of the relevant literature. The project sought the advice of individual consultants, a National Reference Panel and an international Academic

Reference Panel (one of the present writers served on the latter Panel). ANT was piloted in two states and on completion, its publication by a national organization ensured that it was widely disseminated throughout Australia.

Nor did the project team shy away from theory, describing their ‘critical constructivist’ approach as follows:

A critical constructivist approach starts from the experiences and perspectives of the learners and the local community, learning and evolving mathematics that is relevant to their needs – helping students to become strong within their own culture and to learn at the same time how to critically appropriate knowledge from a wider range of experiences. (Johnston, Marr & Tout, 1997:168).

Whether one agrees or disagrees with the ANT approach, it has undoubtedly opened up areas for discussion on which the present English adult numeracy teaching accreditation system is silent. On the vexed question of ‘what is numeracy’, they conclude that “numeracy is not less than maths, but more” (Johnston, Marr & Tout 1997:167) and ANT reflects this positive view.

By 1997, the ANT course had been run about ten times and in most Australian states. Johnston, Marr & Tout (1997:170) report that it has generated much enthusiasm and many questions. Independent evaluation would, of course, be necessary to endorse that view, but it appears from this distance as if, on every count, the Australian project puts the English numeracy teacher development system comprehensively in the shade.

One member of the ANT team, Dave Tout, made a telling comment on the adult numeracy teacher development he encountered (or failed to encounter) on his trip to Europe in 1996 to attend the ALM-3 and ICME-8 conferences⁸. He stated that

I was looking forward to hearing about training or professional development courses and resources, as again this has been a priority in Australia. However, I was surprised that the development of training and professional development opportunities in adult numeracy seemed to have had little prominence and I came away with very little knowledge in this area. (Tout, 1997:14)

This is a sad reflection on the lack of priority that has been accorded to adult numeracy teacher development in England over decades.

Conclusion

Professional development in adult numeracy teaching must have greater prominence if practice in this area is to continue to develop. Teaching numeracy as ‘not less than maths, but more’ requires teacher development worthy of the name. The signs are that this issue is at last beginning to be taken seriously in England at the highest level and this is very welcome. As the 1998/99 report on mathematics in Further Education by the Further Education Funding Council (FEFC) stated:

In mathematics, the need for imaginative, effective teaching is particularly pressing. For a substantial number of further education students, their previous exposure to mathematics has not been a happy experience. Many students enter further education lacking in confidence after years of low achievement in the subject. More perhaps

than other subjects, mathematics needs skillful, inspirational teachers who can rebuild wounded confidence, instill in students the necessary knowledge and skills, and enable them to reach their full potential. (FEFC, 1999: Summary)

The Birmingham and Australian initiatives show that much can be done when committed adult educators are able to work together in a supportive environment. The Australian example shows how much more they can do with a strong theoretical and research perspective and a commitment to developing the mathematical skills and understandings of adult numeracy tutors, where appropriate, as well as students. The gulf, in England, between the ‘licensing’ and professional development of adult numeracy practitioners must be bridged in the new accreditation framework, to encourage the development of practical teaching skills and academic study and research in the field of adults mathematics education. It remains to be seen whether the form of adult numeracy teacher education that emerges in England in the wake of the Moser Report will transform the ‘Cinderella service’ so that it rivals that in Australia. Is a bright future about to dawn for the ‘poor relation’?

Acknowledgements

This paper draws on Coben & Chanda (2000) and builds on Joy Joseph’s paper presented at ALM-3 (Joseph, 1997).

Notes

1. The Qualifications and Curriculum Authority’s (QCA) principal function is to promote quality and coherence in education and training in England. QCA has responsibility for all external vocational and academic qualifications, i.e., academic or vocational qualifications authenticated or awarded by an outside body, other than those at first degree level or higher, including all National Vocational Qualifications (NVQs).
2. On 5 April 1995 The Basic Skills Agency (BSA) replaced the Adult Literacy and Basic Skills Unit (ALBSU) as the national development agency for literacy, numeracy and related basic skills in England and Wales. By ‘basic skills’ the Agency means:

the ability to read, write and speak in English and use mathematics at a level necessary to function and progress at work and in society in general. In Wales basic skills includes the ability to read, write and speak in Welsh where Welsh is the first language or mother tongue. (The Basic Skills Agency)
3. For an interesting discussion of the changing situation in teacher education for the post-16 sector, see Atkin, 2000.
4. RSA is the acronym of the Royal Society for the encouragement of Arts, Manufactures and Commerce.
5. The TDLB has now become part of a larger Employment National Training Organisation.

6. The British system of vocational qualifications based on the assessment of competence comprises National Vocational Qualifications (NVQs) in England and Wales and Scottish Vocational Qualifications (SVQs) in Scotland. Competence-based assessment has generated a considerable literature, both 'for' and 'against' (see, for example: Wolf 1995; Smithers 1993; Jessup 1991).
7. OCN is a regional partnership, affiliated to a national organization (NOCN) of educational organisations in the post-school sector. It acts through a system of peer scrutiny to improve access and quality, and enable the accumulation of credit for learning.
8. The Third International Conference of Adults Learning Maths – A Research Forum (ALM-3) took place at the University of Brighton, England, 5-7 July 1996 (the Proceedings are published as Coben, comp. 1997). The International Congress on Mathematics Education (ICME-8) took place in Seville, Spain, July 14-21 1996. ICME-8, for the first time included a Working Group (WG-18) on 'Adults returning to study mathematics'. Papers from WG-18 are in G.E. FitzSimons (ed.) (1997). ICME-9 will take place in July 2000 in Tokyo, Japan; it also includes a working group in this field (Working Group for Action 6: Adult and Life-long Education in Mathematics).

Bibliography

- ALBSU (1988) 'Editorial' *ALBSU Newsletter*, Vol. 28, Winter 1988. London: Adult Literacy and Basic Skills Unit.
- ANT (see: National Staff Development Committee for Vocational Education and Training and the Commonwealth of Australia, 1995).
- Atkin, C. (2000) 'Training the Trainers: a review of teacher training in England for the further education sector'. In: *Europa Handbuch Weiterbildung*. Newuwied: Luchter Hand Verlag.
- Brittan, J. (1993) *An Introduction to Numeracy Teaching*. London: Adult Literacy and Basic Skills Unit.
- Coben, D. & Chanda, N. (2000) 'Teaching "not less than maths, but more": an overview of recent developments in adult numeracy teacher development in England – with a sidelong glance at Australia'. In: D. Coben, J. O'Donoghue and G. FitzSimons, (eds) *Perspectives on Adults Learning Mathematics: Research and Practice*. Dordrecht, The Netherlands: Kluwer Academic Publishers, Chapter 17. This chapter is an updated version of an article of the same name published in *Teacher Development: an international journal of teachers' professional development (Special Issue on Literacy and Numeracy)*. Triangle Vol.1, No.3. ISSN 1366-4530, pp375-92.
- Coben, D. (Comp.) (1997) *Adults Learning Maths – A Research Forum – 3, ALM-3 Proceedings of the Third International Conference of Adults Learning Maths – A Research Forum (ALM), 5-7 July 1996 at University of Brighton*. London: Goldsmiths College, University of London in association with ALM.
- DfEE (1999a) *Better Basic Skills: Improving Adult Literacy and Numeracy*. Leaflet. London: Department of Education and Employment.
- Eccleston, K. (1996) 'The Reflective Practitioner: Mantra or Model for Emancipation?' *Studies in the Education of Adults* 28 (2), 146-161.
- Evans, J. (1989) 'The Politics of Numeracy'. In: P. Ernest (ed.) *Mathematics Teaching, the State of the Art*. Lewes: Falmer Press, 203-19.
- FEFC (1999) *Mathematics in Further Education: National Report from the Inspectorate*. Coventry: Further Education Funding Council (FEFC).
- FENTO (1999) *Standards for Teaching and Supporting Learning in the Further Education in England and Wales*. London: Further Education National Training Organisation (FENTO).
- FitzSimons, G.E. (Ed.) (1997) *Adults (returning to) Study Mathematics: Papers from the 8th International Congress on Mathematics Education (ICME 8) Working Group 18*. Adelaide, SA: Australian Association of Mathematics Teachers.
- Jessup, G. (1991) *Outcomes: NVQs and the Emerging Model of Education and Training*. London: Falmer Press.
- Johnston, B., Marr B. & Tout, D. (1997) 'Making Meaning in Maths. Adult Numeracy Teaching: a course for teachers'. In D. Coben (Comp.) *Adults Learning Maths – A Research Forum – 3, ALM-3*

- Proceedings of the Third International Conference of Adults Learning Maths – A Research Forum (ALM), 5-7 July 1996 at University of Brighton.* London Goldsmiths College, University of London in association with ALM, 166-171.
- Joseph, J. (1997) 'Numeracy Staff Development for Basic Skills Tutors' in D. Coben (comp.) *Adults Learning Maths – A Research Forum – 3, ALM-3 Proceedings of the Third International Conference of Adults Learning Maths – A Research Forum (ALM), 5-7 July 1996 at University of Brighton.* London: Goldsmith College, University of London in association with ALM, 172-82.
- Maths/Numeracy Research Group (1999) *Report from the Maths/Numeracy Research Group: Increased Participation and Retention.* Birmingham: Birmingham City Council Adult Education Service.
- Maths/Numeracy Research Group (1999) *Report from the Maths/Numeracy Research Group: Increased Participation and Retention.* Birmingham: Birmingham City Council Adult Education Service.
- National Staff Development Committee for Vocational Education and Training and the Commonwealth of Australia (1995) *Adult Numeracy Teaching – Making Meaning in Mathematics ('ANT').* Melbourne, Australia: National Staff Development Committee for Vocational Education and Training.
- Smithers, A. (1993) *All our Futures: Britain's Education Revolution.* A Dispatches Report on education. London: Channel 4 Television.
- Tout, D. (1997) 'Some Reflections on Adult Numeracy'. In D. Coben (Comp.) *Adults Learning Maths – A Research Forum – 3, ALM-3 Proceedings of the Third International Conference of Adults Learning Maths – A Research Forum (ALM), 5-7 July 1996 at University of Brighton.* London: Goldsmiths College, University of London in association with ALM, 13-15.
- Wolf, A. (1995) *Competence-based Assessment.* Buckingham: Open University Press.

A Grounded Approach to Practitioner Training in Ireland: Some Findings from a National Survey of Practitioners in Adult Basic Education

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Introduction

The potential for empowering individuals through adult mathematics education is currently a topic of international significance. Research is currently centred on:

- training practitioners and identifying the skills, knowledge and competences that they need to enable them to create an environment that engenders empowerment on the ground
- policy that engenders empowerment in the individual learner and how policy percolates through the system
- motivating adult learners to access and participate in Adult Basic Education (ABE).

This paper reports on the preliminary findings of a national survey of practitioners in the field of adult mathematics education in Ireland. This survey provides a profile of the practitioner in Ireland, identifying their training needs and providing a contribution to the international debate on practitioner training. This paper also addresses issues of policy in the Irish context and provides a framework for examining empowerment through adult mathematics education at three levels: practitioner level, policy level, and the level of the learner.

Background

The International Adult Literacy Survey (OECD, 1997) highlighted the poor level of adult literacy in Ireland and was pivotal in initiating policy development in this area. There is no national data of the numeracy levels of adults in Ireland outside the results of this survey.

Over the three domains of literacy, 25% of the Irish population were found to score at the lowest level, indicating that a very significant percentage have problems with all but the very simplest literacy tasks. The Irish performance in a comparative context is poor; the percentage of participants who are at the lowest level literacy is higher in Ireland than anywhere else except Poland (Morgan et al, 1997).

Since the publication of these results the adult literacy problem has been elevated to centre stage in educational policy. Provision for adult literacy in the education sector increased from

£0.86m in 1997 to £8.8m in 2000. An investment of £73.5m has been committed to this area in the National Development Plan (NDP), 2000-2006 (DF, 2000).

The plethora of national policy initiatives now emerging in Ireland, for the first time clearly demonstrate a growing conviction at all levels of society, including Government that adult education is a vital component in a continuum of lifelong learning (Department of Finance (DF), 2000; Department of Education and Science (DES), 2000; Department of the Taoiseach (DT), 2000; National Adult Literacy Agency (NALA), 1999; Qualifications (Education and Training) Act, 199. Despite these initiatives a number of lacunae are emerging; Ireland has no clear policy on adult mathematics/numeracy education. The Department of Education and Science has adopted a narrow definition of numeracy that views numeracy as being restricted to number (Farrell, 2001, pers com.). The National Adult Literacy Agency, which has responsibility for the implementation of policy in adult basic education, defines literacy as encompassing numeracy. There is similarity between the situation in Ireland and that outlined for Australia by Cummings (1995). 'That the inclusion of numeracy as a component of literacy: sometimes explicitly included in literacy agendas, sometimes implicitly, sometimes omitted: is not sufficient'

To date there has been no systematic research into the numeracy needs of various stakeholders including employers, practitioners and the learners themselves.

Building learner demand is one of the most pressing challenges in the broad field of adult education today (Wagner, 2000). Government policy alone will not motivate ABE learners nor initiate the process of empowerment; an effective service provision is required.

Bailey and Coleman (1997) highlighted the existence four types of barriers to participation in ABE: situational barriers; informational barriers; institutional barriers and dispositional barriers. The removal of these barriers will require an effective, coordinated policy development across a number of government departments and agencies charged with delivery. Learners should be able to direct their own learning and have an identified structure to enable them to influence programme content, pedagogy and policy.

In an effective ABE system, practitioners play a central role in the process of empowerment. If the ABE sector is to make the quantum leap envisaged for it in the White Paper on Adult Education (DES, 2000), it must have a highly trained corps of practitioners who are dynamic and equipped to lead change. They must play a key role in policy debate and must reflect the distinctive identity of the sector in the field of professional practice and research. This research provides baseline data on the practitioners currently involved in delivering mathematics in this ABE.

National Survey of Practitioners of Mathematics to Adult Learners in Ireland

In the period February to May 2001, a national survey of practitioners involved in teaching mathematics to adult learners was implemented. The survey gathered information on various aspects of practitioner experiences, perceptions and needs, both in their classroom practice and professional development.

Five hundred questionnaires were distributed to practitioners of mathematics/numeracy nationally. The sample included practitioners that represent the spectrum of adult mathematics education delivered in the further education sector in Ireland. 312 (62%) valid questionnaires were returned. For the purposes of this paper the discussion of the results of the research work in progress will be limited to those that pertain to ABE (n=175).

Methodology

The research incorporated the use of qualitative and quantitative methodologies (in- depth interviews, literature review, questionnaire survey). The research instrument was constructed using the following process;

- Draft questionnaire was drawn up based on the literature
- Validation and revision of draft using qualitative methodology
- Revised draft drawn up and piloted
- Final research instrument

Recorded in-depth interviews were carried out and recorded with a number of practitioners, working with different agencies involved in ABE. Each interview lasted about one and a half hours and focused on; previous school experience; career pathways taken; the importance of teaching qualifications; teaching practices; numeracy as a concept and support systems.

The research instrument was revised and piloted (six times), to ensure clarity, and coherence of question flow. For the purposes of the survey the term ‘mathematics’ was defined to incorporate all levels of mathematics from the most basic level including what might be called numeracy; an ‘adult’ was defined as anyone who has left fulltime mainstream education.

Numeracy practitioners were targeted through area adult literacy coordinators and through the National Council for Vocational Awards (NCVA) who provide accreditation for ABE.

Adult Literacy Coordinators were contacted by telephone to secure the participation of their practitioners in the survey. Although 3656 practitioners are involved in adult literacy centres nationally, coordinators estimated that only 5%(approx) actually deliver numeracy. A total of 225 questionnaires were sent out to coordinators for distribution to the numeracy practitioners associated with their centre.

The NCVA as part of its in-service provision invited all practitioners of NCVA Foundation and Level 1 mathematics to attend an in-service seminar on mathematics. An additional 82 questionnaires were issued to the practitioners who attended this programme.

Details of the practitioners represented by the research sample are outlined in Table 1. Data was analysed using SPSS for Windows (version 10.0.5.).

Table 1. A breakdown of the proportion of practitioners represented by the sample.

	Accredited ABE (NCVA*)	Nonaccredited ABE (Adult Literacy Centre)
No. of practitioners delivering adult mathematics/numeracy	156 ⁺	183 [■]
No. questionnaires circulated	82	225
No. questionnaires returned	80	146
% valid returns	96%	76% (114) [°]
% of total sample represented	51%	62%

Key:

* National Council for Vocational Awards

⁺ Figure based on 1999/00 figures from NCVA

[■] Approx. no. based on information provided by Adult Education Co-ordinators

[°] Once Area Coordinators had circulated questionnaires to each numeracy practitioner, those not required were returned.

Results

The results are discussed under the following headings:

- General teaching experience
- Specific experience in teaching adult mathematics
- Teaching practices
- Training
- Attitudes towards mathematics and teaching mathematics
- General classification details

General Teaching Experience

The majority of practitioners work part-time (62%) or as a volunteer (35%). Only 18% were involved on a fulltime basis (Figure 1). 6% of those working fulltime (n=2) and 38% (n=30) of those working part-time also worked as a volunteer.

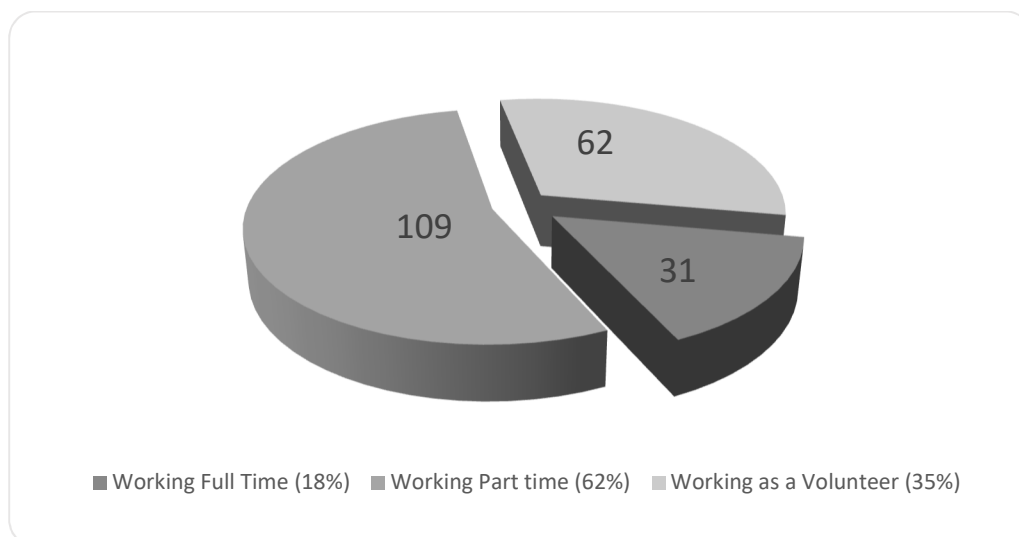


Figure 1: Type of employment of practitioners

More than half the practitioners had no experience of teaching students in mainstream education, however there was evidence of experience of teaching adults. 37% of those sampled had more than 10 years experience, 21% with 6-10 years experience; 41% been involved in teaching adults for 5 or less years (Figure 2).

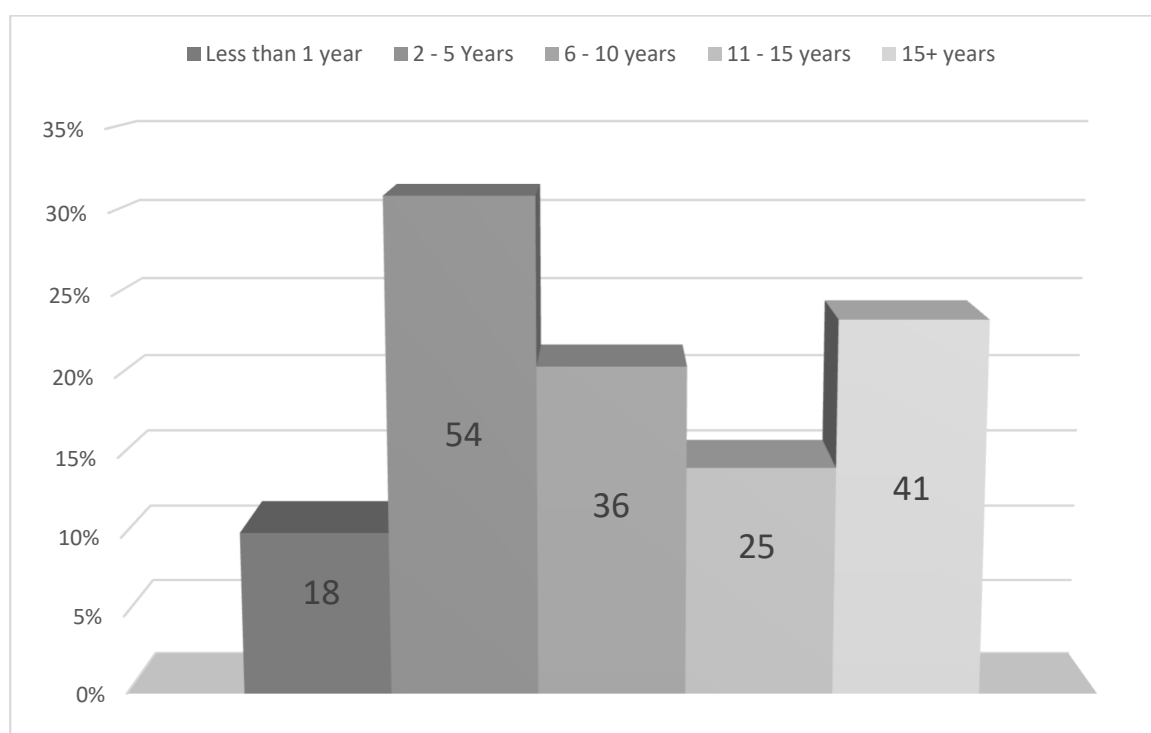


Figure 2: Number of years teaching experience

Specific Experience Teaching Adult Mathematics

Approximately 60% of practitioners have been involved in adult education for more than 5 years; however, practitioners had less experience in delivering mathematics (Figure 3 & 4). 33% of practitioners were delivering mathematics/numeracy for the first time. 46% had less than 5 years experience, only 12% had greater than 10 years experience.

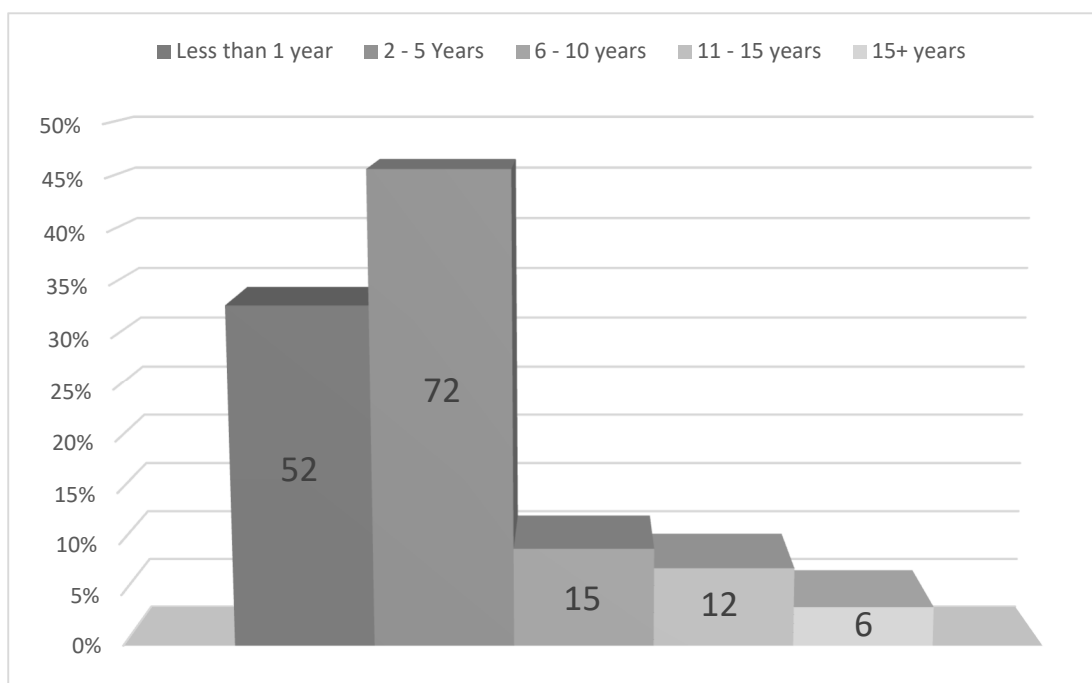


Figure 3: Number of years teaching adults mathematics/numeracy

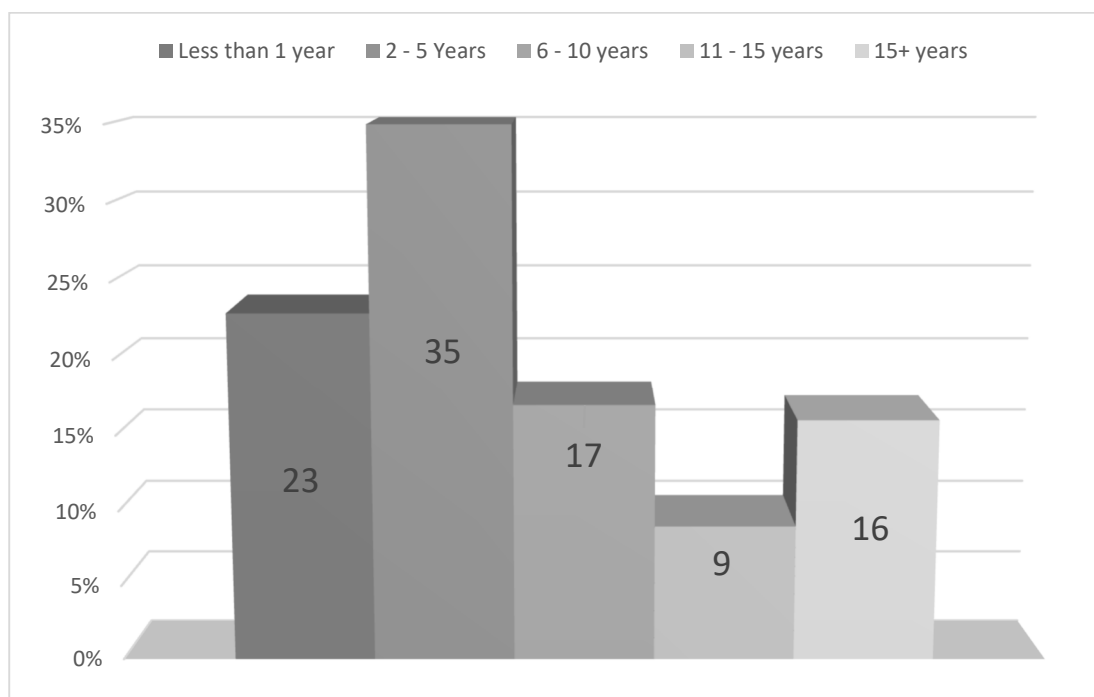


Figure 4: Number of years teaching mathematics/numeracy

Teaching Practices

The views practitioners have on teaching adult learners are outlined in Table 2. Results indicate some differences of opinion on whether adults are more demanding (39% agree, 34% disagree) than mainstream second level students and on whether a chalk, talk and practice approach is effective in the adult classroom (45% agree, 32% disagree).

Table 2: Practitioner views on the adult learner

Practitioner Views on the Adult Learner Agree	Agree with statement (%)
Adults learn best when they are treated as equals	97%
A different approach is required in the classroom when teaching adults	96%
Adults are more demanding than other students	39%
Adults have a complex about mathematics	72%
A chalk, talk and practice approach in the classroom works well with adults	45%

The teaching practices used by practitioners in the classroom are outlined in Table3. Over 90% of practitioners indicated that they always/usually use problem solving (94%) and practical work (90%) as a teaching method. 68% of practitioners always/usually use group work, 59% used blackboard, chalk and talk, 36% use project work. Only about a quarter of those sampled use technology and investigational work.

Table 3: Teaching practices used by practitioners in their classroom.

Teaching Practice	Usually/Always	Never/Rarely
Problem solving*	94% (150)	6% (10)
Practical work	90% (140)	10% (16)
Consolidation and practice	87% (130)	13% (20)
Group work	68% (95)	33% (46)
Blackboard Chalk and Talk	59% (91)	41% (64)
Project work	36% (36)	64% (81)
Technology	28% (34)	73% (91)
Investigational work	24% (28)	76% (87)
Rote learning	13% (16)	87% (107)
Invited guest speaker/role play	<3%	

* Interpreted as exercises of mathematics problems.

Training

68% of practitioners considered that they have average /sufficient training in teaching adults in general. This contrasts with 82% of practitioners indicating that they have had none or insufficient training in teaching adults mathematics specifically

93% of practitioners felt there was a need to develop a training programme for those involved in adult mathematics/numeracy. The vast majority (93%) would attend such a course if it were available.

Practitioners were also asked to indicate the specific training they felt would help them to be more effective in teaching mathematics/numeracy to adults; their priorities are outlined in Table 4.

Table 4: Specific training required by practitioners (ranked)

Specific training practitioners feel would help them to be more effective in teaching mathematics to adults	Valid %	<i>n</i>
Designing resources	67	(117)
Ways of applying mathematics	64	(112)
How adults learn mathematics in particular	63	(111)
Coping with math's anxiety	60	(105)
Using technology	43	(75)
How adults learn in general	40	(71)
Profound understanding of elementary mathematics	36	(63)
Problem solving	34	(60)

Practitioners gave the highest priority (range 60% - 67%) to: designing resources and activities to suit all levels of learners; ways of applying mathematics in different contexts; the way adults learn mathematics in particular; coping with mathematics anxiety. Lower priorities included (range 43% - 34%); using technology; how adults learn in general; more profound understanding of elementary mathematics; problem solving.

Over 90% of practitioners considered that all the elements listed above, except using technology, should be included in a training programme for mathematics/numeracy practitioners (range 98% - 90%). Only 69% thought it important to include the use of technology in delivering mathematics.

Attitudes Towards Mathematics and Teaching Mathematics

60% of practitioners acknowledged that mathematics was their favourite subject in school and feel they have a natural ability with numbers. 44% felt that they never understood all the mathematics they were taught in school. 24% indicated that mathematics made them anxious at school. 19% really struggled with mathematics in school (Table 5).

Table 5: Practitioners feelings about mathematics

Statement about mathematics	Disagree (%)	Neither agree nor disagree (%)
Mathematics was my favourite subject when I went to school	28 (47)	12 (21)
When it comes to numbers I seem to have natural ability	25 (43)	18 (30)
Mathematics made me anxious when I was at school	59 (97)	18 (30)
I never felt I understood all the mathematics I was taught at school	57 (79)	9 (15)
I have middle of the road mathematics ability	31 (53)	10 (17)
I have really struggled with mathematics when I was at school	68 (112)	13 (21)

General Details

ABE practitioners are predominately female (80%). Approximately 60% of the practitioners are aged between 35 and 54 years. Approximately one fifth are over 55. 17% are between 25 – 34 years. A tiny minority (1%) are less than 25 years of age.

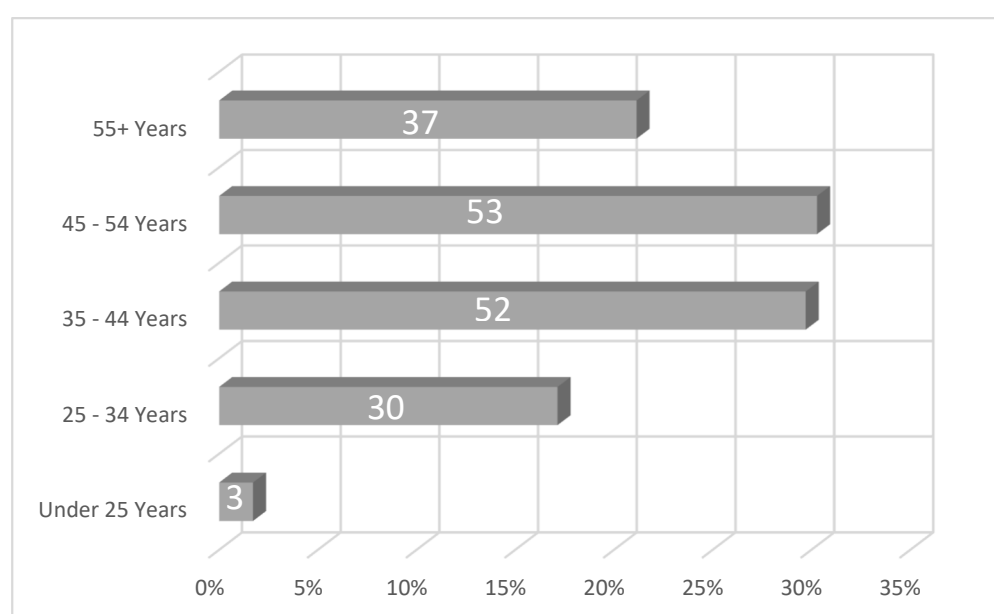


Figure 5: Age profile of practitioners sampled

Qualifications Held by Practitioners

In terms of qualifications, only 7% of practitioners have a degree in mathematics. 46% of practitioners have a degree or post graduate qualification that is not in mathematics. A further 47% do not have a degree. The highest mathematics qualification of over half the respondents (55%) is Leaving Certificate ordinary level.

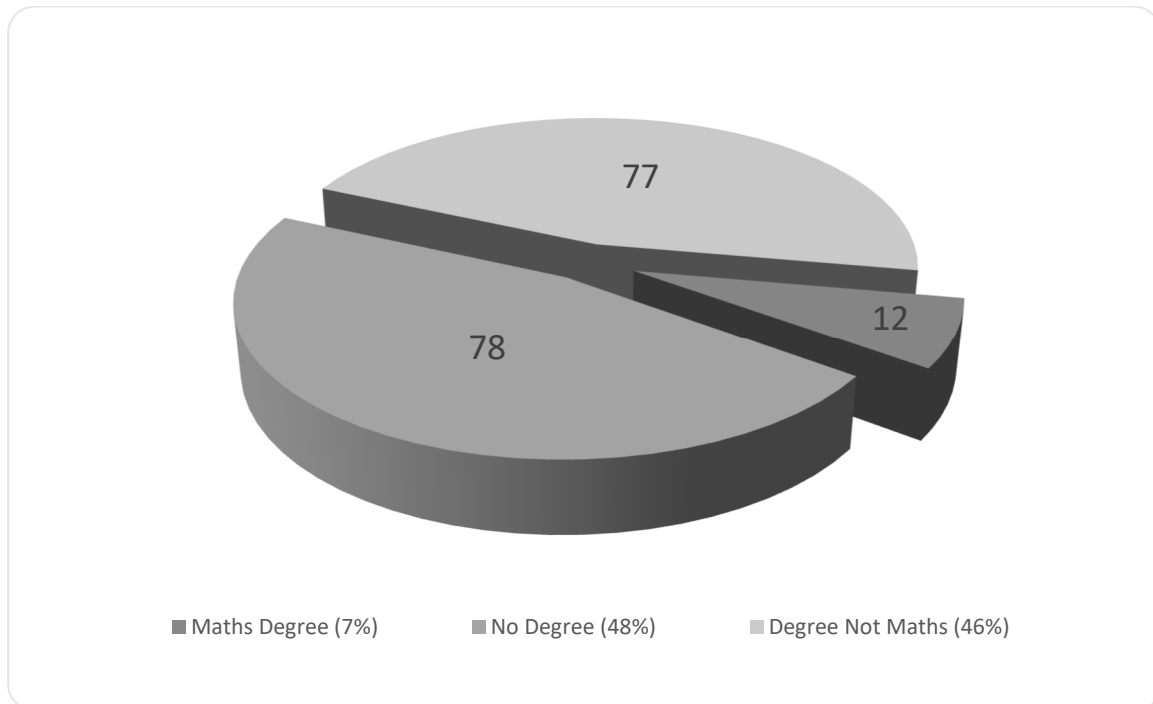


Figure 6: Highest Overall Qualification

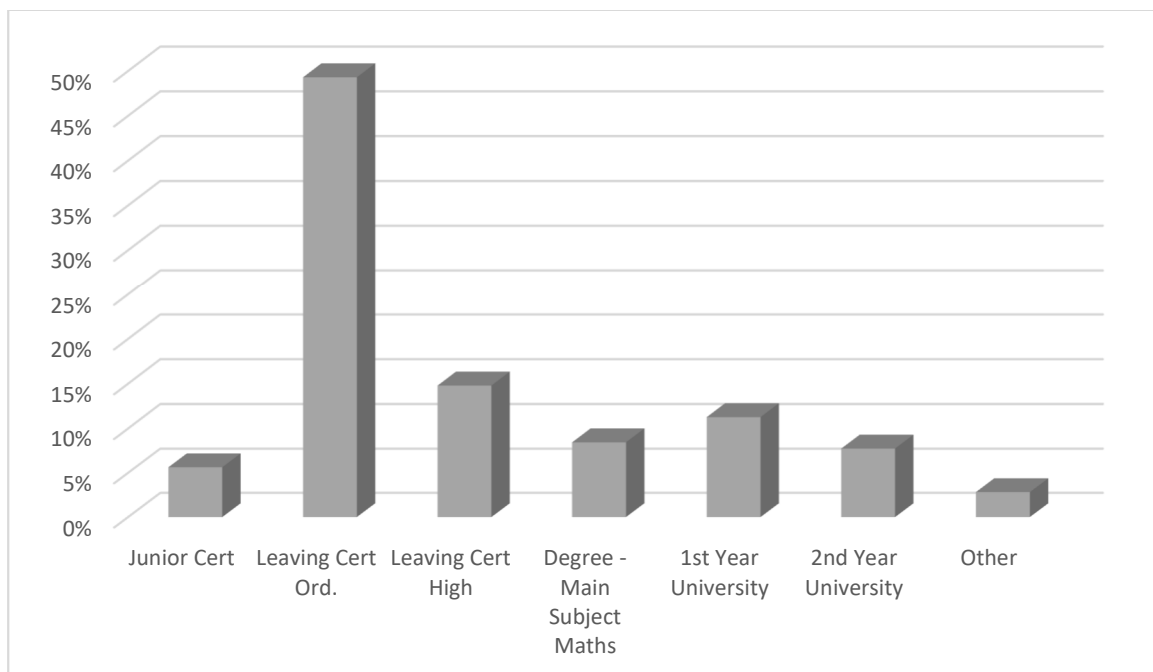


Figure 7: Highest mathematics qualifications held by practitioners

Discussion

The results of this research point to a part-time, volunteer work force in ABE, who have had some training in teaching adults but little or no training in teaching mathematics to adults.

To paraphrase Cooney (1999) who speaks about mathematics teaching in general; effective practitioners of adult numeracy need to know mathematics, know their students, have knowledge of the pedagogy of mathematics and a commitment to their own lifelong learning.

Practitioners in ABE in Ireland know some mathematics. More than half have studied mathematics to Leaving Certificate Ordinary Level, (State Terminal Examination, taken at around 18years). Of these practitioners 46% indicate that they did not understand all the mathematics that they were taught and 19% of the practitioners sampled involved in delivering adults mathematics education struggled with mathematics themselves. Anecdotal evidence to date is that effective adult numeracy practitioners can develop renewed processes of mathematical thinking and engender positive attitudes in adult numeracy students. Ineffective adult numeracy teachers unfortunately can disempower and reinforce the low status of the adult learner. (Benn, 1997; Coben and Chandra, 2000; Cummings, 1995; Bailey & Coleman, 1997). This research demonstrates that practitioners do not have a strong mathematical base to effect competent and confident delivery to adult learners that have acknowledged anxiety about mathematics.

It can be argued on both empirical and philosophical grounds that what teachers learn is framed in the context in which that knowledge is acquired. A teacher's view of mathematics is more or less consistent with the way they experienced learning mathematics themselves. (Cooney, 1999). The paucity of training to teach adult mathematics evidenced by this survey, coupled with an exam-orientated education system at second level, means that the only experience, of mathematics teaching of the practitioners in Ireland is very limited and one focused on getting the right answer and memorising the formula; rather than on generating an understanding of mathematics.

The support that practitioners have shown for a training programme that would effectively equip them for delivering mathematics to adult learners, and the willingness of practitioners to participate in such a programme were it available, demonstrates a clear commitment by practitioners to their own life long learning. Quality training that will empower the practitioner to engage in good practice for teaching adults mathematics and at the same time facilitate empowering mathematics in their own students is essential.

It is clear that empowerment is becoming a central thesis of this paper. Since that is the case it is necessary to look closely at the process of empowerment in ABE in Ireland. For the purposes of this paper empowerment is defined as 'the act of taking away demotivators and barriers in the system', (Persico, 1991:61). This definition has its roots in workplace organisations but offers an approach to examine in a holistic way, the current structure of the sector.

There are three levels in the ABE system through which the process of empowerment of the individual, through adult mathematics/numeracy education, is initiated: policy level, practitioner level, and the level of the learner.

Using this framework, Ireland has a top down policy of implementation (linear model- see Figure 8) driven by a perspective that prioritizes literacy above a combined literacy and numeracy agenda. A fragmented ABE provision is rapidly trying to respond to reactive, albeit well-funded policy initiatives in the absence of a theoretical and research framework on which to base their approach.

The Process of Empowerment – Linear Model

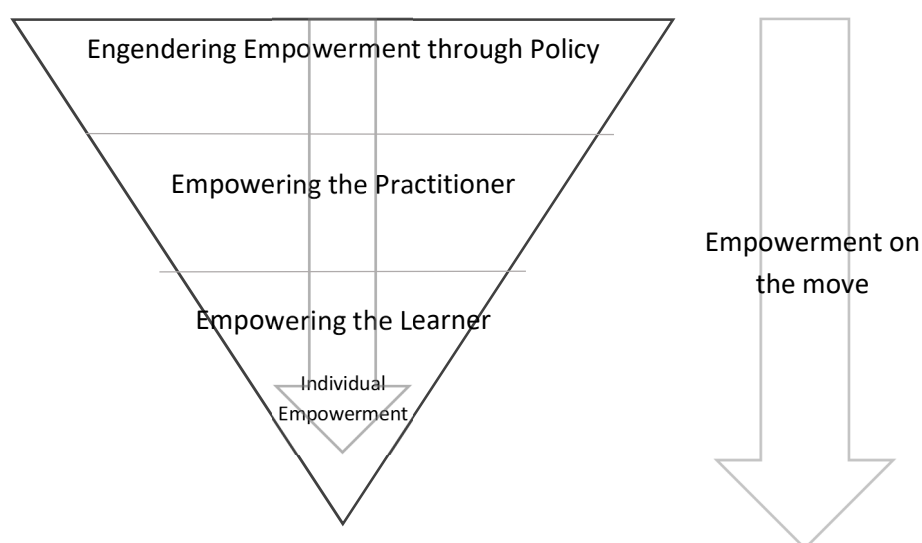


Figure 8: Top down approach to the process of empowerment

Moving from this linear model at one end of a scale, it is suggested that a better approach to the initiation of individual empowerment in adult learners is to have a more integrated process (Integrated model –see Figure 9).

In this integrated model both learners and practitioners should have an identified structure to enable them to influence programme content, pedagogy and policy. The findings emerging from this study support the conviction that the practitioner plays the pivotal role in the process of empowerment in ABE. Highly trained, confident practitioners could, for example, through a recognized forum, inform policy, moving policy development to being proactive rather than reactive and ensure policy is implemented at centre level. Practitioners can facilitate the initiation of empowerment in individual learners allowing them to address their own needs and encouraging them to take responsibility for their own learning.

In reality the process will include a hybrid of the two models. A dynamic interaction between individual stakeholders will shift the balance one way or another, depending on for example; new revised policy initiatives, policy prioritisation and funding; practitioner experience, practitioner training, and the stage of individual empowerment of the learner.

The Process of Empowerment - Integrated Model

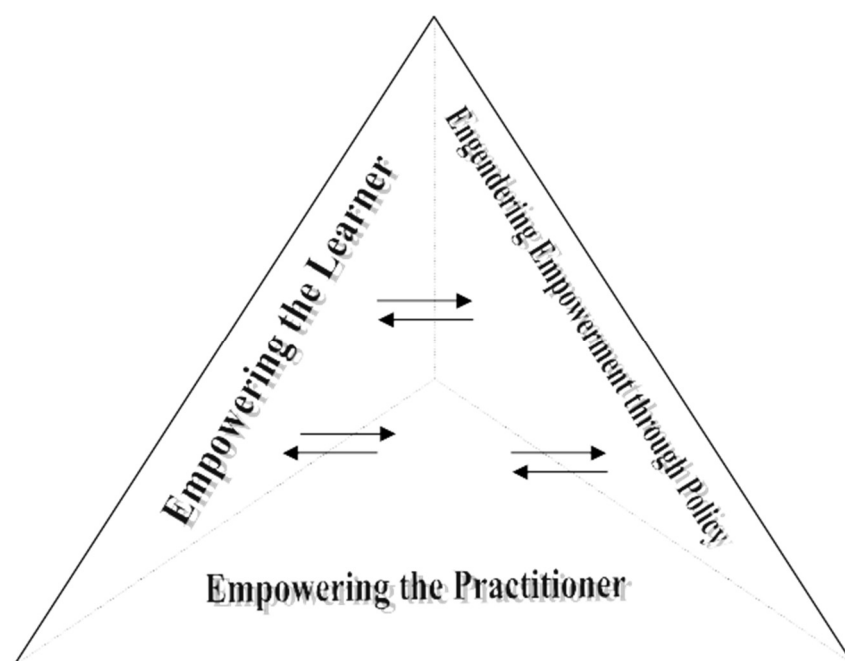


Figure 9: Integrated approach to the process of empowerment

Conclusion

The current national policy emphasis on ABE, coupled with the funding that has been allocated to this sector provides an opportunity in Ireland for addressing and removing barriers to the process of empowerment. In order to progress the ABE system to a more integrated model of empowerment; the authors assertion based on their research is that:

- All the elements of an effective ABE system must individually and collectively engender empowerment,
- There is a need for a systematic and democratic exploration of the nature of numeracy. Government policy and priorities must recognise that numeracy is not a single concept that can be incorporated within literacy or be strongly guided by the school mathematics curriculum,
- Practitioners need to be provided with the tools they require to become ‘a highly trained corps of practitioners who are dynamic and equipped to lead change, to play a key role in policy debate and to reflect the distinctive identity of the sector in the field of professional practice and research.’
- There must be a proactive approach to targeting specific groups of adult learners who must be encouraged to engage and develop confidence in the ABE system, that is an integral part of a national framework of lifelong learning.

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References

- Bailey, I. and U. Coleman (1997). *Access and Participation in Adult Literacy Schemes*. Dublin: National Adult Literacy Agency.
- Benn, R. (1997). *Adults Count Too: Mathematics for Empowerment*, Leicester: The National Organization for Adult Learning (NIACE).
- Coben and N. Chanda (eds.) (2000). 'Teaching 'Not less than mathematics, but more'; an overview of recent developments in adult numeracy teacher development in England - with a sidelong glance at Australia'. In: *Perspectives on Adults Learning Mathematics: Research and Practice*. Dordrecht: Kluwer Academic Publishers, 307- 327.
- Cooney, T. (1999). 'Conceptualizing Teachers' Way of Knowing'. *Educational Studies in Mathematics*, Vol. 38, 163-187.
- Cumming, J. (1995). *Adult Numeracy Policy and Research in Australia - the Present Context & Future Directions*. Department of Employment, Education, Training and Youth Affairs, Australia
- Department of Education and Science (1999). *Qualifications (Education and Training) Bill 1999*. Dublin: Stationery Office.
- Department of Education and Science (2000). *Learning For Life - White Paper on Adult Education*. Dublin: Stationery Office.
- Department of Finance (2000). *National Development Plan 2000-2006*. Dublin: Stationery Office.
- Department of the Taoiseach (2000). *Programme for Prosperity and Fairness*. Dublin: Stationery Office.
- Farrell, J. (2000). E-mail communication, Department of Education and Science, Subject: Numeracy; received 23rd May 2001.
- Morgan, M., B. Hickey and R. Kellaghan (1997). *International Adult Literacy Survey: Results for Ireland*. Dublin: Educational Research Centre.
- National Adult Literacy Agency (NALA) (1999). *The National Adult Literacy Agency, Strategic Plan 1999-2001*. Dublin: National Adult Literacy Agency.
- OECD (1997). *Literacy Skills for the Knowledge Society*. Paris: OCED.
- Persico, J. (1991). *Leadership and Empowerment in a Total Quality Organization, Total Quality Management*, Vol. 2(1), 57-61.
- Wagner, D. (2000). *EFA 2000 Thematics Study on Literacy and Adult Education*. International Literacy Institute, University of Pennsylvania-UNESCO.

Professional Development of Adult Numeracy Teachers—What Form Does It Take?

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Abstract

At ALM10 a Discussion Group on 'Professional Development' was facilitated by Lena Lindenskov (Denmark) and Terry Maguire (Ireland). The discussion group focused on the question; 'What would a good numeracy teacher, who had undertaken our (fantasy) numeracy PD course, know, believe and be able to do after it'. The discussion highlighted the need for effective, tailored professional development for adult numeracy teachers which must be developed in tandem with the wider policy environment including economic and educational considerations.

This workshop extended the discussion initiated at ALM9 and gave participants the opportunity to experience some of the hands on activities (simulations) that have been used with adult numeracy teachers during Professional Development courses in Denmark and Ireland. Activities were chosen from the following:

- *Activities for developing and refining 'mathematical awareness' among teachers; activities include investigating everyday life and everyday materials*
- *Trying out and reflecting on how to adapt good materials originally developed for children*
- *Investigating adults' different approaches and methods to problem solving*
- *Doing project work on self-chosen area of concern*

A central aspect of the workshop was to examine methods than enable teachers of adult numeracy to reflect on their own teaching, both in terms of their adult learners and the goals of programmes of professional development in the context of their own lifelong learning.

Introduction

There has been very little research carried out on what constitutes effective professional development for teachers³¹ of adult numeracy. However, the almost universal increases in funding seen in most countries that participated in IALS and SIALS survey (OECD 1995 and 1997) coupled with the development of numeracy curricula for adults (for example in Britain, Denmark) and the move in many countries to reassess literacy and numeracy requirements of adults through a National Reporting Scheme, and/or the need to integrate literacy and numeracy with the Vocational and Education Training Sector, has brought the issue of teacher competence and qualification to the forefront internationally. Historically, research into teacher training has been limited and only occasionally reported, for e.g. through the ALM international research fora, (Johnston et al, 1997; Joseph, 1997; Ayoub, 2003). ALM9 recognised the growing importance of professional development as an issue of concern among both practitioners and researchers and for the first time incorporated a discussion group to focus on professional development. Representatives from five different countries attended the discussion over two days (Lindenskov and Maguire, 2003). At the same conference a separate, but overlapping discussion highlighted a number of other related and important points including the need to develop creative approaches to teaching mathematics and numeracy, the need for the integration of information technology, the need to raise questions about the level of numeracy qualifications held by teachers and the need to know what numeracy qualifications are actually necessary (Polkinghorne 2003). In addition it became apparent that many countries differed both in their approach and commitment to professional development and much can be learnt through collaboration and discussion.

Since the results of IALS and SIALS were published, both Denmark and Ireland have seen increased resources being allocated to Adult Basic Education however both countries differ significantly in their approach to the development and requirement for professional development for adult numeracy teachers.

Professional Development for Adult Numeracy Practitioners in the Ireland

The results of the International Adult Literacy Survey (IALS) highlighted that a significant proportion of the adult population in Ireland have unacceptably low levels of literacy and numeracy. Rectifying this situation was seen as a matter of national priority (NDP, 2000) and policies were developed and significant resources were allocated to develop provision in Adult Basic Education. In the case of adult mathematics education, the concepts espoused in the policies handed down have not been adequately problematised, and the policies have been conceived in isolation from the teacher body, who eventually have to try and teach in accordance with the aspirations these policies contain. This primary flaw is further

³¹ The term teacher is used here to include teacher (Denmark) and tutor (Ireland).

compounded by the nature of the organisation of the service providers in this sector. They are highly individualistic and typically operate in isolation from one another.

Recent research highlighted that the teachers of adult numeracy in Ireland were a group characterised by a high degree of heterogeneity. Working conditions for teachers were very variable, as were the levels of expertise amongst the teachers. Many suffered from 'maths anxiety' themselves and were heavily influenced by their own school experience of mathematics. As a group, they held a diverse range of views regarding adult learners and adult mathematics and there was little or no consistency in their conceptualisation of numeracy. The single unifying factor amongst the teachers of adult numeracy in Ireland was that none of them had access to professional development directly related to adult mathematics education (Maguire, 2003).

The pivotal role played by teachers of adults in Ireland was clearly recognised in the White Paper on Adult Education (2000). The White Paper emphasised that the government aimed to ensure that qualifications for the teaching and practice of Adult Education will be accorded formal recognition. It has been three years since the publication of the White Paper and as yet, none of the changes promised have been put in place. Professional development in Ireland for teachers of adult learners remains the remit of individual service providers.

Numeracy provision in Ireland lags behind the provision for literacy, up to now numeracy has not been problematised in the Irish context and that there is no explicit or nationally accepted policy with regard to numeracy. Current provision for the training of numeracy teachers has developed from the adult literacy sector and has not been specifically developed for numeracy. An added complication is the fact that in the majority of cases in the vocational education sector, adult numeracy provision and practice is organised and supported by staff whose experience and training is in the field of literacy rather than numeracy.

The National Adult Literacy Association (NALA) is funded through the Department of Education to support the development and implementation of training for adult literacy workers and in addition to the accredited programmes outlined below, NALA provides a range of non-accredited programmes for adult numeracy teachers annually. To date, only 5.9% (216) of the 3656 teachers working in the literacy schemes around the country have completed any training in numeracy. Other service providers for e.g. the Prison Education Service provide limited professional development to numeracy teachers. All teachers of the prison education service undergo at least a half days training in numeracy as part of their initial training. In addition 'numeracy' days are organised through the prison education in-service committee to encourage networking and sharing of resources of numeracy teachers in the prison service, and these take place annually. In-service is also provided to teachers who deliver National Council for Vocational Awards (NCVA) accredited modules. This provision is however limited (by instruction from the Department of Education and Science) to in-service on assessment issues in relation to individual modules (Maguire and O'Donoghue, 2001). In terms of accredited programmes numeracy remains a small component of existing provision which had been geared towards literacy rather than numeracy.

National strategy for Ireland was launched by the National Adult Literacy Association in 2004. This strategy builds on the research completed through the University of Limerick (Colleran, 2002; Maguire, 2003) and for the first time (in Ireland) defines numeracy separately from literacy. The strategy places a strong emphasis on developing effective professional development for teachers of numeracy. The success (or not) of the strategy will inevitably depend on the availability of resources to translate it into practice. The current economic decline in Ireland does not bode well in the short term.

Professional Development for Adult Numeracy Practitioners in the Denmark

In contrast to Ireland Denmark has taken a very coordinated approach to curriculum development, and professional development. Denmark until recently did not have a single expression corresponding to the English term 'numeracy'. However Lindenskov and Wedege (2000) introduced and defined a new Danish word *numeralitet* which meaning lies close to some of the conceptual understandings of the English term 'numeracy', but differs from others. The Danish word *numeralitet* was later adopted by the Ministry of Education. The two-pronged general definition of *numeralitet* describes a math-containing everyday competence that everyone, in principle, needs in any given society at any given time:

- Numeralitet consists of functional mathematical skills and understanding that in principle all people need to have.
- Numeralitet changes in time and space along with social change and technological development.

Prompted by the results from SIALS, Literacy skills for the knowledge of society, (OECD 1997; Pilegaard 2000), which highlighted that 28% of the adults in Denmark had poor quantitative literacy (less than level 2), the opening address of the Prime Minister's to the Danish Parliament in October 1999 gave adult and further education special mention,

“The educational system must do a better job of targeting those with short-term education. (...) This applies especially in the most basic areas of knowledge of Danish and arithmetic/mathematics.”

In May 2000, a government bill was passed which focused on provision for preparatory adult education, which, included courses that aim towards the development of the participants' numeracy. A new curriculum 'Preparatory Adult Education Mathematics' -(in Danish abbreviated as 'FVU-matematik') and teacher education programme were developed in tandem and were underpinned by the aim that the curriculum would make adults' mathematics visible and open the doors named 'mathematics' to adults (Coben, 2000; Lindenskov and Wedege, 2000). The curriculum developed stressed the functionality of numeracy. The curriculum focused on the dynamic interrelations between:

- The activities of counting, measuring, locating, designing, playing, explaining (Bishop, 1988)
- Authentic media and data
- Mathematical concepts and operations

The special teacher education course developed alongside 'FVU-matematik' had *numeralitet*

as its core concept in four themes:

1. numeralitet outside education,
2. numeralitet in curricula and learning materials,
3. adult numeralitet and difficulties and
4. the new Danish adult numeralitet curriculum.

The need for teachers to have completed this special teacher education (numeracy) course if they are teaching FVU-matematik' was made a legislative requirement through the Executive Order of the Ministry of Education no. 680 (July 2001) which set out the qualifications teachers are required to have as follows:

A. Teachers with mathematical qualification are required to have

1. mathematics as one of the main subjects in teacher training colleges for primary and lower secondary school - or equivalent mathematical qualifications acquired by formal education or by teaching practice -
2. *in addition teachers are required to have completed and passed specialised teacher training course approved by the Ministry of Education (Our emphasis)*

B. Teachers with educational qualifications are required to have:

1. at least equivalent to educational qualification from teacher training colleges for primary and lower secondary school or from teacher training courses for adult education
2. completed at least 500 lessons teaching practice in adult education (Until relevant specialised teacher training courses are established and approved by the Ministry, you can be granted an exemption from A.2)

Until 2003 six special teacher trainer courses for teachers of 'FVU-matematik' have been completed, (these courses continue to be delivered).

The Workshop

The experience that teacher educators in Denmark have gained from the delivery of these special teacher trainer courses and the experience gained from the research work carried out with teachers in Ireland (Maguire, 2003) highlighted that teachers rarely use practical problems and hands on activities and lack the confidence and the skills to introduce these kinds of activities in the classroom.

The workshop allowed participants to experience some of the activities that have been used in training courses in Denmark and to reflect on how these activities could be used for a range of purposes. These kinds of activities enable the teacher to give adult learners the opportunity to experience:

- that mathematics also is something more than just pencil and paper work, and more than just algorithm-procedures.

This means that many teachers will have to change their own view and attitudes to mathematics!

- that mathematics is not only an abstract subject, but also concrete and practical.

This implies recognition of the fact that different ways and methods can be used to solve problems involving mathematics!

- that work with mathematics also can involve experiments and investigations.

This implies that mathematics is not only based on pure thinking. Empirical studies can help too!

These activities that have been used in training courses in Denmark are also important for the teacher as:-

- it gives the teacher an opportunity to see the adult learner work in practical situations which provides an opportunity for the teacher to discover learner competences that can't be discovered by working only with pencil and paper items
- hands-on items can give the teacher a broader spectrum of practical situations that provide an ideal opportunity to challenge the learners in order to develop conceptual understanding reaching more kinds of situations
- the practical situation gives the teacher an excellent time for observation and talk with the learner for use as formative assessment.

The three TIMSS activities and the Get It Together (EQUALS, 1989) activities used in the workshop originate from work with children. School based mathematics activities are not always considered to have relevance in the adult context. The Danish experience to date indicates that the activities are also engaging to adults. An added advantage is that many adult learners are themselves parents and school mathematics from primary and lower secondary have relevance in their own lives. However it is important that the practical activities are well-planned and include guidance for using the equipment; the intended mathematical content; expected learner performance; how the item is placed in a broader perspective, and which common misconceptions the activity might evoke.

TIMSS Activity 1 - Plasticine.

Originally this activity was designed to give the learner an opportunity to show competences both in mathematics and science. The learner is asked to use a balance and two weights on 20 g and 50 g to make lumps of plasticine weighing 20g, 10g, 15g and 35g. The task gives learner opportunity to work with the principle of the balance in a practical way using mathematical problem solving in non-routine situations. It is intended that learners make observations and talk about problem solving strategies.

TIMSS Activity 2 - Calculator.

In this activity the learner has the opportunity to work with multiplication using a calculator to discover and explore number-patterns. By using the calculator on 34×34 , 334×334 and 3334×3334 the learner works with multiplication, the position-system, analysing and factoring numbers. This is an example of an activity that gives the learner an opportunity to solve a problem either theoretically or by the trial and error method through experimentation.

TIMSS Activity 3- Dice.

Converting the number generated by a traditional dice to a new set of numbers gives the learner the opportunity to work out a new algorithm, to observe characteristic features of different numbers, to record, analyse and interpret the result. Experience both in Denmark and Ireland has highlighted that people of all ages find that the use of games in this context is very engaging.

Get It Together Activity– Languages of the World.

During this activity each of six participants is given a card with a statement about one or more languages, for example *About as many people speak Hindustani as speak Spanish. About 200 million people speak Bengali. Your group's task is to rank the top ten languages spoken in the world today. Participants must take turns to work out with the other participants in the game how their statement contributes to finding the answer to the problem.* Participants have the opportunity to use and hear mathematical words when they exchange information, bring forward ideas, discuss ideas and finally decide – if possible – on a single solution to the task. Number sense is being trained in a context of everyday language: more, very few, about, only two, less, percent, different, most, very common. The activity affords the teacher the opportunity to spend more time watching the learners, and to get sense of learners' number sense and problem solving behaviour.

Get It Together – What's the Pattern.

During this activity each of six participants is given a card with a statement for example *The first and second numbers are the same. What is the seventh number in the pattern?* One learner in the group might act as an observer, who can record critical points for the group to reflect on together. For example the period of reflection could focus on what was learned, on how to learn from each other, on the participation of individual learners and on the different strategies discussed.

Participants at the workshop at ALM10 reflected on their own attitudes to the activities and how they could be used with adult learners. The following points were made in the discussion that followed:

- Initially some participants experienced some anxiety about working in a group and were anxious about whether their own skills and knowledge were up to completing the tasks.
- Some participants did not enjoy using plasticine as it made the activity seem overtly childish although the task itself was challenging.
- Participants used a range of methods and different strategies in completing the same task.
- The potential for the activities to be adjusted for different contexts.
- The range of mathematical, problem-solving skills that could be developed.
- The potential usefulness of these activities in the classroom.

Supporting Reflection in Teacher Training Courses

Both the Danish and Irish experience of teacher training has found that teachers want to be given 'fish on a plate', rather than being taught how to 'catch the fish' themselves.

Cooney and Krainer (1996) describes this as the 'consumer mentality' that teachers often bring to in-service with them. They have an expectation of materials that can be directly copied and handed out/used with their learners. The consumer attitude often causes problems and resistance amongst teachers who have come with different expectations. In terms of delivery, the opposite extreme is to present research results and inform teachers that it is their task to put theory into practice. A compromise in terms of course content needs to be agreed with participants to maintain teacher motivation. The correct mix of 'fish on the plate today' that will satisfy their consumer mentality, together with increasingly advanced 'fishing lessons' will be required.

The activities that are used in the teacher training professional development courses can be used by teachers in their classroom (fish on the plate). However a process of reflection must also be built into an effective professional development programme (fishing lessons). Two approaches to support reflection in teacher training programmes were discussed during the workshop.

In Denmark the process of reflection is stimulated by asking teachers to complete a questionnaire as an integral part of every activity. Briefly the teachers are required to describe the thinking and work processes they have used during the activity and where in everyday life are similar thinking and work processes relevant. The teachers are also asked to reflect on the range of approaches that could be used in completing the activity, to identify the number sense and number skills and other basic mathematical concepts and skills needed both to start the activity and that can be developed through the activity. Finally teachers are asked if they would use this activity in their own classroom and if so how they might adapt it to develop for example mathematical language skills or to increase the level of difficulty. Teachers are given the opportunity to discuss their responses with each other.

In Ireland, Reflect participatory methodologies were piloted as part of a teacher training programme. The activities were very successful in stimulating discussion debate and reflection among teachers. The Reflect approach values each individual's existing knowledge and experience. Reflect incorporates '*reflection for the purpose of change and subsequent action in a cyclical process*'. The participatory tools, incorporating visualisation approaches (maps, calendars, diagrams, matrices etc) are used to create an atmosphere in which everyone can contribute. The visual representations are intrinsically mathematical and provide good starting points for adults who may have associated literacy problems. Similarly, they provide useful tools for teachers of numeracy who may lack confidence in their own mathematical skills (<http://www.networkedtoolbox.com/pages/about-reflection-action>).

Conclusions

Clearly effective professional development of teachers in adult numeracy and its operationalisation must address a number of issues at once. It must satisfy the requirements outlined by practitioners to the question, (*'What would a good numeracy teacher, who had undertaken our (fantasy) numeracy PD course, know, believe and be able to do after it'*). It

must take cognisance of the extensive research that is available in relation to professional development of mathematics. It must recognise that the learners are not children but adults, and that they may require different pedagogies in practice to ensure the operationalisation of the provision incorporates the appropriate goals and gives teachers the opportunity to engage in professional dialogue about problems they deem important. It must integrate mathematics and pedagogy if professional development is to translate into more effective practice in the adult classroom. What ever form it takes it must recognise and address the six roles that an adult numeracy teacher plays: as a learner for numeracy; a teacher of numeracy; as a collaborator with other teachers; as agents of change working within a particular socioeconomic environment; as a lifelong learner and as a learner of teaching itself (Maguire, 2003). Most importantly it should not be viewed as a discrete, compartmentalised entity, but within much broader parameters where its relationship with the wider political, social and economic context is kept to the fore (Coolahan, 2000).

Updates by 2018

In inspecting what have happened since we wrote this article, we wish to stress the importance of adult learners' beliefs, dispositions and interests. Today we want to add that teachers should be able to investigate these aspects as well as adults' different approaches and methods to problem solving.

The Danish teacher training courses presented in the article are still running. The intention for adult preparatory education and adult vocational training is that the teachers' formal qualifications will provide relevant skills to understand and act on the setting's official aims and recommended instruction strategies, and to engage in contextualized mathematics which is relevant to the setting. It is a formal requirement from the Ministry of Education for teachers in adult preparatory education to complete a minimum of a 4-year, full-time teacher education degree program, and a four-month full-time degree program, including the two programs, "Adult numeracy" and "Adults' mathematical difficulties," provided by university colleges around the country. But, evaluation has shown that the formal requirement is not met by all providers (EVA, 2012). For adult vocational training, teachers must complete a minimum of vocational education degree program, and 3 years of relevant employment. During the first 4 years as the teacher, a 1-year, full-time degree in andragogy should also be completed. The Ministry of Education recommends that the degree include two programs called "Adult numeracy" and "Adults' mathematical difficulties."

In Ireland, the National Adult Literacy Agency, based on the research of Maguire (2003) subsequently developed a national framework for meeting the professional development needs of teachers of adult numeracy in the Irish further education and training sector. This framework currently guides the content of professional development provision for teachers of adult mathematics learners in Ireland. Figure 1.

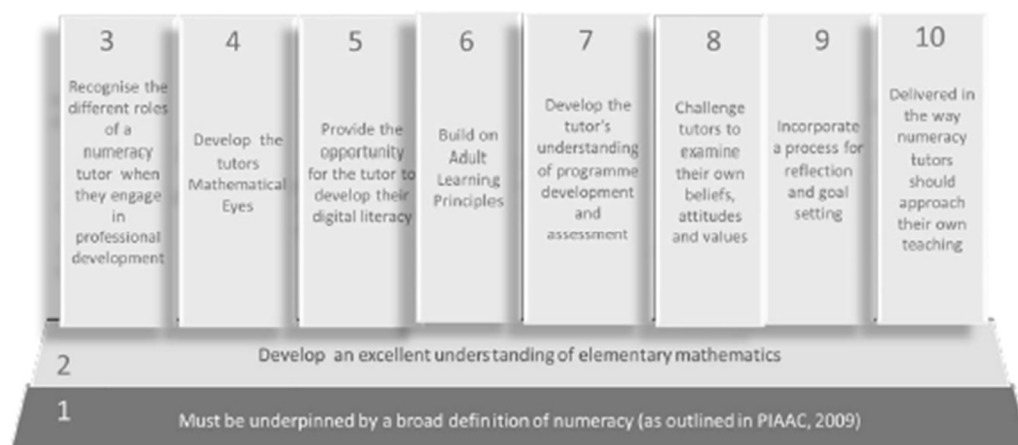


Figure 1. Core components of professional development for tutors of adult numeracy in further education in Ireland.

Maguire and Simth, (Forthcoming) have subsequently reviewed and further developed this framework to incorporate the need for teachers of adult mathematics to be good communicators and to understand the big ideas in mathematics as a framework for developing their own understanding and competence in mathematics.

References

- Ayoub, T., 2003. The Importance of Adequate Teacher Training for Teachers of Adult Numeracy." In: *Proceedings of Policies and Practices for Adults learning Mathematics: Opportunities and Risks, the Ninth International Conference of Adults Learning Mathematics - A Research Forum (ALM-9)*, held in Uxbridge College, England, 17-20 July 2002.
- Bishop, A. J., 1988. 'Mathematics Education in its Cultural Context.' *Educational Studies in Mathematics*, 19(2), 179-191.
- Coben, D., 2000. 'Mathematics or Common Sense? Researching 'Invisible' Mathematics through Adults' Mathematics Life Histories.' In Coben et al. (eds.) *Perspectives on Adults Learning Mathematics*. Dordrecht: Kluwer Academic Publishers
- Colleran, N., 2002. Improving Adults' Quantitative Problem Solving Skills: Theory, context and programme design. *Unpublished PhD Thesis*, University of Limerick, Ireland.
- Coolahan, J., 2000. "Teacher Education in Ireland and Western Europe: A Comparative Analysis." In: *Drudy, S. and Oldham, E. (eds.) Educating the Educators. Proceedings of a conference on Teacher Education. April 2000*. Educational Studies Association of Ireland. Held in Cardinal Press, Co. Kildare.
- Cooney, T., and K. Krainer, 1996. Inservice Mathematics Teacher Education: The Importance of Listening In: *International Handbook of Mathematics Education*, Kluwer Academic Publishers: Netherlands.
- Danish Evaluation Institute, EVA, 2012. *Evaluation of preparatory adult education*. In Danish: EVA, Danmarks Evalueringsinstitut. *Evaluering af forberedende voksenundervisning (FVU)*, Copenhagen: EVA.
- Department of Education and Science, 2000. 'Learning to Life' White Paper on Adult Education. Stationery office, Dublin.
- Department of Finance, 2000. *National Development Plan 2000-2006*, Stationery Office, Dublin.
- EQUALS (1989). *Get It Together*. Berkeley, California: Lawrence Hall of Science.
- Executive Order of the Ministry of Education no. 680 (July 2001) In Danish: Bekendtgørelse nr. 680 af 10/7/2001 om ændring af bekendtgørelse om undervisning m.v. inden for forberedende voksenundervisning (FVU-bekendtgørelsen) (Matematik, lærer kvalifikationer) <http://www.retsinfo.dk/GETDOC1/ACCN/B20010068005-REGL>
- EVA, Danmarks Evalueringsinstitut (2012). *Evaluering af forberedende voksenundervisning (FVU)*, Copenhagen: EVA.
- Johnston, B., B. Marr, and D. Tout, 1997. "Making Meaning in Maths. Adult Numeracy Teaching: a Course for Teachers." In: *Adults learning Maths - A Research Forum 3, ALM-3 Proceedings of the Third International Conference of Adults Learning Maths- A Research Forum (ALM)*. 5-7 July 1996 at University of Brighton. London: Goldsmiths College, University of London in association with Adults

- Learning Maths - A Research Forum held in Coben, D.,(ed.), Goldsmiths College, University of London, London.
- Joseph, J., 1997. "Numeracy Staff Development for Basic Skills Tutors." In: *D. Coben (Comp.) Adults Learning Maths- A Research Forum -3, ALM -3 Proceedings of the third International Conference of Adults Learning Maths - A Research Forum (ALM)*. 5-7 July 1996 at University of Brighton. London; Goldsmiths College, University of London in association with Adults Learning Maths - A Research Forum.
- Lindenskov, L.; Maguire, T. 2003. 'ALM Discussion Group. Professional Development.' To be published in Evans, J. et al. (Eds.) *Policies and Practices for Adults Learning Mathematics: Opportunities and Risks, The 8th International Conference on Adults Learning Mathematics (ALM8)*. 17-20 July 2002. London.
- Lindenskov, L.; Wedege, T. 2001. *Numeracy as an Analytical Tool in Adult Education and Research*. Centre for Research in Learning Mathematics, Publication no.31, Roskilde University.
- Maguire, T. and J. O'Donoghue, 2001. "Developing a National Framework for Adults' Mathematics Education in Ireland: A Pilot Study." In: *Schmitt, M. J.; Safford-Ramus, K. (comps), proceedings of ALM-7, the Seventh International Conference of Adults Learning Mathematics - A Research Forum*. National Centre for the Study of Adult Learning and Literacy (NSCALL) Harvard University Graduate School of Education, Cambridge, M.A. in association with ALM.
- Maguire, T., 2003. Engendering Numeracy in Adults Mathematics Education with a focus on tutors: A grounded Approach. Unpublished PhD Thesis, University of Limerick, Ireland.
- Maguire, T., and A. Smith. (Forthcoming) Maths Eyes – A concept with potential to support Adult Mathematics Education. TSG 6. Proceedings of the thirteenth International Congress on Mathematical Education. Springer.
- TIMSS, 1995. Mathematics Achievement in the Middle School Years: IEA's Third International Mathematics and Science Study (TIMSS) <http://nces.ed.gov/timss/pubDetails.asp?PubID=2> accessed on: Jan- Apr, 2001 .
- OECD - Organisation for Economic Co-operation and Development (1995). *Literacy, Economy and Society. Results of the First International Adults Literacy Survey*. Statistics Canada.
- OECD - Organisation for Economic Co-operation and Development (1997). *Literacy skills for the knowledge of society: Further results from the International Adults Literacy Survey*. Statistics Canada. (SIALS)
- Pilegaard J.T., A. Holm, A. Andersen, S. Hastrup, M. B. Heinskov and J. E. Jacobsen, 2000. "Danskernes læse-regne-færdigheder – udvalgte resultater", AKF (in Danish, with an English summary).
- Polkinghorne, R., 2002. "Consultation on Research on Adult Numeracy at ALM-9." In: *Proceedings of, Policies and Practices for Adults learning Mathematics: Opportunities and Risks, the Ninth International Conference of Adults Learning Mathematics - A Research Forum (ALM-9)*. Held in Uxbridge College, England, 17-20 July 2002.
- Weng, P., 1996. *Matematik og naturvidenskab i folkeskolen - en international undersøgelse*. (The Third International Mathematics and Science Study, TIMSS) København: Danmarks Pædagogiske Institut.

A Review of Level 4 Training for Adult Numeracy Specialists

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Introduction

In England a new strategy was introduced in September 2002 which requires all new adult numeracy specialists to have, or work towards, new qualifications based upon the Level 4³² subject specifications constructed by FEnto (Further Education National Training Organisation) (DfES 2002). This paper describes the structure of this new qualification and some of the modes of delivery that have been used. It concludes with a summary of the reactions of some Level 4 trainees and some of the views of the Numeracy Team at the LLU+ about the specifications. Throughout the workshop participants were able to interact with the teaching materials and discuss issues arising from this approach to adult numeracy teacher training.

The background and purpose of the of the qualification

The qualification was introduced to raise the vocational skills of numeracy specialists for quality reasons. It was made clear in guidance documents that it was in addition to a teaching qualification such as a PGCE / Cert Ed³³.

The development of this new qualification can be traced through the key documents that have been produced as part of the UK government's strategy for improving adult literacy and numeracy skills. Though this paper concentrates entirely on the numeracy qualification the policy that led to its development also led to similar qualifications for adult literacy and ESOL³⁴ specialists.

In *A Fresh Start*³⁵ the case for improving the provision for adult literacy and numeracy in England is made, and as part of that case are the initial recommendations to both increase the number of adult literacy and numeracy teachers, and the standards of training.

Without enough good teachers there is little hope of achieving the proposed targets. At present, too many teachers teach part-time, and some are inadequately prepared. To achieve our aims, many more teachers will need to be trained to teach for the new curriculum. We shall require over 15,000 full-time equivalent teachers in this area, compared with under 4,000 at present. Teacher training programmes will have to be commensurate. And a new qualification for teachers should be developed jointly by

³² Level 4 indicates this is a degree level course, and the term 'Level 4' within numeracy (and literacy) teacher training in England has become used as an abbreviation for the *Subject Specifications for teachers of adult literacy and numeracy*.

³³ Post Graduate certificate in education or Certificate in Education

³⁴ ESOL – English for speakers of other languages

³⁵ *A Fresh Start* is commonly referred to as 'The Moser Report'

QCA, the Further Education National Training Organisation (FENTO) the BSA and others. (DfEE, 1999 para1.38).

Later in the report the case is made more strongly.

The quality of teaching is crucial. We are aware of the many skilled and dedicated teachers working in this area, but specific training for teachers is nowhere near universal. This cannot continue.

The fact that basic skills teaching remains an area where there are few career opportunities or opportunities for professional development means that it still largely remains an unattractive career path. This is not to say that people don't want to teach basic skills. But it does make it more difficult to recruit new teachers.

In essence the best-trained teachers in our education system are teaching enthusiastic young children at Key Stage 1. Those with least opportunities for professional development, and with most job insecurity, are teaching adults who have often failed at school and need intensive help. (DfEE, 1999 paras 4.16/19/20).

Following the publication of *A Fresh Start*, the Department for Education and Employment (now the Department for Education and Skills - DfES) took up the proposals made by Sir Claus Moser and published their strategy in *Skills for life* in 2001. The strategy took forward the proposals to increase recruitment of teachers for adult literacy and numeracy and to develop new qualifications.

The national literacy and numeracy strategy for schools has shown that good teachers are a pre-requisite for success. The same must become true of adult teachers of literacy, language and numeracy. Initial and in-service professional development for all these teachers will therefore ensure that they have a sound knowledge of the practicalities of literacy, such as teaching spelling, comprehension and the development of writing strategies, and numeracy, such as the latest strategies for estimation, addition and multiplication.

. Teachers specialising in literacy and numeracy skills will be expected to begin working towards new literacy and numeracy qualifications from September 2001. The Learning and Skills Council will encourage this training so that all teachers of literacy and numeracy are adequately trained to help their learners. (DfEE, 2001, paras 136, 137)

The subject specifications were then produced by the Further Education National Training Organisation (FENTO) and the DfES as the basis for developing courses. In the foreword to the subject specifications, written jointly in the names of David Hunter (Chief Executive, FENTO) and Susan Pemberton (Director, Adult Basic Skills Strategy Unit) the case for the new qualifications is repeated more strongly.

. . . . One of the key commitments set out in the *Skills for life* strategy has been to raise achievement by improving the status and quality of training available, and raising the level of qualifications, for all teachers of adult literacy and numeracy. A key strand of this work has been undertaken by the Further Education National Training Organisation (FENTO). FENTO has developed this suite of subject specifications for teachers of adult literacy and numeracy.

. . . These specifications are designed to be used in specialist teacher training, in specialist modules for continuing professional development (CPD) and in other staff in-service training. . . . These subject specifications are the first step towards

recognising that teachers of adult literacy and numeracy have a challenging and professional role with the same curriculum status as other curriculum areas. In line with the Government's strategy they signal our belief that adults developing these skills deserve to be taught by skilled and competent teachers with the appropriate specialist level 4 teaching qualifications.

For the first time in post-16 education, teachers of adult literacy and numeracy have the opportunity to work towards professional teaching qualifications. . . . The intention is to raise the status of the profession and confirm that teaching literacy and numeracy is a professional activity that does not differ in demand or expectation from teaching any other subject area. (DfES/FENTO 2002)

Following the publication of the Subject Specifications a number of courses and qualifications were developed by higher education institutions and qualification authorities. The remainder of this paper describes the early developments of a number of courses based on the subject specification for teachers of adult numeracy by the Numeracy Division at LLU+ based at London South Bank University (LSBU). These courses were designed to be assessed through either the regulations of City and Guilds or LSBU.

The content

The content of the course consists of:³⁶

Numeracy Learning and Development in Context

- Social Factors and Issues
- Personal Factors Affecting Learning

Personal Numeracy Skills

- Numbers and Numeric Operations
- Geometry and Spatial Awareness
- Probability and Statistics
- Working with Algebra

The workshop provided an opportunity for the participants to have a taster of the sorts of activities that have been developed for the delivery of this qualification.

Further details and a description of the activities discussed are summarised in table 1.

³⁶ Full details of one of the courses developed to meet the Subject Specifications can be found in the City and Guilds course handbook for the 9486 course at <http://www.basic-skills.org.uk>

Table 1. A summary of the structure and content of the course

Unit Social Factors		
Content	Assignment Tasks	Example Activity
<ul style="list-style-type: none"> • Policy documents • Theories of learning • Review of general theories • Maths / Numeracy theories 	<p>1. Case study of provision An opportunity to examine the effectiveness of numeracy provision within a limited geographical area</p> <p>2. Profile of a learner A description of one numeracy learner identifying the factors which have affected their numeracy learning in the past.</p>	<p>Some history related to basic and key skills</p> <p>A kinaesthetic exercise in which participants read information on wall posters in the training room which provided information about previous adult numeracy related government schemes and answered questions on the target groups of these schemes</p>
Unit Personal factors		
Content	Assignment Tasks	Example Activity
<ul style="list-style-type: none"> • Personal Maths histories • Dyslexia / dyscalculia • Learning styles • Language and numeracy 	<p>1. Identification of errors An exercise which looked at errors and misconceptions in arithmetic calculations and suggested activities to overcome them</p> <p>2. Strategies for dealing with specific problems A description of particular maths difficulties and the main aspects of dyslexia and dyscalculia and the strategies, including a learning styles approach, which may be used to overcome them.</p>	<p>Words Activity</p> <p>The trainees, in small groups, were given a list of words on a display card. Some of the words were obviously ones encountered in a mathematics or numeracy context.</p> <p>They were asked to identify the significance (if any) of all the words and what difficulties or confusions the words could produce in the context of a numeracy class-room</p>
Unit Numbers and Numeric Operations		
Content	Assignment Tasks	Example Activity
<ul style="list-style-type: none"> • Numeral systems – roman/binary/other languages • Rules of arithmetic 	<ul style="list-style-type: none"> • Working with numeral systems A number of exercises performing calculations with unfamiliar numeral systems • Explanations of rules of arithmetic Worked examples demonstrating the commutative, associative and distributive laws of arithmetic • Reflection on teaching A discussion relating a study of different numeral and number systems to teaching arithmetic in the adult numeracy classroom 	<p>Binary and Denary numerals</p> <p>A tactile exercise matching cards with binary numbers to their equivalent in the familiar denary system.</p> <p>The exercise conducted in small groups allowed discussion to develop and for those more familiar with the binary system to support others in the group,</p>

Unit Geometry and Spatial Awareness		
Content	Assignment Tasks	Example Activity
<ul style="list-style-type: none"> • Geometry and Euclidean Proof • Cartesian Co-ordinates • Linear programming • Non-linear systems • Idea of calculus 	<ul style="list-style-type: none"> • Discuss proof of Pythagoras' Theorem • Proofs using Co-ordinate geometry • Give example of linear programming problem • Explain the use of non-linear scales • Describe basic principles of calculus 	<p>Timeline problem (non-linear scales exercise)</p> <p>A list of events from the 'big bang' through the evolution of living organisms to the modern period of science and technology. These were to be represented using some form of scale or illustration.</p> <p>A group discussion concluded the activity, allowing issues about linear and non-linear scales to be explored.</p>
Unit Probability and Statistics		
Content	Assignment Tasks	Example Activity
<ul style="list-style-type: none"> • Basic probability • Uses of conditional probability • Descriptive statistics • Hypothesis testing – • difference of means, correlation, chi-squared 	<ul style="list-style-type: none"> • How to introduce probability to learners • Use of conditional probability • Critical analysis of use of statistics in education • Examples of hypothesis testing 	<p>Card sorting exercise for hypothesis testing</p> <p>A number of cards with a description of an event and a graph or table with a summary of the data. The cards to be sorted into three categories – a correlation, no correlation, or unclear. The cards were in pairs, where similar situations were described on, but one had an obvious link and one a more ambiguous outcome.</p>
Unit Working with algebra		
Content	Assignment Tasks	Example Activity
<ul style="list-style-type: none"> • History of algebra • Use of algebra • Working with algebraic expressions • Investigations 	<ul style="list-style-type: none"> • Critical look at algebraic problems • Identification of link between arithmetic operations and algebra • Algebraic investigation 	<ul style="list-style-type: none"> • Use of algebra discussion exercise

The training courses

The context within which the two authors work is an innovative unit for teacher training in the post 16 sector (LLU+). This new qualification was delivered in a number of different ways during 2003 and 2004, aiming to meet the needs of a variety of trainees and targets set through different aspects of the UK governments Skills for Life policy initiative (DfES 2001).

The different models of training and number of trainees were described and these are summarised in Table 2.

Table 2. An overview of the groups and delivery modes used in Level 4 training by LLU+ Adult Numeracy Teacher Trainers

Group	Description	Number*	Delivery	Total delivery	Completion status **	Awarding Body	Notes
A	Central London College Trainees who had just completed C&G 7407 Stage 2	8 (0)	20 weeks 1 day / week 6 h / day	120 hours	completed training assessment not completed	C&G	Two learners from a different college just taking Personal Numeracy Skills part
B	Certificate in Adult Literacy, Numeracy and ESOL Education	10 (1)	30 weeks $\frac{1}{2}$ day / week 3 hours / $\frac{1}{2}$ day	90 hours	completed training assignments completed	LSBU	Two learners taking Stage 3 teaching and learning skills as well as Subject Specs
C	Fast Track Course 1 For experienced teachers with A level maths or equivalent	8 (2)	15 weeks 1 day / week 5 hours / day	75 hours	completed training assignments submitted	C&G	Two trainees had maths degrees The remaining 6 had mixture of skills with 2 particularly worried about their maths skills
D	Intensive Level 4 Course For experienced teachers with A level maths or equivalent	20 (0)	2 weeks 5 days / week 5 hours / day	50 hours	course just complete	C&G	1 exemption Remainder all with relatively high maths skills
E	Fast Track Course 2 For experienced teachers with A level maths or equivalent	18 (2)	15 weeks 1 day / week 5 hours / day	75 hours	course just completed	C&G	2 exemptions Most with high maths skills, 3 with some worries re experience

* Brackets indicate the number of candidates with full exemption from the Personal Numeracy skills ie those with a mathematics degree.

** As at May 2004

Approaches and activities to teaching and learning

Following the description of the course and the ways in which it had been delivered there was an opportunity for the participants in the workshop to use the tactile materials, such as card sorting, and discuss the approaches and activities that had been introduced in the training.

In order to understand the reactions of trainees and give some more background for discussion the approach of the LLU+ in their training was described. One important, and to some controversial, aspect of this new qualification is that it introduces more advanced mathematics skills (in the Personal Numeracy Skills, described above). This mathematical content is not part of the content of the adult numeracy core curriculum (BSA, 2001) or equivalent qualifications, which most of the trainees will be teaching to adult students.

The courses developed at LLU+ have included approaches and activities to enable trainees to make connections between the more advanced mathematical skills in the level 4 qualification and the approaches and skills required for teaching adult numeracy. For example, throughout the training a number of techniques were used as a way of modelling teaching approaches including the use of

- tactile matching and sorting cards,
- kinaesthetic activities such as a carousel of information,
- ICT with spreadsheets and interactive whiteboards, and
- discussion of mathematical issues.

In addition examples of the way in which the content of the training was delivered were outlined:

- in the number section, by asking trainees to reflect on how their working with other number systems may have parallels for numeracy learners;
- in algebra, by considering how ‘real’ standard problems are in text books to enable a critique of problems in mathematics;
- in geometry, by considering how elements of geometry can be applied;
- in probability, by describing how probability would be introduced to learners;
- in statistics, by undertaking a critique of the use of statistics in education.

What do the trainees think?

Following a discussion on these approaches, some examples of the views collected from trainees’ evaluations were distributed.

- | |
|--|
| <ul style="list-style-type: none">• Opportunity to reflect upon practice was useful• Too many assignments• I thought it might be more practical to help me in my day-to-day teaching• Some practical examples (but not enough)• Would like maths that is relevant to basic skills ... and approaches to teaching fractions, decimals etc.• It was great to discover that mathematicians are lively, fun people and not geeks• It has been disappointing that there is a lack of actual ‘how to teach the skill in the classroom’ |
|--|

- We are happy with the materials
- We think it is good to understand concepts and not just mechanics, in order to relate to classroom approach
- It has been helpful to go through the thinking process
- We would like maths books for us as teachers, rather than for students
- There is a need for help with teaching, especially in relation to differentiated learning

In addition a summary of all evaluations was given

On a positive note the trainees

- felt that it has been beneficial to work with other colleagues and share ideas
- welcomed the challenge of the training
- thought that the majority of activities have been very interesting.

On the other hand, the trainees

- felt that much of the mathematical content is not relevant to their work in adult numeracy
- would prefer to spend considerably more time discussing adult numeracy and teaching issues
- think that the assignments are difficult to fit into their work loads .

It was pointed out that the majority of trainees were willing ‘volunteers’ and that others who may be ‘drafted in’ might not be as positive about their training.

What do we think?

Having developed and delivered these courses the authors reported that the numeracy team felt that the following should be taken into account in any review of the specifications and qualifications. Overall there has been a concern shared both by the teacher trainers and the trainees about some of the content included in the original subject specifications. As Griffiths has said elsewhere ‘As far as I am aware there are no public documents giving the rationale for the content of the subject specifications’ (Griffiths 2004), though the specifications have had to be followed in all course developments.

- There is too much detail in particular elements of the specifications – for example why is calculus included?
- There is a tendency to ask for too many assessment activities.
- Elements of the personal factors unit needs some rethinking as it assumes some ‘deficiency’ in learners rather than considering different ways of thinking in more learners
- A specific unit on the teaching and learning of numeracy is needed
- Qualifications should not require evidence expected from teaching qualifications eg SoW/Lesson plans etc although we would expect some reflective thinking, probably via a simple diary / journal.

- A wider concern has come to our attention through the requests we receive for a number of courses. There is an obvious need for some sort of ‘conversion course’ for those with experience of other sectors (eg Primary schools) or other post-16 subjects (eg ESOL). The content of the specifications and the existing qualifications do not address the needs of these groups well. There is a particular problem with the teaching practice element of qualifications whereby there is a ‘Catch-22’ situation where trainees cannot complete qualifications without teaching numeracy but cannot get work teaching numeracy without the qualification. (Stone 2004)

Conclusion

In this workshop the discussion took place throughout, as the participants were working with the materials. Information was exchanged, and the relevance of the qualification in an international context of teacher training for adult numeracy was explored.

References

- BSA (2001) *Adult Numeracy Core Curriculum*. London: Cambridge Training and Development on behalf of The Basic Skills Agency
- DFEE (1999) *A fresh start: improving literacy and numeracy. The report of the Working Group chaired by Sir Claus Moser*. London: Department for Education and Employment
- DFEE (2001) *Skills for Life. The national Strategy for improving adults literacy and numeracy skills*. London: Department for Education and Employment
- DfES & FENTO (2002) *Subject Specifications for teachers of adult literacy and numeracy*. London: Department for Education and Skills.
- Griffiths, Graham (2004) ‘The Spectre of the Sub Specs : A review of level 4 training for adult numeracy specialists’ in *Numeracy Briefing* Issue 2 2004
- Stone, Rachel (2004) ‘How to teach how to teach numeracy’ in *Numeracy Briefing* Issue 1 April 2004

Workshop

What Should Pedagogy for Adult Numeracy Consist of? Or It Ain't What You Do but the Way That You Do It

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The current context of professional development of adult numeracy teachers in the UK gives a welcome opportunity to discuss what should a training programme for teachers consist of. We are, at the moment, one of the parties involved in the revision of the Subject Specifications for adult numeracy and the content of an appropriate pedagogy for adult numeracy is on our minds.

In writing the draft documents that we have so far produced we have been developing our thinking of what is appropriate in such a document and how it may be used. Exemplifying general teaching and learning standards, such as 'apply appropriate initial assessment procedures' does not appear to be a positive move forward. In our view it is more sensible to start from what appears to be specific to adult numeracy learning and teaching. Another issue for us is the level of detail of such descriptions which may produce no particular disagreement with the profession over content but may cause problems if it is expected that all descriptors need to be evidenced in some audit process.

We argue that any document which purports to describe a pedagogy for adult numeracy will be situated in a particular time and context and should be thought of as indicative rather than definitive. It is our view that a pedagogy of adult numeracy is not a static object but a dynamic entity that should be constantly revisited and revised, and that any description of it should be understood as a time constrained, best shot that should be tested and subjected to serious critique. We welcome such critique and look forward to discussions that move forward the development of teachers.

In recent years there has been a sea-change in teacher training in England for adult language, literacy and numeracy. The professional development of teachers has come to the fore in many countries including the UK. International perspectives on these have been discussed in a number of places. In mathematics education, the US has seen a special edition of the CBMS / AMS / MAA publication, *The Mathematics Education of Teachers* (2001), the UK has seen a lot of work in primary school teachers education in issues of the journals of BRSLM (<http://www.bsrlm.org.uk/>) and AMET (<http://www.amet.ac.uk/>). In Australia, the National Staff Development Committee for Vocational Educational and Training has developed some extremely useful and influential training packages and materials for adult numeracy. In addition Maguire and O'Donoghue (2004) and Maguire et al (2004), have discussed some issues of professional development of adult numeracy teachers in Eire, Denmark and the UK.

In many ways the current UK context offers some optimism for the development of adult numeracy education. The requirement that all new teachers should be appropriately trained, and the existence of targets to encourage all teachers to take part, has created the space for real development in adult numeracy education. Indeed, the majority of participants on programmes run by the LLU+ have expressed some very positive views related to their training (see Griffiths and Kaye (2004)). Nevertheless, the same learners also expressed some concern over the lack of particular elements of content of these programmes, namely, an adult numeracy pedagogy.

Some key questions have occupied our thoughts: what skills and knowledge are required of adult numeracy teachers and how should they be covered in teacher education programmes?

The chart below highlights some key stages in the evolution of the attempt to standardise training and professionalise the post compulsory sector in England and Wales.

Year	Development
Up to 2001	No compulsory teacher training qualification required to teach numeracy in the post-16 sector although a small selection of short specialist programmes are available (e.g., C&G 9281, 9285)
2001	Success for All – government strategy requires all adult/further education teachers to hold or be working towards a full certificate in education (or post-graduate CertEd). Approved qualifications must meet national standards for teaching + supporting learning
2002	Numeracy, literacy teachers now required to demonstrate that they meet set specifications in their subject area, in addition to a generic teaching qualification
2006	Generic teaching standards and subject specifications under review for implementation in September 2007

While efforts to raise the professional status of numeracy teachers are welcome, the attempt to do this via the standardisation of programmes causes some concern. Following the introduction of the Subject Specifications for adult numeracy teachers in 2002 (DfEE (2001)), it soon became apparent that making teachers ‘do some hard sums’³⁷ and giving

³⁷ It was mentioned to the authors that some of the original subject specifications were included to ‘satisfy the professors of hard sums’ i.e. meaning that some mathematicians would be concerned if a certain level of mathematics was not included. Clearly, something was missing. At LLU+ the feedback from our own teacher training programmes was that while the course sessions were fun and participants were exposed to imaginative variety of teaching methods, they did not feel they were learning as much as they would have liked that would be useful to them in the numeracy classroom. To this end, we began enriching our programmes on offer with opportunities to explore mathematics and numeracy at a basic level and to discuss and evaluate ways to teach it. Therefore, when a request came from the

them some background information on personal and social factors affecting learning was not really equipping them to teach their subject. Aside from the evaluations from our own trainees, this concern has been documented by Office for Standards in Education (Ofsted) (Ofsted (2003)) and by the National Research and Development Centre for adult literacy and numeracy (NRDC) (Lucas et al (2003)).

The recognition that general pedagogical skills and knowledge and subject knowledge need to be accompanied by subject pedagogical knowledge has led to an integration of the generic standards for teaching and the specific skills and knowledge that it is felt numeracy practitioners need to teach their subject. LLU+ is playing a role in this revision and this has led us to consider the following questions:

- What do we mean by a pedagogy for adult numeracy?
- What should be in it?
- What role should it play in teacher training?
- And who decides?

The purpose of this paper is to explore the issues raised by these questions.

Adult Numeracy Pedagogy—What Does It Mean?

Dictionary definitions of ‘pedagogy’ include the following:

- pedagogy noun [U] SPECIALIZED the study of the methods and activities of teaching Cambridge <http://dictionary.cambridge.org/>
- Ped-a-go-gy ... [uncountable] formal the practice of teaching or the study of teaching Longman <http://www.ldoceonline.com/>

The following is a definition from Wikipedia:

Pedagogy is the art or science of teaching. The word comes from the ancient Greek Paidagogos, the slave who took children to and from school. The word "paida" refers to children, which is why some like to make the distinction between pedagogy (teaching children) and andragogy (teaching adults). The Latin word for pedagogy, education, is much more widely used, and often the two are used interchangeably.

Nevertheless to many, pedagogy is a slippery concept, one that maybe always existed but was never fully defined. The use of the word ‘art’ implies a creative process. ‘Science’ and ‘study’ suggest a publicly accepted and researched (but not necessarily static) body of knowledge, while ‘practice’ indicates a set of skills, with echoes of apprenticeships and mentors.

In the arena of teacher education, Shulman (1987) introduced the term ‘pedagogical content knowledge’ in a paper that investigated a number of elements that might be described as pedagogy. Shulman was making a distinction between the practical issues of methods and

government Department for Education and Skills (via the London Strategic Unit) for us to draft a paper outlining a pedagogy for adult numeracy, we had lots of ideas to put into it. This then became a working document which has led into our contribution to the current revision of the subject specifications for numeracy teachers.

techniques and the theoretical perspectives and rationale for those techniques. The author also emphasised, and continues to do so, the subject specific nature of teaching and learning.

I think that there is a great deal to be learned from the generic approaches. But at the same time, I've been struck by how incomplete these programs are and how much they leave unexamined that is absolutely essential to improving teaching. Teachers never teach something in general -- they always teach particular things to particular groups ... in particular settings. (Shulman in interview
<http://www.nsdc.org/library/publications/jsd/shulman131.cfm>)

The importance of context and how it might affect both what is taught and how it is taught gives rise to the following problem: it is impossible to define a body of knowledge and skills that would address every single teaching context that a numeracy practitioner might find themselves in. Therefore some generalisation is inevitable. But is it possible to distil those skills and knowledge that will equip a teacher for every possible situation?

A further issue is the nature of the difference between a pedagogy for adult numeracy and a curriculum for trainee numeracy teachers. This is particularly challenging if you believe, as we do, that a pedagogy for numeracy should not be an immutable set of standards and content, a 'cure-all' that is applicable to all circumstances. There are parallels here with epistemologies of mathematics itself. If one takes a fallibilist perspective, i.e. that mathematics is invented rather than discovered, and as such is socially constructed, time and place-dependent and subject to change and revision, then the art, science, study or practice of teaching it must be similarly open to critique and revision. Furthermore, if pedagogy, like mathematics, is invented rather than 'out there' in some sense, then the fact that it is created from a particular perspective and a particular culture should be made transparent. In other words, any pedagogy should be itself reflexive, inasmuch as the very critique of it should form a part of it. This gives rise to the second and third questions outlined in our introduction. What exactly is it that should be in a pedagogy for numeracy, and how does it relate to what we want numeracy teachers to do or be able to do during and by the end of their training?

Adult Numeracy Pedagogy—What Should Be in It?

One possible starting point for a consideration of pedagogy content is the schools teacher training curricula for primary and secondary school mathematics in England. Each curriculum is structured in three sections:

- A. Pedagogical knowledge and understanding required by trainees to secure pupils' progress in mathematics
- B. Effective teaching and assessment methods
- C. (C)Trainees' knowledge and understanding of mathematics.

Some examples from the Secondary Mathematics ITT curriculum follow (see TDA website <http://www.tda.gov.uk/upload/resources/doc/a/annexg.doc>):

Section A

2. Trainees must be taught that pupils' progress in mathematics depends upon them teaching their pupils:

e. to be accurate and rigorous, including the importance of :

using mental and written methods to give approximate answers to computations, *e.g. to make a mental approximation prior to computations done on a calculator.*

Section B

7. Trainees should be taught to plan mathematics teaching, identifying the knowledge, skills and understanding which pupils are to acquire, and build on and, where appropriate, relating them to other areas of mathematics, including:

- i. giving sufficient attention to oral and mental work;
- ii. making effective use of purposeful enquiry within mathematics;
- iii. consolidating and practising skills on a regular basis; **etc.**

And the Primary Mathematics ITT curriculum as described in the 4/98 Circular (DfES (1998)):

Section A

3. Key aspects of mathematics underpinning progression

In order to understand how to develop pupils' mathematics, all courses must ensure that trainees know and understand the following key aspects of mathematics. They must be taught how and why the different elements work, how they are connected and how they underpin pupils' progress in developing understanding of, and skills in, mathematics.

a. *Structures and operations*, including:

- i. the structure of number *e.g. order and size*
- ii. the conceptual links between different aspects of number *e.g. place value, zero, fractions, powers of ten, and how the relationship between these provides a conceptual framework for decimals; etc...*

Section B

6. As part of all courses, trainees must be taught:

- a. how to teach accurate and rapid **mental calculation**, through ensuring that pupils:
 - i. identify and use the properties of number and the relationships between them: size (including estimation and approximation), order and equivalence;
 - ii. understand the operations of addition, subtraction, multiplication and division;
 - iii. have instant recall of number facts, including multiplication tables;

Section C

13. Subject knowledge and understanding

<p>As part of all courses, trainees must demonstrate that they know and understand:</p> <p>b. mathematical reasoning and proof</p> <ul style="list-style-type: none"> • the correct use of $=$, \equiv, \Rightarrow, \therefore; • the difference between mathematical reasoning and explanation, as well as the proper use of evidence; • following rigorous mathematical argument; • familiarity with methods of proof, including simple deductive proof, proof by exhaustion and disproof by counter-example 	<p><i>To underpin the teaching of the Key Stage 1 and Key Stage 2 programmes of study, including:</i></p> <p><i>for example:</i></p> <ul style="list-style-type: none"> • demonstrating and checking a particular case; • the dangers of drawing conclusions after an event has occurred a few times; • recognising the difference between something that happens occasionally and something that will always happen; • using experimental evidence to determine likelihood and to predict; • proving, for example, that numbers divisible by 6 are also divisible by 3 (deduction); • proving, for example, that there are only 11 unique nets of cubes (exhaustion); • disproving, for example, that any quadrilateral with sides of equal length is a square (counter-example)
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The following points are of interest:

The first is that the division of teaching and assessment methods from pedagogical knowledge relating to mathematics follows the distinction that Shulman makes (1987). In both curricula there is an attempt to identify those aspects of pedagogical knowledge and understanding that are specific to the teaching and learning of mathematics, while in Section C there is an attempt to highlight those aspects of mathematics knowledge and understanding that are relevant to the teaching of it at Key Stages 1 and 2. This feels much more useful than the some of the content of the Subject Specifications for Adult Numeracy Teachers, where it seems there has been little or no attempt to relate mathematical knowledge with numeracy pedagogy. However, in the schools documents there appears to be some lack of clarity in the decisions made regarding what goes in what section. This will be returned to later.

Neither curriculum claims to be comprehensive, and initial teacher training providers are expected to include additional content of their own devising on their courses. Nor is either curriculum claiming to be a course model, so, for example, it is not expected that sections A, B and C above would be taught discretely. In the Secondary School Mathematics document (for those teaching 11-18 year olds), section C is minimal, as it is expected that trainees will have covered a large amount of mathematics during their degree programmes (although it is suggested that an audit of mathematics skills is carried out).

The terminology used causes us some concern: the use of the words ‘required’ and ‘secure’ in the title of section A implies that there is a fixed and defined set of knowledge and understanding that will guarantee the progress of pupils (and as if student progress itself is a linear process that can be ticked or not ticked on a checklist). Similarly, we question the assumption in the title of Section B that teaching and assessment methods can be effective in themselves. Even if a measure of effectiveness could be agreed upon, surely that would depend on where, when and how such teaching and assessment methods were utilised. The skill of the teacher comes in choosing what approaches to employ and when, and the knowledge of a wide range of methods in the first place.

Finally, the use of the words ‘should’ and ‘must be taught’ throughout make us wonder why it is felt that such a level of prescription is necessary. Certainly in the UK, while teacher training qualifications have been required for school teaching, as is noted above, it is only more recently that the government has attempted to describe what should be in such programmes. Indeed, the circular 4/98 which describes the mathematics and pedagogy that primary school teachers should all be aware of, did not produce a particular backlash although there is some evidence that providers were concerned with the workload involved in evidencing skills.

... the subject knowledge requirements of the Standards are very demanding, but there is a very great latitude in the way that they are being interpreted, particularly in relation to the evidence institutions expect tutors to produce for their own internal purposes or for Ofsted inspectors, who sometimes have a rather narrow content based focus. In both cases we are up against the obsession that the system has at all levels with summative assessment, which runs counter to all the evidence that formative assessment is much more effective in raising standards. We must constantly counter the arguments for more testing by providing evidence that there are other much more effective ways of raising standards, both in teacher education and in school mathematics. (French (2003))

The proceedings of the British Society for the Research in the Learning of Mathematics (BRSLM) have many references to the professional development of primary school teachers (eg Rowland et al (2003)) yet there is no suggestion that the knowledge and skills included in the circular were problematic to teacher educators. We suggest that it is likely that the content of the circular had developed from practice and had little content that was considered controversial. This is in stark contrast to the specifications for adult numeracy which appear not have developed from practice and have an emphasis on higher level personal mathematics skills that do not appear to have much direct relevance to classroom teaching. We also question the focus in teacher training programmes on formal assessment of trainees’ personal mathematics skills, possibly at the expense of exploring the related pedagogical issues.

It could be argued that one of the most important aspects of an ITE course should be about helping students to gain an understanding of some of the “big ideas” in mathematics. This is difficult to achieve if monitoring the acquisition of a large amount of mathematical content becomes the dominant feature of all courses. (Harries and Barrington (2001))

Kaye (2006) highlights those aspects of mathematics and numeracy that are not focussed on discrete skills, such as historical and cultural perspectives. Such perspectives on the nature of mathematics also need to be reflected in any account of how the subject should or could be taught, and as such, have an important place in numeracy teacher education.

The difference between essential and useful knowledge and skills for trainee teachers is a thorny one, and is illustrated by the example given below. On a course we run at LLU+ for experienced numeracy practitioners training to be teacher trainers, we designed an activity to help them to try to identify and classify the pedagogical and subject knowledge, skills and understanding that it might be useful for trainee teachers to have in the curriculum area of fractions.

We came up with a list of elements, some taken from textbooks on primary school mathematics teaching (Mooney et al. (2002)) and others arising from observations of trainee teachers that we had carried out on previous teacher training programmes, and asked the

trainee teacher trainers to classify them according to three headings that were similar to those given in the schools curricula above. Our own classification was as follows:

Teaching Fractions—Examples of underpinning skills and knowledge for trainee teachers

Subject knowledge
<ul style="list-style-type: none"> • Definitions of a fraction, e.g. part of a whole, a comparison between a subset and a whole set, the point on a line between 2 whole numbers, the result of division of whole numbers, comparing the sizes of 2 measurements or sets of objects, as a proportion etc • Conceptual understanding of operations of fractions (e.g. what does $\frac{3}{8} \times \frac{4}{5}$ actually mean?) along with the algorithms for these operations • Alternative methods for finding fractions of quantities (e.g. $\frac{3}{4} \times \frac{100}{1}$ or $100/4$ then \times by 3) • The history of fractions and their application in real life • The relationship between fractions, decimals and percentages and different ways of calculating these equivalencies • The significance of being able to manipulate fractions in algebra
Pedagogical knowledge
<ul style="list-style-type: none"> • Typical learner errors and misconceptions relating to fractions and their sources • Possible progression (e.g. different models of what to teach in what order) • When to use ‘deep’ or ‘surface’ approaches (simply teach the algorithms or focus on conceptual understanding?) • When to use holistic or sequential approaches (e.g. use a fraction wall to compare fraction sizes or find the lowest common denominator and convert to equivalents) • Factual knowledge of elements and sample activities in the Core Curriculum relating to this area • Possible contexts in which to teach fractions • Rationale for teaching fractions • Research related to teaching and learning fractions • Analyse the language associated with fractions • Links with other curriculum areas, e.g. measures, shapes
Teaching methods
<ul style="list-style-type: none"> • Have a repertoire of teaching and learning strategies relating to fractions and be able to select the most appropriate ones for a given group of learners, with reference to learning theories and other factors • Identify strategies for responding to common errors and misconceptions in this area • Be aware of the range of resources available for the teaching and learning of fractions and be able to select, design or adapt ones that are fit for purpose • Be able to design formal and informal assessments for learning related to fractions • Analysis of teaching activities related to fractions, including identifying underpinning theories of learning

The identification and classification of these elements was felt by those that tried it to be a useful exercise for preparing for teacher training, inasmuch as it provoked much discussion and argument over the choice and placing of the content. However, it raised many further issues regarding pedagogy and teacher training.

One was how to classify the elements given. Every time we run this activity, different groups argue over different ways to classify the content, and we ourselves find changes in our views over time. But perhaps the classification itself is not important. In a teacher-training programme, one would hope that such content would be integrated in any case, rather than taught discretely in the sections given. A more important question, perhaps, is the justification of such content in the first place. Most of the practitioners with whom we have used this exercise have agreed from a ‘common sense’ perspective that the elements listed above capture some of the skills and knowledge that are useful for numeracy teaching. But we wonder if there is a case for more research in this area in order to provide a rationale for such content.

Another issue is that the list is by no means comprehensive (and is in fact a shortened version of a much longer list). Does this matter? Perhaps the aim of a pedagogy/teacher training curriculum should be that it initiates a process rather than attempts to cover everything there is to know about teaching a particular topic or area. However, the very model itself on which the above table is constructed is questionable. Why choose fractions as a discrete entity? Should ‘fractions’ be taught as an isolated topic? In selecting a numeracy curriculum area in which to exemplify subject and pedagogical knowledge, we have unintentionally prescribed the very thing we wished to avoid, namely that curriculum areas are taught in isolation from each other. There is some attempt to rectify this by the inclusion of elements such as ‘links with other curriculum areas’ and ‘possible contexts in which to teach fractions’, but nevertheless, there are still many assumptions apparent in the construction of such a document. Also missing, for example, is the diagnosis of existing learner knowledge, experience and understanding of this area, and how to utilise that in further learning, or teaching by theme or a wider, shared, authentic purpose rather than treating the curriculum as separate sets of skills that are to be acquired in a linear fashion.

This highlights the issue of how to construct an outline of a pedagogy for numeracy. A model that starts with generic skills (e.g plan a lesson) becomes meaningless unless accompanied by exemplification for numeracy, which carries with it the danger that the exemplification itself is seen as prescriptive. But starting with the subject content (in this case, fractions), brings with it its own dangers, such as the implication that this topic be treated as distinct, or the lack of emphasis on the importance of starting from where the students are.

In our own pedagogy document, commissioned by the London Strategic Unit, we used the following section headings:

- Key areas and principles
- Examples of underpinning theory and policy
- Exemplification in numeracy teacher training
- Exemplification in numeracy classroom.

Some examples are given in the table below, with some further examples in appendix 1.

Key Areas (and principles)
Language and numeracy

<ul style="list-style-type: none"> • Mathematical thinking can be developed through the use of language. Teachers therefore need strategies for developing learner use of numeracy language, for moving key terms and phrases from passive to active vocabularies, for promoting mathematical discussion in the classroom and for developing the numeracy communication skills of presentation, justification and explanation • Issues related to the learning of mathematics in English as Another Language, and the implications of this for teacher and curriculum
Examples of underpinning theory and policy
Mathematics Register (Fullerton 1995); Laborde (1990); Pimm (1991)
Exemplification in numeracy teacher training
<ul style="list-style-type: none"> • The use of language and literacy teaching strategies to help learners develop their numeracy vocabulary • Exploring strategies for promoting discussion and communication in the numeracy classroom Strategies and resources for using numeracy as a vehicle for literacy and language development
Exemplification in numeracy classroom
Writing key terms on the board and asking learners to repeat them (e.g. ‘width’, ‘metres squared’) Activities which encourage the active use of numeracy terms such as one learner guessing a shape by asking another learner to describe its properties. Eliciting the informal maths understanding of learners and translating this into formal maths notation.

The aim was to capture the skills that numeracy teachers use when teaching, and the background knowledge needed to support those skills. As with the design of the activity about the skills and knowledge required to teach fractions outlined above, several questions arose during the construction of this document. One was the decision about what to put in and what to leave out. Another was how to justify our choices. Wherever possible, we included relevant rationales and links to relevant theory research policy documentation. However, it is impossible to be truly objective in terms of coverage. We, as much as anyone, have our own ideas about what ‘ingredients’, in terms of skills and knowledge, a numeracy teacher needs. It was therefore important to be transparent about our standpoint. And because of this, we were at pains to emphasise that the pedagogy document was a ‘working’ one, not one set in stone.

Some of the content arose from issues experienced during the considerable number of numeracy lesson observations that we as a team have carried out with our course participants, for example, teachers only using one method of calculation and not being open to methods that learners themselves may already use. However, this gave rise to the issue of how to avoid being too prescriptive. As mentioned earlier, we did not want to produce a ‘how to teach numeracy’ manual, mainly because we believe that there is no single ‘correct’ way to teach numeracy. Our aim, therefore, was to produce a broad spread of areas of skills and knowledge that it would be useful for numeracy practitioners to have in order to be able to

select the most appropriate teaching methods for their particular contexts at this particular time.

What role should a pedagogy for adult numeracy play in teacher training?

Training courses for primary and secondary school mathematics teachers that we have been involved with, either as trainers or participants, have involved spending considerable amounts of time ‘playing’ with numbers and mathematical concepts and exploring the implications of teaching them. Up to now in England there has been very little of this on teacher training programmes for adult numeracy, the focus being on developing trainees’ personal mathematics skills at a higher level, and on ‘generic’ teaching skills. Such ‘generic’ skills include; finding out about institutional support structures for students with specific needs; describing and evaluating initial assessment procedures; discussing summative assessment and so on. This is partly because on most general teacher training programmes in the post-16 sector in England, numeracy teachers are thrown in with, for example, construction teachers, hairdressing teachers, business administration teachers and others, and so the pedagogical skills looked at can only ever be generic ones.

With the revision of the national standards and specifications, the emphasis is now on considering how general teaching skills can be interpreted in a numeracy context, with the idea of increasing the number of teacher training programmes on offer especially for numeracy practitioners. This gives rise to the question of how to plan and run such programmes, and in what ways a pedagogy for adult numeracy can be integrated.

Some approaches that we have taken to integrating numeracy teaching-related issues into our training programmes at LLU+ are as follows:

- Modelling activities and methods that can be used (or adapted for use) in numeracy teaching, with evaluation by trainees.
- Exploring mathematics at a basic level and discussing teaching implications. For example looking at connections between topics and curriculum areas and how this might affect course planning.
- Setting up numeracy teaching planning activities for trainees to carry out (e.g. plan an activity to introduce the topic of volume), with appropriate support, intervention and feedback from the trainer (e.g. helping trainees in selecting a suitable context based upon learner needs and experience).
- Examining examples of numeracy lesson plans and finding ways to enhance or improve them, for example, adding in language based objectives and appropriate activities to cover these.
- Considering case studies of particular learners or groups of learners and how they might be supported in learning numeracy.
- Evaluating resources, including real and ICT-based materials, and discussing how they may be used or adapted. This may include critiquing the supposed ‘level’ that a resource is pitched at and how it may be scaffolded or changed to accommodate students working at other levels.
- Analysing common learner ‘errors’ and discussing how to respond to them.
- Role-play, for example peer-teaching.
- Looking at mathematics from historical or cultural perspectives and discussing how this might impact on the teaching and learning of it.
- Engaging with relevant theory and research relating to the teaching and learning of numeracy and mathematics.

- Reflecting on own practice.

Although we try to avoid being too prescriptive, a significant minority of our course participants want just that – an all-purpose ready-made ‘how to teach’ bundle of ideas for activities that they can take wholesale and use with their students. We try to encourage trainees to evaluate any teaching methods that we use and consider how they would adapt them for their own contexts. It is noticeable, however, and perhaps understandable, that newer teachers simply want to apply any new methods acquired much as they are. Those who are more experienced are more apt to reflect on alternative approaches and their applications in different contexts.

Does this mean that training programmes which are packed full of ‘ideas for teaching numeracy’ are appropriate for beginner teachers? The danger is that by demonstrating, for example, the use of clock faces to introduce halves and quarters, it may appear that we are advocating that this is the only way to introduce halves and quarters. This suggests then that the modelling of teaching methods and activities should therefore be tempered with critical evaluation and consideration of alternative approaches. However, it is important to recognise that professional development for teachers is a process, not a race towards an end ‘product’. The aim of teacher education, we feel, should surely be to enable practitioners to become reflective and critical regarding their own practice, and that of the teaching community as a whole, and to assist them in developing the tools with which to be critical, rather than turning out supposed ‘ideal’ teachers. This may mean trying out all sorts of ‘packaged’ methods and resources as a new teacher before being able to analyse and evaluate them in the light of experience.

Another issue is that some of the numeracy-related pedagogical content of our courses has only an indirect link with teaching, and our trainees sometimes find it difficult to see the relevance of such content, for example, a discussion of views on the epistemology of mathematics. Making explicit the rationale for the inclusion of such content (in this case, possible links between people’s views of mathematics and the ways they teach it – e.g. Ernest (1989), Lerman (1990)) is therefore essential.

Numeracy pedagogy does not only have implications for input in teacher training sessions. The roles of lesson observations and mentoring play a key part in the professional development of practitioners. By what criteria are we judging the effectiveness of a lesson? There is a danger with observations that we simply want to turn the observee into a version of ourselves. Perhaps the emphasis in observation should be less upon what the observed teacher does *per se*, but more on what their rationale is for the methods and approaches selected, and how they evaluate these afterwards. Can numeracy pedagogy here play a role in establishing such a rationale, and the criteria against which to (self) evaluate?

Conclusion

As previously mentioned, a focus on the professional development of teachers is a welcome move. While it is not expected that any set of descriptors or training programmes will work with all teachers in all circumstances, it is helpful if such work has the confidence of the profession at large. The version of training that followed the introduction of the original subject specifications for adult numeracy did not have that confidence and necessitated a change of tack. The work so far on the revisions of the subject specification shows that the profession can accept a set of descriptors, although there are concerns about how much is reasonable to evidence. For us the precise content of any specification is not that important. The development of teachers is something that will take time and, as mentioned above, no one course will ever produce the perfect numeracy teacher. Rather the professional

development of teachers is a ‘lifelong’ process and will require ongoing work. Given such circumstances it seems appropriate that whatever the redefined specifications are, we allow institutions some flexibility in interpretation and accept that over time guidance will need to change.

In addition, notions of what constitutes ‘good practice’ and an appropriate curriculum for numeracy learners will change over time. Part of any curriculum for numeracy teachers should therefore incorporate the critical skills needed to question the very content of such curricula, evaluating it against theory, research, experience and the cultural, social and political climate of the time. Cooney (2001) claims that the aim of teacher education is to develop ‘clever’ teachers who can envision a different world of teaching but operate within their own classroom constraints. We prefer to replace the word ‘clever’ with ‘critical’, but the premise remains the same. The question then remains as to what guidance teacher educators need, and whether a prescribed set of standards can in themselves achieve this aim.

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References

- Askew, M., Brown, M., Rhodes, V., Wiliam, D. & Johnson, D. (1997). *Effective Teachers of Numeracy: Report of a study carried out for the Teacher Training Agency*. London: King's College, University of London.
- Black, P. & William, D. (1998a). Assessment and Classroom Learning. *Assessment in Education*, 5 (1), pp.7-71.
- Black, P. & William, D. (1998b). *Inside the Black Box: raising standards through classroom assessment*. London: School of Education, King's College London.
- Chinn S. & Ashcroft, J. (1998). *Mathematics for Dyslexics*. London: Whurr Pub.
- Conference Board of the Mathematical Sciences (2001) *The Mathematical Education of Teachers* AMS | MAA .
- Cooney, T. (2001). Considering the Paradoxes, Perils and Purposes of Conceptualizing Teacher Development. In F-L Lin, and Cooney, T. (Eds.) *Making Sense of Mathematics Teacher Education*. (pp. 9-16), Dordrecht: Kluwer Academic Publishers.
- DfES (1998) Circular 4/98 see <http://www.tda.gov.uk/partners/ittstandards/transitional/circular498.aspx> accessed July 2006.
- DfES (2001). *Subject Specifications for teachers of adult literacy and numeracy*. Nottingham: DfES/FENTO.
- DfES (2002). *Success for All: reforms in further education and training*. London: Department of Education and Skills publications.
- Dunn R. (1984). Learning Styles: State of the Science. *Theory into Practice*, 23(1), 10-19.
- Ernest, P. (1989). The impact of beliefs on teaching. In C. Keitel with P. Damerow, A. Bishop, and P. Gerdes (Eds.) *Mathematics Education and Society*, Paris, UNESCO, 1989: 99-101. Expanded version reprinted in P. Ernest (Ed.) *Mathematics Teaching: The State of the Art*, London, Falmer Press, 1989: 249-254. Translation reprinted as ‘L’Impatto del Modeli Mentali sull’Insegnamento della Matematica’ in *L’Educazione Matematica*, 3 (2), 1992, 65-70. Reprinted in Bloomfield, A. and Harries, T. Eds (1994) *Teaching and Learning Mathematics*, Derby: Association of Teachers of Mathematics.
- French, D. (2003). Subject knowledge and pedagogical knowledge. *Mathematics Education Review* No16. AMET <http://www.amet.ac.uk/mereview/16-Apr-2003.html>
- Fullerton, O. (1995). Who Wants to Feel Stupid All of the Time? In P. Rogers & G. Kaiser (Eds), *Equity in Mathematics Education – Influences of Feminism and Culture*, (pp.37-48). London: The Falmer Press.
- Gal, I. (1998). *Numeracy Development: A Guide for Adult Educators (Literacy: Research, Policy & Practice)*. London: Hampton Press

- Griffiths, G and Kaye, D. (2004). A review of level 4 training for adult numeracy specialists. *Proceedings of the 11th International Conference on Adults Learning Mathematics (ALM-11)*
- Harries, T. and Barrington, R. (2001). An exploration into the mathematics subject knowledge of students on QTS courses. *Mathematics Education Review No14*. AMET
<http://www.amet.ac.uk/mereview/14-Sep-2001.html>
- Haylock, D. (2005) *Mathematics Explained for Primary Teachers*. Paul Chapman
- Ifrah, G. (1994) *The Universal History of Numbers*. Harvill Press
- Knowles, M., Holton, E & Swanson, R (1998) *The Adult Learner* Butterworth Heinemann
- Kaye D (2006) Numeracy Briefing
- Laborde, C (1990) 'Language and Mathematics.' In P. Nesher & J. Klipatrick (Eds) *Mathematics and Cognition: A research Synthesis by the International Group for the Psychology of Mathematics Education*, (pp.53-69). Cambridge: Press Syndicate of the University of Cambridge.
- Lerman, S. (1990). Alternative Perspectives of the Nature of Mathematics and Their Influence on the Teaching of Mathematics. *British Educational Research Journal*, 16(1), 53-61.
- Mooney, C., Ferrie, L., Fox, S., Hansen, A., Wrathmell, R. (2002). *Primary Mathematics, Knowledge and Understanding*. Exeter: Learning Matters.
- Lucas, N. et al. (2003). *New initial teacher education programmes for teachers of literacy, numeracy and ESOL 2002/03: an exploratory study*. London: NRDC
- Ma, L. (1999). *Knowing and Teaching Mathematics: teachers' understanding of fundamental mathematics in China and United States*. Mahwah, NJ: LEA.
- Macguire, T., Lindenskov, L., and Seabright, V. (2004). Getting Adult Numeracy Tutors to Engage. In Professional Development – International Approaches *Proceedings of the 11th International Conference on Adults Learning Mathematics (ALM-11)*
- Maguire, T and O'Donoghue, J. (2004). A model of professional development for tutors of adult numeracy. Background discussion paper submitted to TG6 at 10th International Congress on Mathematical Education. Copenhagen, 4-11 July 2004.
- OFSTED/ALI (2003) *Thematic review Literacy, numeracy and English for speakers of other languages: a survey of current practice in post-16 and adult provision (HMI 1367)*. London: Ofsted.
- Pimm, D. (1991). Communicating mathematically. In D. Durkin & B. Shire (Eds.) *Language In Mathematical Education – Research and Practice* (pp.17-23) Buckingham: Open University Press.
- Rowland, T., Huckstep, P., and Thwaites, A. (2003). Observing subject knowledge in primary mathematics teaching. *Proceedings of the British Society for Research into Learning Mathematics*, 23(1), pp.37-42.
- Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57 (1), 1-22.
- Skemp, R. (1993 edition). *The Psychology of Learning Mathematics*. London: Penguin.
- Swan M (2005) http://www.ncetm.org.uk/files/224/improving_learning_in_mathematics.pdf
- Vygotsky, L. (1994). *Thought and language*. Cambridge: MIT Press.

Appendix 1: Examples of content from the working document describing a pedagogy for adult numeracy

Key areas (and principles)	Examples of underpinning theory and policy	Exemplification in numeracy teacher training	Exemplification in numeracy classroom
Assessment in numeracy What counts as evidence in numeracy assessment (and for what purpose)? The role of informal assessment in numeracy such as observation of tasks or discussion is essential in terms of subsequent choices made with regard to teaching approaches. Using results to inform planning and using feedback to promote further learning in numeracy are also areas which should be explored in some depth, as there can be missed opportunities for numeracy learning through the way that feedback is given.	M4L Stage 2 Pathfinder on formative assessment Black and Wiliam (1998) Iddo Gal (1998)	Evaluation of written or ICT based numeracy assessments in terms of validity. Exploration of other forms of assessment in numeracy, and of ways to use information gained, e.g. through role play, or watching then pausing videos of lessons to discuss what learning is taking place and what the teacher could do next.	Using mini whiteboards to ascertain learning or understanding – e.g. draw a picture of $\frac{2}{5}$ Using feedback from numeracy assessment to promote further learning – e.g. ‘Does your answer seem sensible?’ ‘Explain your method. Can you think of a different way to work this out?’ Using open ended questions to ascertain understanding of concepts, e.g. if this is $\frac{1}{4}$ then draw what you think the whole shape might look like.
Understanding of numeracy curriculum areas and the connections between them A thorough ‘at homeness’ with the Adult Numeracy Core Curriculum areas and the possible progression for each area beyond Level 2 (e.g. how some areas of Number lay the foundations for algebra). A wider understanding of the contexts and applications of curriculum topics, both at Core Curriculum and higher levels.	Maths Explained for Primary School Teachers – D. Haylock (2005) (Paul Chapman) Primary Mathematics – Teaching Theory and Practice – Mooney et al (Learning Matters) Other maths texts at higher levels	Exploring concepts at Core Curriculum Level, e.g. using squared paper (or folding pieces of paper) to explain or interpret what is meant by $\frac{2}{5} + \frac{1}{3}$, $\frac{2}{5} - \frac{1}{3}$, $\frac{2}{5} \times \frac{1}{3}$ and $\frac{2}{5} \div \frac{1}{3}$.	Being familiar with a range of methods for, say, long multiplication, and thus being able to show flexibility in the teaching of this and start with the methods that learners feel most comfortable with. Being better able to explain/discuss place value and the role of zero with learners as a result of teacher exploration of non-positional number systems
History of mathematics Knowledge of different number systems, systems of measurement and their development over time.	Arguments from the British Society for the History of Mathematics, e.g. critiques related hierarchy Georges Ifrah (1994)	Exploration of different topics	Discussing with learners a brief history of Number, in order to situate current curriculum content.

Professional Development of Workforce the Adult Numeracy Teaching

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During the past 5 years, there has been a growing awareness of the need to focus attention on the development of those who teach adult numeracy. However, our theoretical frameworks are varied and our knowledge of what works or what its impact is remains limited (See, for example, Condelli et al, 2006). This presentation explores some of the issues involved in moving that agenda forwards for teachers of adult numeracy. (In using 'numeracy' I include the full range including mathematics).

The Workforce

The Adult Numeracy Teaching Workforce (ANTW) across the world is diverse, has entered the adult education and training workforce from a range of backgrounds from educational to vocational to simply having concerns for the socially excluded. In many countries, there is no agreement about minimum criteria for doing the job and across many continents those doing it are paid very little or indeed are volunteers. The Mackay et al (2006) study into the professional development needs of the sector in Australia acknowledged the lack of reliable data on the workforce itself, let alone its level, qualifications or development needs. This in itself is a major challenge.

Studies of the characteristics of the workforce (Mackay et al, 2006; Young et al, 1995; Lucas et al 2004) indicate that those involved in Adult Education programmes are predominantly part-time, ageing females with diverse qualifications and experience. Interestingly in the Lucas et al study, of a subgroup of the workforce aiming at an HE level qualification, most were already qualified as teachers, with only 12% having no or minimal training.

Adult Numeracy Teaching – A Semi-profession

The adult numeracy teaching workforce could not yet be described as a profession and views about professionalism differ. Professions are occupations that usually require extensive training and the study and mastery of specialized knowledge. Many have established regulatory Professional Bodies. They have codified their conduct, and what they require for entry into their profession. Some of these codes are detailed with a strong emphasis on their particular area or expertise. Most have an overlap of ethics and quality

standards. Typically many professions *require* members to continue to engage in professional development activity and ongoing professional learning in order to retain their membership of that profession and their ‘licence to practice’.

There are of course both benefits and problems associated with the formation of professions. On the positive side, members acknowledge and use a formal knowledge base, they share an explicit value base and make a commitment to high standards of practice with an acceptance of accountability for their actions.

On the negative side, membership of a profession can be exclusive and elitist, with expectations of special perks and privileges, the creation of barriers to those not members of the profession, and potentially a failure to identify with clients from a different social background.

Historically in technologically advanced countries, vocational and adult teachers have been held in significantly lower esteem than school teachers. For example, school teaching has secured itself as a graduate entry profession across Europe, whereas the same is not true of vocational/adult teaching. In part it is a result of the different contexts in which individuals work, with much of vocational education being provided historically from within the workforce itself.

The current workforce reform movement in English schools has led to the creation of new roles, including the teaching assistant role. The teaching assistant role is gradually moving from a status of ‘volunteer mum’s army’, to that of professional – with the establishment of qualifications frameworks, career structures negotiated with employers’ bodies, and the establishment of a professional development framework. *Crucially*, teaching assistants themselves are embracing their roles as professionals in their own right, rather than merely teacher apprentices.

For adult numeracy teachers within the UK, the Institute for Learning (IfL) has established itself to play a leading role in creating a new professional status. It claims that:

The Institute is in the process of creating a new professional identity for teachers in the post-compulsory sector, based on shared values, a unified expectation of behaviour and conduct and a career long commitment to professional development. This response to the White Paper draws heavily on the Institute’s experiences including;

- Developing the framework for the conferral of Qualified Teacher Learning and Skills (QTLS) professional status.
- Creating the mechanism for the award and renewal of the Licence to Practise.
- Establishing the requirements for remaining in good standing.

- Designing a supportive model of continuing professional development, linked to the requirements for remaining in good standing and the renewal of the Licence to Practise.
- Developing a code of values and practice to underpin professional identity.
- Setting up the technical architecture (database, secure hosting, online tools and CPD portal) for the registration of teaching practitioners and relationship management.

Alternative approaches to professionalism can be found in both the literature about expert practice and in the literature about communities of practice. Early literature about expert practice focuses on the development of cognitive skills, the development of complex and rapid memory recall and the key function that practice plays in developing expertise. Later research acknowledged the importance of self-regulation and personal belief systems in development as an expert teacher. Such research shows that experts continuously:

- Set new goals and design new tests for themselves
- Look for new ways of doing things and monitor the work setting
- Engage in deliberate planning for flexibility

As participants in a ‘community of practice’, individuals can be viewed as initially peripheral to that community but, as they engage in discourse and practices of that community, moving more to the centre of that community they become expert. Crucially, in this approach to expert practice, members have to interact and mutually engage in the discourse and activities of that community if learning and development is to take place. They have to be active participants in the community and construct their identities in relation to that community for expertise to grow (Wenger, 1999).

In these interpretations of professionalism, qualifications are not seen as a prerequisite, and ‘official’ membership is not required, rather membership is something the individual chooses and progresses as s/he becomes more expert or more engaged in the particular discourses and practices of that community.

The IfL (2007) suggests that adult numeracy teachers operate within a context of *dual professionalism*. They define personal development as important for fulfilling social expectations professional development as important for fulfilling economic expectations. They go further and suggest that current funding regimes for developing teacher expertise tend to focus on the latter at the expense of the former. In articulating this concept, the IfL highlights some of the tensions posed by the varying interests of different stakeholders in the professional development of the ANTW.

Stakeholder Interests in Professional Development

Governments across the world are concerned about levels of adult literacy and numeracy and almost all now have a range of adult numeracy education provision, in colleges, in the workplace and in the community. However, as O'Donoghue & Maguire (2004) identified, few countries yet have a comprehensive national provision for the training and development of the adult numeracy teaching workforce.

National interest in raising the quality of numeracy and mathematics teaching stems from an imperative to improve skills of the population which in turn is driven by a desire to improve economic advantage and performance. Increasingly governments acknowledge the need to improve social cohesion and also emphasise adult numeracy within a lifelong learning approach. The European Commission, for example, launched a work programme in 2002, on "the open learning environment, active citizenship, equal opportunities and social cohesion" (European Council of Ministers, 2002). In the UK, the ALI/OFSTED report (2003) and the Smith report (2004) have accelerated the provision of training and development opportunities as part of the 'Skills for Life' Strategy. Similar activities can be seen in Denmark, Holland and Norway.

Individual Communities are largely concerned to raise the knowledge and skills of the local population to enhance both social cohesion and economic capacity. Here at ALM we have heard over the years, of a range of community based projects with such goals. The interest of individual communities in professional development of the teaching workforce is less consistent, their voices harder to hear. Cowan (2006) suggests that for communities to make progress on adult literacy promotion, 'coalition building' between organisations within that community is needed.

Employers focus on professional development as a strategic approach to increased productivity and growth, particularly within a knowledge economy. Their prime motive is to ensure that their employees have and develop the skills needed to enhance performance. The teaching workforce within such organisations is relatively small and largely provided by those whose prime work function is different. Hence, whilst companies have a strong commitment to employee development overall, this may focus much more significantly on the development of higher level skills and leadership with the commitment to development of teaching workforce within the organisation much less obvious.

In some countries, workplace professional development engagement is at particularly high levels.

Competence development has a high priority in Denmark. 9 out of 10 companies offer competence development to their employees within a year of employment
Competence development can take many forms in Danish workplaces:

- External courses
- Courses and training periods at the workplace
- Longer courses and programmes, such as MBA programmes (Danish Ministry of Science, Technology & Innovation)

Voluntary Sector Organisations such as UNESCO or the International Commission on Workforce Development³⁸ are establishing themselves in the global professional development market, through the provision of on-line training and development opportunities. In the latter case, ICWFD's mission is to redress social disadvantage:

The ICWFD is committed to bridging the digital divide in marginalized and disadvantaged sectors of society worldwide by providing training in vital job skills to reduce poverty and unemployment, enhance employability, empower youth to become productive citizens, enable entrepreneurship, stimulate broad-based economic development, and accelerate social transformation. (ICWFD, 2007)

Activists/Researchers/PD providers/ Professional Bodies have a commitment to adult maths learning, belief in the importance of capacity building through professional development, but with varying stakes in the different approaches to achieving this and with varying access to funding which in turn drives other interests. So we might view the paucity of research into development of this workforce, not because this group has no commitment to such development, rather because funding mechanisms have to date precluded such a focus. Often such bodies are key drivers for professional development and may themselves be registered as voluntary sector organisations. Again, in the UK the Joint Annual NIACE & UCU Conference in May 2007 was entitled 'Professionalising the literacy, language and numeracy workforce' and provided

“an opportunity for all staff working in literacy, language and numeracy whether part-time or full time, managers, providers and planners to hear about planned changes as well as current practices which are effective in raising standards of teaching and learning”.
(Conference flier)

Individual adult learners have a key stake in teacher development, needing the best possible teaching to improve their numeracy/mathematics skills. Like individual communities their voices are difficult to hear, and their interest may often be represented by governments or work organisations, with a consequent focus on functional numeracy (eg Wake, 2005) and

³⁸ ICWFD is a California-based 501(c)3 non-profit public charity organization.

the need to have teachers who can prepare them effectively for relevant qualifications or work skills.

Like teachers themselves, learners want to have teachers who can teach them effectively in ways and in contexts that meet their individual needs. Hence a key stake for learners is in developing the capacity of teachers to respond to their individual needs in meaningful ways.

Teachers themselves appear currently to have a limited stake in professional development. Whilst research suggests that teachers are committed, want to do a good job and want to know and learn more about their students, investment in professional development may have few other tangible rewards.

TeacherNetUK suggests that for schoolteachers:

CPD is relevant to all teachers. It is about making progress in the teaching profession—increasing teachers' skills, knowledge and understanding

For this workforce, without a profession and without obvious career pathways, the relevance of CPD may not be so obvious!

Effective Professional Development

Over the past 20 years research into teaching (as opposed to learning) has been an increasing focus of educational research, although the majority of this work has focused upon school teaching, rather than teaching within adult education.

Two foci of professional development are briefly discussed in what follows:

Development by individual acquisition of skills and knowledge

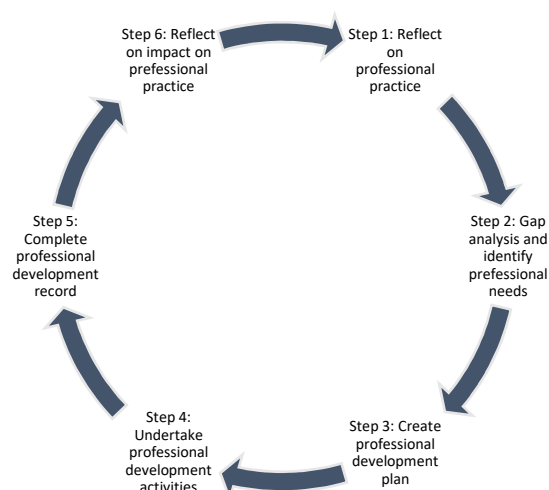
Development through participation in activity

From Adult Education literature, Knowles (1980) six fundamental assumptions about the unique characteristics of adult learners surely apply equally to the teaching workforce as they do to their students:

- The learner's self-concept.
- The role of the learner's experience.
- Readiness to learn.
- Orientation to learning.
- Need to know.
- Motivation.

Much of the literature on adult learning highlights the central role of the reflective process which is generally seen as an essential factor for constructing teacher knowledge. Schon's (1996) work on 'the reflective practitioner' laid the foundations for much focus upon this notion within teacher education and Kolb's (1984) framework for experiential learning is much used by providers as the basis for approaches to professional development.

So for example, the IfL Consultation on CPD (2006) offer the following model:



Towards a New Professionalism. From 'Establishing a Model of Continuing Professional Development for Teaching Practitioners in Post-Compulsory Education and Training' (IfL, 2006)

Tools to support the reflective process such as the maintenance of learning diaries, reflective evaluation writing, use of videorecorded/webcammed teaching sessions and the like are commonly in use across teacher development programmes. Availability of activities which meet the particular development needs of individual teachers is variable and one of the challenges for providers and a common focus for research activity is how to do this effectively.

Research and literature on Learning Styles (Sadler-Smith 2004) or multiple intelligences (Gardner 1989) would suggest that design of development or processes, need also to take account of the preferred learning styles of individual teachers, and address a range of their intelligences. Interestingly, recent thinking about leadership focuses on the way leaders use their emotional intelligence (see for example, Emmerling & Goleman 2003).

Professional development literature with a different focus centres on professional development within organisations and the concept of the learning organisation. (See for example, Argyris & Schon, 1996, and Senge et al, 2000.)

This whole organisational approach to learning underpins the approach of many private sector organisations who increasingly value the competitive advantage gained through engagement of all employees in ongoing professional development creating a transformational learning system.

Senge's five disciplines – personal mastery, mental models, shared vision, team learning, systems thinking, suggest professional development must encompass both individual and team development based on dialogue and within a shared vision.

The ideas implicit within this approach have also been extended to thinking around the growth of social capital, informal learning and lifelong learning and the educative power of organisations and groups. Field (2005) claims that the concept of social capital underpins the educational and training policies of the European Commission.

In the context of the adult numeracy teacher workforce, individual teachers can use their social capital to gain access to skills and knowledge in various ways. The literature on schooling and social capital suggests that strong networks and educational achievement are mutually reinforcing. We may conclude therefore that the greater the stock of social capital in the adult teacher workforce, the greater the capacity for mutual learning and improvement.

Such a line of argument suggests that more efforts need to be made to encourage networking amongst the workforce and a greater emphasis on organisational approaches to professional development.

Within the vocational field, the concept of apprenticeship is common and accepted as an effective way in which novices learn their trade – a model of initial development in Teacher Education used in many countries. Mentoring and coaching approaches to development has been adopted more widely within Education, not just for teaching roles. The role of the advanced skills teacher, or numeracy consultant as an expert within the school classroom context has been documented. (Brown & Coben, 2007) This suggests that adult teaching might have more to learn and research about the way in which novice teachers can learn, in situ, to become more experienced through the role of the visiting expert who can offer mentorship and coaching and direct feedback on practice.

The 'What' of Teacher Development

Teacher knowledge

From a situation where expert teachers' practices were accepted as being based on tacit knowledge and taken for granted understandings of their actions and those of their students, there has been an attempt in many countries to try to 'uncover that tacit knowledge' and to articulate what it is that people need to learn in order to become effective teachers; to articulate what good teaching is.

A broadly used, though still contested and oft modified framework due to Schulman & Grossman (1988) forms the basis of many current approaches to initial and continuing teacher development. Their model defines 7 domains of knowledge:

- knowledge of subject matter,
- pedagogical content knowledge,
- knowledge of other content,
- knowledge of the curriculum,
- knowledge of learners,
- knowledge of educational aims, and
- general pedagogical knowledge

Enhancing teacher knowledge is the focus of much professional development activity (and research) with the following not untypical:

Depending upon your philosophic standpoint, what constitutes knowledge in each of these domains, what it means for teachers to have knowledge and the ways in which they might gain knowledge—if indeed they do—is a contested debate!

Teacher practices

Teachers acquire/develop knowledge and through their practice demonstrate observable skills and behaviours.

This can be seen reflected in the articulation of ‘competences’ for teaching, expressed in many countries’ professional accreditation requirements which place an emphasis on, for example, clarity of exposition, attention to questioning techniques, approaches to engaging attention etc.

Teaching approaches and styles include three described by Askew et al (1997)

- the connectionist approach, where the teacher makes connections between different areas and representations of mathematics
- the transmission approach, where individuals teach skills mastery in discrete steps, emphasising procedure
- the constructivist approach, where the teacher scaffolds learning and asks questions to help learners articulate their meaning making as well as others which have been documented,
- discovery approach, where the teacher establishes an appropriate environment where learners discover for themselves
- mentoring and coaching approaches, where the teacher works in (usually) a one to one context with the learner supporting specific skill development and reflection

Teacher behaviours and skills, and hence attempts to change behaviours are a common focus for professional development but often embed unclosed values for particular approaches. (See for example Malcolm Swann - active teaching, TERC etc).

Teacher beliefs, values, emotions and self concept

In her book ‘Differentiated Coaching: A Framework for Helping Teachers Change’, Jane Kise (2006), puts forward a framework for teacher development based upon a conviction that the most meaningful change takes place when teachers’ beliefs, feelings and personality are taken into account. This aligns with a growing body of research into teacher beliefs and development of self-efficacy (See for example, Fang, 1996). The concept of self and role are fundamental notions within Symbolic Interaction theory which suggests that adult teachers’ role identity provides a set of behavioral expectations which motivate them to act in specific ways that are maintaining and protective of that role and hence self-esteem. Requiring teachers to change their behaviors requires them to re-evaluate their role and hence their self-concept. This may require them to resolve tensions that arise and to reconstruct their fundamental beliefs – not a fast process.

Such matters appear to be rarely addressed by professional development programmers.

Contexts for Teaching

Educators work with learners along a continuum

Informal

Formal

Conversation based	Negotiated curriculum	Set curriculum
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Much of the adult numeracy teaching workforce is engaged in teaching in contexts which are non-formal and often in situations where students have gained mathematical knowledge and skills through non-formal learning contexts.

This suggests that in designing development we need to consider the extent to which individuals are working within formal or non-formal contexts and our approaches differ accordingly. So for example, recommendations to match teacher development with curriculum needs may not be relevant in informal learning contexts.

This spectrum also applied to development activity itself and teachers may engage in a diverse range across the informal-formal continuum, with the more formal leading to specific qualifications. One challenge is how to facilitate the acquisition of the formal

qualification through engagement in informal development activity. In that context the report *Making Learning Visible* (Cederfop 2000) provides an overview of the position of non-formal learning across Europe.

The importance of development activity relating to authentic teaching experiences for the ANW is reported in well documented. This is more difficult to ensure when development takes place away from the normal workplace. The abstract vs embedded nature of numeracy is a further differentiator in context for the teacher. In some situations, teachers are preparing learners to deal with external tests which may require them to perform in abstract contexts. In others, all the numeracy may need to be embedded. Many development activities are unlikely to be appropriate for both teaching contexts.

Finally, teachers operate within different political and financial contexts. There is some evidence (Smith & Gillespie, 2007) that these constraints do have an impact on what teacher development is even possible at the outset. Certainly, the lack of serious funding for teacher development in this sector is a major barrier to progress.

What Do We Know about What Teachers Themselves Want?

Evidence about the expressed development needs of teachers is rather thin at present although what there is has some common features and shares some features with those needs expressed by school mathematics teachers, teachers in other subject disciplines and other workforce groups.

In Australia, Mackay, Burgoyne, Warwick and Cipollone (2006) investigated the current and future professional development needs of the Language, Literacy and Numeracy Workforce and considered the particular needs of three sectors – vocational trainers, specialist teachers and volunteer tutors and found that attitudes to professional development and the issues surrounding effectiveness were strongly related to the sector in which teachers worked.

Numeracy specialist and volunteer teachers viewed development of their teaching practice as the most pressing need. To improve both their current and future practice, all sectors want to learn more about managing the changing profile of learners and hence access to appropriate resources and learning materials to meet the needs of specific learner groups. They also saw information and communications technology skills development as an emerging need, use of which could help them be more effective teachers.

All groups favored face to face development activities and had a strong preference for short ‘hands-on’ training sessions given by expert facilitators together with opportunities for informal sharing of ideas with peers. Significantly, they were very concerned that they themselves had a stake in designing the development activities – a view shared by teachers in the USA (Smith et al, 2002).

The motivations of adult specialist numeracy teachers was also highlighted by the results of a recent UK study (Hudson et al, 2006). Participants gave a wide variety of reasons but these focused on

- Career development
- Developing their own mathematical understanding
- Increasing their pedagogical knowledge

More generally, a research report carried out by academics at Manchester Metropolitan University and Education Data Surveys in 2002 for DfES on teachers' perceptions of continuing professional development (CPD) found that. Key features of worthwhile CPD were considered to be

- perceived relevance and applicability to school / classroom settings
- focus on teaching skills and subject knowledge; (few took part in research, secondments, international visits or award bearing courses, although these were highly valued)

Teachers also felt there should be a better balance between meeting individual and organisational needs. In 2005, the Wellcome Trust commissioned a survey of science teachers and managers in state maintained schools in England to determine teachers' views about CPD. The resulting report, *"Believers, seekers and sceptics: what teachers think about continuing professional development"*, revealed strong support for CPD particularly to update subject knowledge.

The overall message from these (and other) studies of teacher conceptions of professional development is one where development to improve skills and performance in current role is associated with 'events' which are of high quality and which provide new knowledge about the subject, about learners or teaching strategies. Also valued is the opportunity to share experience with peers – often seen as the most valuable part of course or conference attendance.

What Is on Offer?

Formal qualifications

A number of countries are working to establish formal qualifications frameworks for the adult numeracy teacher workforce as part of a wider suite of qualifications for teaching in the lifelong learning sector. As an example, in the UK, qualifications include:

Level 3/4 Award in Preparing to Teach in the Lifelong Learning Sector

Level 3/4 Certificate in Teaching in the Lifelong Learning Sector

Level 5/6/7 Diploma in Teaching in the Lifelong Learning Sector

These cover the following general domains:

Domain A Professional values and practice

Domain B Learning and teaching

Domain C Specialist learning and teaching

Domain D Planning for learning

Domain E Assessment for learning

Domain F Access and progression

and the following Specialist Maths/Numeracy domains:

Social reasons for learners gaining mathematics skills and knowledge
Personal factors influencing learning of mathematics

Topics in Maths (Subject Knowledge)

Interestingly, a different but related set of professional standards are being developed for those engaged in teaching numeracy and mathematics within the work based learning sector and is due to report in November 2007.

Informal learning approaches

Much more common are informal development activities which do not lead to a qualification. A brief survey of professional development offerings advertised recently, or described in projects revealed a wide range of methods in use although the predominant approaches to professional development for the adult teacher workforce are those starred below:

- Workshops on mathematics and/or pedagogy (individual or series) *
- Short courses *
- Conferences
- Study materials *
- Apprenticeship
- Mentoring
- Coaching
- Internships
- Workshadowing
- Maintaining learning logs/learning diaries *
- Change of role
- Peer review and observation
- Learning sets

- Study circles
- Reading groups
- Practitioner research
- e-discussion boards and forums*
- Social e-networks
- Engagement in curriculum development and/or other projects examining/assessing for an external body

How Do We Know Whether Professional Development Has Been Effective?

Establishing criteria that help us to know whether professional development has been effective is perhaps the most significant challenge, particularly in the light of a lack of clarity about the purposes of professional development.

In their study of Teacher Change and the implications for Adult Basic Education, Smith Gillespie (2007) identified individual, organisational and contextual factors which influenced the degree to which teachers changed and developed.

Individual characteristics included: teacher motivation, teacher concerns, teacher self efficacy, teachers' cognitive styles/ways of knowing, reflectiveness, formal education and experience.

Organisational factors included: System within which Teachers operated, Leadership, Coherence between PD and curriculum change demanded, Collegiality and Working conditions.

They noted that the degree of change which teachers could make was often heavily influenced by the wider context and cultural and organisational expectations including those of parents, administrators etc.

They went on to examine how change was related to the content of professional development and the process by which it was delivered. They considered at two types of teacher development – more traditional forms which tended to comprise 'events' that by and large took place outside the workplace context, and forms which they called 'job-edded' where development was located within the workplace as part of an effort to create ongoing professional communities.

They noted that research has shown approaches of the former type were more effective if they:

- Were of longer duration
- Make strong connections between what is learned and the teacher's own work context
- Focus on subject knowledge
- Include a strong emphasis on reflection and analysis

- Include a variety of activities
- Encourage teachers from the same workplace to attend together
- Focus on quality and features of professional development

By contrast Job-embedded professional development approaches which included such techniques as study circles, learning sets, action research groups etc. were more effective if they:

- Focus on helping teachers study their learners' think, rather than just trying out new techniques
- Involve collaborative learning activities among teachers
- Involve activities where teachers make use of learner performance data
- Are supported by facilitators who organise development framework

In an earlier study, Smith et al (2002) had investigated the relative merits of three specific types of professional development intervention with ABE teachers. They found, and were surprised that teachers who learned and did more to address learner persistence, after participating in the professional development, were more likely to be those who:

- began their teaching in the field of ABE,
- had fewer years of experience in the field,
- did not have master's or doctoral degrees.

It appeared that it was not the model of professional development that had a major impact on learning, rather impact related to length of time engaged and the quality of the provision. Furthermore, teachers who had some voice in decision-making seemed more able to advocate for and take action than teachers who had little voice in program decisions.

What Has Not Been Considered?

- Some approaches – shadowing, e-learning, peer review of LTA, internships, use of consultant experts
- Change implies re-evaluating beliefs, values, roles
- Embedded values within current offerings
- Changed practice or better at current practice?
- Teacher control
- Understanding impact on learners
- Role of learner feedback
- Timeliness of perspective enlargement

What Does All This Imply for Professional Development Planning in the Future?

Earlier sections have identified a number of factors that need to be taken into account in considering the future planning of and research into professional development of the workforce:

- a) We do not really know who comprises the workforce. Encouraging governments to collect data on this would appear to be an important step.
- b) We do know that workforce is diverse, and we do know that professional development crucially must meet their individual needs. This has to imply flexibility of provision.
- c) An approach being developed by some countries is the formal professionalisation of the workforce. For some this may provide advantages, for others disadvantages. A starting point could be greater universality of initial training provision.
- d) Such formalisation leads to the creation of formal qualifications. In a context where much development happens in informal contexts through informal mechanisms, we need to find effective ways to formalise the informal without destroying its nature.
- e) Professional development can meet individual personal needs of the teacher but it also meets both social and economic needs. This can lead to tensions.
- f) Some approaches to development are seen as more effective than others by providers and researchers, but these may not be the same as those preferred by teachers themselves. Approaches which focus on short workshops remain the most common.
- g) The content of development may focus on mathematics or on pedagogy or both. Rarely does content appear to address issues of belief, values and self efficacy despite these issues being acknowledged as crucial for personal and team development.
- h) There is an implicit assumption in most of the discussion about teacher development that teacher change is sought. We need to be clear about what we are seeking to do in promoting professional development and to acknowledge that at times, enhancing current practice may be more effective an outcome than changing practice.
- i) We know almost nothing about the impact of professional development on student learning. Evaluating effectiveness must surely take account of this dimension.

Gravani (2007) suggests that for teachers and tutors, professional learning is characterised by a number of dimensions including professionalism, mutuality, emotionality and formality. She suggests that we need to shift our attention from the delivery of short courses to an understanding of the complexity of the process by which professional learning is developed. This view argues for a more comprehensive framework of professional development.

Developing a Framework for the Professional Development of the Workforce

Underpinning any framework for professional development must lie some agreement about the vision, some shared values and some agreement about the purpose and expected outcomes.

At a local level we might wish to see

- Confident, competent teachers?
- Noticing, learner focussed teachers?
- Teachers who have an impact?
- Teachers who share and develop their social capital?

- Shared values and ethical approaches?

Regionally we might wish to see

- Increased funding for professional development
- Commitment from employers and communities for professional development
- Policy developed by governments that supports professional development
- Increased funding for research so that we can make best use of resources

Whilst some teachers will wish to develop themselves for personal satisfaction or for wider benefits to society, others will need to see that this provides some personal reward. Hence, the development of mechanisms of formal recognition and career paths will need to form an element of the framework. Achieving formal recognition through informal means will be a parallel requirement. Having such a framework has the potential to enhance the status of the workforce and the work towards professional development entitlements. It could restrict the workforce and so the hurdle for initial entry must not be high.

Teachers want relevant professional development. Successful activity starts with their needs and evolves as their needs change. It results in outcomes which have been sought facilitating this requires expertise. If the needs of the individual teacher and the needs of their ‘employer’ and both the social and economic needs of nations are to be met, a partnership is needed where goals, outcomes and desired impacts can be established and through which evaluation can of effectiveness can be made.

Approaches will need to be negotiated and appropriate methods selected according to need. Above all, the selves of the workforce will need to be central and their values and beliefs disclosed.

Teachers value connectivity with their peers. Ensuring that mechanisms are in place to facilitate this will be a crucial part of the framework. It is clear that professional development is a long-term enterprise that presumes changes in what teachers know and how they practice. Establishing mechanisms that enable teachers to maintain engagement over time will need to be a priority.

A Research Agenda

Researching the professional development of the adult numeracy teaching workforce is in its infancy and we have much to do.

We need to integrate our theoretical perspectives on professional development if we are to advance our professional development practices. Somehow we will need to find a way to integrate theories which focus on ‘acquisition’ and theories that focus on ‘participation’. We will need to be clear about the purpose of professional development and the outcomes we

are expecting. We will need to reach some common ground over the duality of purpose for adult numeracy teaching. We will need to establish some common values.

Whilst we have some insights into the effects of different approaches to professional development, we need more knowledge about the effects these have on teachers themselves, on their knowledge gains and on their skills.

We will want to find out more about which approaches have the capacity to change teachers' beliefs and transform their practice.

Technology has the capacity to transform our global social and educational networks. As ICT skills develop and access becomes more widespread we will need to understand more about how teachers and their students can benefit from engagement in on-line communities and how this helps them learn and develop.

Beyond that, we know little about the impact on learners of teacher development. Whilst we in ALM may favour particular approaches to teaching, there is little evidence that these lead to better student outcomes or to collective growth.

As professional development frameworks start to emerge and the workforce becomes more professionalised, we may expect that teachers get better at their jobs and that the population at large becomes more numerate. We will need to evaluate the effects of large scale professionalisation, both on teachers themselves and on learners, their communities and their workplaces.

Finally, we need to examine the ways in which we can scale up particular initiatives which appear to be effective. So for example, Ann McDonnell's peer coaching approach may prove to be very effective. How and with what impact could this scale up if indeed it can scale up.

The Role of ALM

As an international practitioner-researcher network, ALM is particularly well positioned to begin to address these issues. It can:

- Share formal qualifications frameworks
- Establish ways to formalise the informal
- Seek funding to run a global, coaching, mentoring and supervision network
- Facilitate Information and Knowledge exchange amongst the workforce in partnership with e-providers
- Establish research programmes focussing on professional development and its impact

References

- ALI/OFSTED. (2003). *Literacy, Numeracy and English for Speakers of Other Languages: A review of current practice in post-16 and adult provision* (No. HMI 1367). Coventry: Adult Learning Inspectorate, OFSTED and Her Majesty's Inspectorate.
- Argyris,C., & Schon,D.(1978) *Organisational learning: A theory of action perspective*, Reading, Mass: Addison Wesley.
- Brown,M., Coben,D., Rhodes,V.,Swain,J.,Ananiadou,K. & Brown, P. (2007) A Study of Effective Proactice in Adult Numeracy Teaching, a paper prepared for publication by BSRLM.
- Cowan, M.A., (2006) Beyond Single Interests: Broad-Based Organising as a vehicle for promoting Adult Literacy. *Review of Adult Learning and Literacy*, 6, 8. NCSALL Condelli,L., Safford-Ramus, K., Sherman, R., Coben, D., Gal, I., Hector-Mason, A., (2006) *A Review of the Literature in Adult Numeracy: Research and Conceptual Issues*. US Department of Education, 27 March 2006.
- Cedefop (2000) *Making Learning Visible, Cedefop Reference Series, No 3013*.
http://www.trainingvillage.gr/etv/Upload/Information_resources/Bookshop/102/summary3013_en.pdf. Accessed 24 June 2007
- DfES (2001) Skills for Life: The national strategy for improving adult literacy and numeracy skills. London, Department for Education and Employment (UK).
- DFES(2003) The Skills for Life Survey: A national needs and impact survey of literacy, numeracy and ICT skills. London. Department for Education and Skills.
- Emmerling, R. J. & Goleman, D. (2003). Emotional Intelligence: Issues and Common Misunderstandings. *Issues and Recent Developments in Emotional Intelligence*, [On-line serial], 1(1). Available <http://www.eiconsortium.org>
- European Council of Ministers, 2002, Council Document 5828/02.
<http://register.consilium.eu.int/en/02/st05/05828en2.pdf>
- Fang, Z. (1996). A review of research on teacher beliefs and practices. *Educational Research*. 38 (1) 47-64.
- Field,J. (2005) 'Social capital and lifelong learning', the encyclopedia of informal education.
www.infed.org/lifelonglearning/social_capital_and_lifelong_learning.htm
- Gardner, H., & Hatch, T. (1989). Multiple intelligences go to school: Educational implications of the theory of multiple intelligences. *Educational Researcher*, 18(8), 4-9.
- Gravani, M.N. (2007) Unveiling Professional Learning: Shifting from the Delivery of Courses to an Understanding of the Process. *Teaching and Teacher Education: An International Journal of Research and Studies* , 23,5,688-704 July 2007.
- Goleman, D. (1996). *Emotional Intelligence: why it can matter more than IQ*. London: Bloomsbury.
- Hudson, C. with Colley,J.Griffiths, G. & McClure,S. (2006). Maths4Life Pathfinder Report, *National Research and Development Centre for Adult Literacy and Numeracy. Chapter 6 Level 4 numeracy teacher education* 52-58 NRDC.
- Hustler, D., McNamara, O., Jarvis,J., Howson, J., Mary Londra, M.,& Campbell,A. (2003)Teachers' perceptions of continuing professional development DfES Research Report RR429/RB429, HMSO.
- ICWFD (2007) International Commission on Workforce Development Home Page
<http://www.icwfd.org/index.html>
- Institute for Learning (2006) A model of CPD for teaching practitioners in PCET. Consultation Document July 2006. http://www.ifl.ac.uk/cpd_portal/cpd_consultation_jul_06.html accessed 24th June 2007
- Kise,J.A.G.,(2006) *Differentiated Coaching: A Framework for Helping Teachers Change*. Corwin Press.
- Knowles, Malcolm S. (1980) *The Modern Practice of Adult Education*. Revised and Updated. Englewood Cliffs: Prentice Hall Regents.
- Kolb, D. (1984) *Experiential Learning: Experience as the Source of Learning and Development*, NewJersey: Prentice Hall.
- Lave, J. and Wenger, E. (1991) *Situated Learning:Legitimate Peripheral Participation*, Cambridge:Cambridge University Press.
- Lucas,N., Casey,H. & Giannakaki, M.,(2004). Analysis of participants and evaluation of teacher training needs from core curriculum training, *Research Report, National Research and Development Centre for Adult Literacy and Numeracy*.
- Maguire,T. & O'Donoghue,J., (2005) Matching professional development approaches to the needs of tutors of adult numeracy. In Lindberg(ed) (2005). *Proceedings of the 11th International Conference on Adult Learning Mathematics in Kungälv (Goteborg Sweden)*, Goteborg University Department of Education.

- Mackay,S., Burgoyne,U., Warwick, D.,& Cipollone,J. (2006) *Current and Future professional Development Needs of the Language, Literacy and Numeracy Workforce. An Adult Literacy National Project Report*. National Centre for Vocational Education Research (NCVER), Jan, 2006.
- QIA. (2007) Skills for Life Improvement programme <http://www.sflip.org.uk/> accessed 24 June 2007.
- Raabe, B, Frese,M & Beehr,T.A. (2007) Action regulation Theory and Career Self-Management. *Journal of Vocational Behaviour*. 70 (2), 297-311.
- Sadler-Smith, E and Smith, P J (2004), 'Strategies for accommodating individuals' styles and preferences in flexible learning programmes', *British Journal of Educational Technology*, 35(4), 39–412.
- Smith,A. (2004) Making Mathematics Count, the Report of Professor Adrian Smith's Inquiry into Post-14 Mathematics Education, London, DfES.
- Smith,C., & Gillespie M. (2007). Research on professional Development and Teacher Change: Implications for Adult Basic Education. Review of Adult Learning and Literacy 7 , 7 Lawrence Erlbaum Associates.
- Smith, C., Hofer,J.,Gillespie, M.,Solomon,M., & Rowe,K. (2002) How Teachers Change: A Study of Professional Development in Adult Education. NCSALL Report. National Centre for the Study of Adult Learning and Literacy. Harvard Graduate School of Education.
- Schon, D.A. (1996). *Educating the reflective practitioner: Toward a new design for teaching and learning in the professions*. San Francisco: Jossey-Bass, Inc.
- Senge,P.(1990) *The Fifth Discipline, The art and practice of the learning organization*, London: Random House.
- Storberg-Walker,J. (2006) *Exploring the role of Communities of Practice in Judicial Continuing Education*. Paper presented at the Academy of Human Resource Development International Conference, Columbus, OH, Feb 22-26, 2006, 548-555 (Symp 27-1).
- Wake,G.(2005) FUNCTIONAL MATHEMATICS: MORE THAN “BACK TO BASICS”Nuffield Review of 14-19 Education and Training. Aims, Learning and Curriculum Series, Discussion Paper 17. August 2005.
- Wenger (1999) *Communities Of Practice Learning, Meaning, And Identity* Cambridge University Press (UK), 1999.
- Young,M.B., Fleischman,H., Fitzgerald,N.,and Morgan,M.A., (1995) National evaluation of adult education programs (Executive summary, Contract No. LC 90065001). Arlington,VA:Development Associates.

Initial Findings from Research on the TIAN Project: A New Professional Development Model for Adult Education Math Teachers

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Teachers Investigating Adult Numeracy (TIAN) is a collaborative project of the Center for Literacy Studies at the University of Tennessee and TERC, a Cambridge, MA, non-profit organization focusing on math and science. Funded by a grant from the National Science Foundation and support from six participating states, it draws on the work of two other projects: EMPOWER and Equipped for the Future. In developing and testing the TIAN professional learning model, our central question has been: What will it take to help the adult education workforce move closer in belief and practice to a more complete definition of mathematics proficiency?

This paper introduces the project's assumptions and goals, describes the elements of the intensive model, and presents the accompanying research methodology and early findings on what changed for the teachers who participated in the professional development program.

An Overview of the TIAN Project

Teachers Investigating Adult Numeracy has been a 4-year collaboration (2005–2009) developing and testing a model for in-service professional learning that uses teacher investigations and reflective learning to engage adult education teachers in considering how to implement purposeful and effective mathematics instructional approaches.

TIAN's primary focus is on teacher learning. The goals of TIAN are:

1. to increase and deepen teachers' mathematical content knowledge
2. to increase the number and range of teachers' instructional approaches
3. to increase teachers' knowledge and use of state mathematics content standards
4. to increase states' capacity to provide quality mathematics instruction.

A total of 40 Massachusetts and Ohio adult basic education teachers participated in the pilot phase of TIAN in 2005–2006, and 76 teachers from Arizona, Kansas, Louisiana, and Rhode

Island took part in the field test in 2006–2007. In 2008, teachers from all six states were involved in a variety of activities to support, extend, and share what they had learned.

Assumptions

Our TIAN work has been based in two assumptions about learning and teaching mathematics.

1. Math is more than procedures

Research tells us that mathematical proficiency includes, but is more than, being fluent with procedures, and that effective math learning and teaching should also attend to conceptual understanding, strategic competence, adaptive reasoning, and a productive disposition (Kilpatrick et al., 2001). While this particular research refers to children learning math, we, the authors of this article and principal investigators of the Teachers Investigating Adult Numeracy project, believe this definition of mathematics proficiency holds true for all ages. In this article, we describe the professional learning model we believe is needed to support teachers who wish to base their instruction on this definition, describe the research conducted concurrently during the testing of the model, and present some of the initial data about the impact of the model on teachers involved in the field test years.

We have found that teachers tend to teach the way we were taught. That means teachers who were taught only math procedures now teach only procedures, while those who were taught skills, concepts, and strategies in an environment that encouraged reasoning, communication, and problem-solving, now teach that way. In adult basic education and GED classes, teachers often teach a range of subjects and few have extensive training in mathematics. As a result, in visits to ABE (Adult Basic Education) programs, we are much more likely to see teachers experimenting with innovative approaches to instruction in reading, writing, and social studies than in math. Additionally, the instructional emphasis in mathematics has been on procedures (Ward, 2000).

2. Quality professional development has some essential features

Research on teacher professional development tells us that effective teacher professional development in mathematics and science occurs over time and is not a “one-shot” activity. The intervention should be built upon activities that help instructors advance their own conceptual understanding of mathematics and the way adults learn so that instructors use this knowledge in planning instruction for learners. It helps instructors connect content and materials to authentic and real-world numeracy/mathematics situations (Sherman et al., 2006).

It is important that the professional development reflect the research on how adults learn (e.g., multiple problem-solving strategies, collaborative learning, and access to prior knowledge). The mathematical content within the professional development should reflect national (e.g., National Council of Teachers of Mathematics, American Mathematical

Association of Two-Year Colleges, and Equipped for the Future) or state standards. In the TIAN model teachers engage with mathematics content as learners as well as instructors and connect the mathematics they are learning and teaching with their state's standards.

The Professional Development Component: The Intervention

The professional development provided by TIAN is both extensive and intensive. Two processes for teacher change that have been shown to be effective in mathematics education play a central role in the model. The first is the opportunity for teachers to do mathematics themselves with an emphasis on learning with understanding (Ball, 2000; Hill et al., 2005). In the first year, participating teachers attend three institutes; in each, they spend two days doing mathematics together, sharing their work, and analyzing how they can apply what they learn in their classrooms. They experience new approaches first-hand. The institutes and teacher meetings held between institutes are structured in ways that ask teachers to be learners of mathematics.

The second process shown to be effective is the opportunity to conduct close examination and discussion of student work. Between institutes, the participants teach lessons on data and algebra that they adapt from math materials they develop and from a curriculum called EMPOWER, developed by TERC (see http://adulthoodnumeracy.terc.edu/EMPOWER_home.html). They document their planning and instruction in two detailed work samples in which they describe what they have done and why and how three students at different levels responded to the instruction. Some of these samples have been posted on the website to illustrate to other teachers how the lessons played out in an adult education classroom. (See http://adulthoodnumeracy.terc.edu/TIAN_worksamples.html.)

TIAN's mathematical content centers on two strands of mathematical proficiency: algebra and data. While a comprehensive instructional program in ABE mathematics must also include the development of number and operation sense and geometry and measurement, we chose to focus on algebra and data analysis for several reasons. Algebra, the "gatekeeper" subject is, as Robert Moses (2001) believes, essential for full citizenship. Understanding the presentation of basic statistics in the media is also essential. Moreover, algebra and data analysis have received added emphases on the most recent edition of the GED exam and in the most recent sets of adult-focused standards. However, we have found both to be areas with which current teachers are uncomfortable, and which are often taught only to high level students. TIAN helps teachers build their confidence and competence in algebra and data by involving them in doing math as well as learning how to teach math.

TIAN gives participating teachers opportunities to learn new instructional approaches, including:

- Working collaboratively on open-ended investigations
- Sharing strategies and understandings orally and in writing

- Justifying answers in multiple ways
- Using contexts that are meaningful to adults
- Exploring a variety of ways for entering and solving problems.

These instructional approaches are intended to increase students' opportunities to learn and are supported by research on principles of effective teaching (Brophy, 1999; Bransford et al., 2000; Hiebert & Grouws, 2007).

One of the challenges of beginning to use new approaches to instruction, particularly approaches that are not based on rigidly sequenced published materials, is assuring that necessary content is covered at appropriate levels. Adult education mathematics content standards and curriculum frameworks can provide that structure. TIAN training includes each state's standards, and teachers are helped to connect their instruction to their state standards. In the United States, each state is responsible for the provision of basic education for adults. Thus, the state education staff plays an important role in shaping how the professional development project begins and how the work of the teachers is expanded and sustained. The model assumes that the teachers in the TIAN cadres will serve as change agents not only within their classrooms and programs, but across their state, and the state staff leads the organization of the cadre into regional groups. To support this process, during the second year of the state's involvement, two goals were added:

- To increase the number of mathematics teacher leader/facilitators who would support their state's efforts in improving instruction.
- To increase the ways in which the state could support the teacher leader/facilitators in expanding the number of teachers included.

With the new goals in mind, the TIAN Project provided additional web-based resources (TIAN Bundles) and supported leadership development among the six first-year cadres in various ways as the teacher leaders/facilitators led local groups, co-facilitated trainings, or met in study circles.

In November, 2008, the TIAN Project staff invited representatives from the six states to a 3-day leadership institute in Cambridge MA: The TIAN Facilitating Mathematics Professional Development Institute. The training was the culmination of the pilot and field test phases of the TIAN model, leaving each state with an increased capacity to further develop their adult education math instructional programs.

The Pilot and Field Test of the Professional Development Component

Six teacher cohorts from six states, for a total of 116 teachers, participated in testing the TIAN model, 40 in the pilot and 76 in the subsequent field test.

The teachers in the pilot cohorts were from Massachusetts and Ohio. We recruited these two states because we knew that they had state adult mathematics standards and were actively

working to improve mathematics instruction in adult education. The teachers in field test cohorts were from Arizona, Kansas, Louisiana, and Rhode Island. We chose these four states from the twenty states that applied to be part of the field test. Our choices were based on our interest in having a diverse group of states regionally and in terms of level of state support.

The teachers and classrooms in the pilot and field test cohorts were somewhat similar. All were teaching in adult learning centers/programs sponsored by a school district, a city, a community college, a community based organization, or a correctional facility. The majority (75% pilot, 59% field test) taught in open-entry, open exit programs, where new students entered when space became available, rather than on a semester or course basis. A large majority (90% pilot, 80% field test) taught other subjects as well as math. Class size varied, with 4–18 students in pilot classes and an even wider range in the field test classes, with 2–37 students. Pilot classes averaged 9 and the field-test classes averaged 11 students. There was a notable difference between the two groups in math class time. On average the pilot teachers' students spent 3.6 hours/week “doing math” in class, whereas the field-test teachers' students averaged 5.9 hours/week. All teachers participated in a year's worth of TIAN activities. One such activity, from the third TIAN institute, is described below.

Potatoes are sitting in a bowl next to some vegetable peelers; a pile of pennies with coin wrappers are on a nearby table; on another table a bunch of envelopes are waiting to be stuffed. Twenty ABE teachers break into teams of four and rush to one of the stations to begin to do a “sample of work” to determine how long it would take to help out at a community event by peeling 50 pounds of potatoes for a huge potato salad, rolling 10,000 pennies, and stuffing 1,000 envelopes. Everyone is left to their own devices, and all five teams take different tacks: some have one team member do the work, while another records how long it takes to stuff 10 envelopes; others count how many envelopes can be stuffed in a minute; others test out what can be done in two or five minutes. Some build up to 10,000 by calculating in their heads, others round numbers with confidence. Some use good old-fashioned paper and pencil computation or cross-products, others punch numbers into calculators. Everyone is on-task and having a good time.

Once they have completed the tasks at each station, the groups post their results on newsprint. The facilitator asks them to describe strategies, and why the strategies work or don't. People seem amazed that there are so many ways to arrive at a reasonable answer. If one estimate is way off, the whole group focuses on why. The facilitator pushes the participants to compare, contrast, and make connections between the various strategies.

The teachers have “lived” the lesson they will be trying out in their classes. Next, the teachers examine student work. They read a classroom vignette that describes a dilemma that came up for a group of students and are asked, “What would you do next as a teacher that would be helpful?”

If you compare this active open-ended exploration of ratio and proportion with the typical way ratio is presented—setting up two ratios and cross-multiplying—, you get a sense of what goes on in a TIAN Institute as well as the extent to which we are encouraging teachers to stretch their mathematical understandings and classroom practices.

The Research Component

TIAN's three goals for teacher-participants are:

- Goal 1. To increase and deepen teachers' mathematical content knowledge,
- Goal 2. To increase the number and range of teachers' instructional approaches,
- Goal 3. To increase teachers' knowledge and use of state mathematics content standards.

To determine our success in meeting these three goals we collected and analyzed a variety of data from participating teachers. In this article we are reporting on our initial analysis of results regarding the last two of these goals; we will report on the first goal in another paper.

Initial Findings on Goal 2: Number and Range of Teachers' Instructional Approaches

To examine changes in the number and range of teachers' instructional approaches we have considered data from three sources:

- A 75-item questionnaires completed by the teachers before the first institute and after the last institute. These instruments asked for information about the teacher's students, the teacher's own math background, beliefs and math teaching practices, and use of state math standards.
- Classroom observations conducted with a sample of participating teachers before the first institute and after the last institute. We used an open-ended protocol in which we asked trained observers to take ongoing notes of teacher and student activities, paying particular attention to a set of student and teacher activities of particular interest based on our objectives. Additionally the teachers were interviewed before and after the observation about the class, their goals for that particular class, and their assessment of how the class went. After reviewing the first set of observations of field test teachers, we drafted an analysis rubric which lists a set of teacher and student activities and other features to be identified from observers' notes. These rubrics were used to guide preliminary analysis of the observations. In the pilot year we conducted initial observations with half the participants. Due to resource constraints, in the field test we did initial observations of about 1/4 of the participants.
- Phone interviews conducted with a sample of teachers one year after their participation in TIAN. In these interviews, teachers were asked about their current teaching situations and how TIAN had affected how they taught.

From pre-post questionnaires we found statistically significant increases reported in:

- finding real-life applications in algebra

- willingness to be flexible about sequence of topics presented
- using exploratory as opposed to didactic approaches to instruction
- encouraging students to use exploratory approaches to understand mathematical concepts versus learning rules
- having students write about and demonstrate mathematical understanding in a variety of ways.

From pre-post questionnaire items on important factors in planning a math lesson, we found that after participation in TIAN, teachers reported an increase in their consideration of individual student goals and consideration of pedagogical issues such as using a variety of strategies and interactive materials.

In pre-post classroom observations, 6 of 14 field test teachers showed changes that included increased use of real-life contexts, small groups, and hands-on activities. In follow-up interviews conducted a year after participation in TIAN, 11 of 17 teachers reported using real-life materials or hands-on materials in their most recent math class and 9 of 17 had students work in small groups. All 17 teachers reported lasting changes in their understanding of how to teach math.

Initial findings on Goal 3: Changes in Teachers' Knowledge and Use of State Mathematics Content Standards

We examined three data sources to determine how TIAN teachers reported any changes in how they used their state standards to plan mathematics instruction:

- A pre/post written assessment on state standards completed at the first and last institutes
- Questions in the pre/post questionnaire (for the field test)
- Phone interviews (one year later) with a sample of participants

Before their participation in TIAN, 11 of the 64 of the field test teachers who had completed the pre and post assessment showed evidence of a clear understanding of their state's math content standards. After TIAN 21 teachers showed a clear understanding of the standards. Before TIAN, 17 teachers had no or very limited knowledge of their state standards. After TIAN no teachers reported no or limited knowledge.

From the pre-post field test questionnaires, we found teachers reported significant change in the influence of state standards on their decisions about what to teach (mean of 2.2 to 2.61 with 2 = "some influence" and 3 = "strong influence"). Seven of 17 field test teachers who were interviewed a year after participating in TIAN reported using standards regularly in planning instruction.

The results from these data from field test teachers indicate to us that TIAN has been successful in increasing participant's knowledge of and use of their state math content standards.

Discussion

This article provides an introduction to the TIAN project and some initial indications of what teachers are taking from it. When we look across the results we report here, we see strong indications of change in the number and range of participating teachers' instructional approaches in mathematics. At the end of their year's involvement in investigating math they had moved away from lots of drill, a strict sequence of skills, the exclusive use of workbooks. They reported that they now used more hands-on activities, had students explore possible solutions, and increased communication about math. Nearly all the teachers we interviewed a year after their participation in TIAN continued to talk about this kind of change in their understanding of math instruction and in their practice. While this data is preliminary, it indicates that we should continue to develop this professional development model for adult education math teachers.

There are other questions we hope to answer based upon the data we have available. We believe the success of the model most likely depends on the extent to which the state level office staff provides support; we suspect states that make the greatest investment will see the most change. We also are interested in the extent to which the model supports increase in teacher math content knowledge.

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References

- Ball, D. (2000). "Bridging practices: Intertwining content and pedagogy in teaching and learning to teach." *Journal of Teacher Education*, 51, 241–247.
- Bransford, J., Brown, A., & Cocking, R., Eds. (2000). *How People Learn: Brain, Mind, Experience, and School: Expanded Edition*. Washington, DC: The National Academies Press.
- Brophy, J. (1999). *Teaching*. Brussels: International Academy of Education; Geneva: International Bureau of Education. Retrieved April 11, 2008 from <http://www.ibe.unesco.org/publications/EducationalPracticesSeriesPdf/prac01e.pdf>
- Hiebert, J., & Grouws, D.A. (2007). "The effects of classroom mathematics teaching on students' learning." In F.K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 371-404). Charlotte, NC: Information Age Publishing.

- Hill, H., Rowan, B., and Ball, D. (2005). "Effects of teachers' mathematical knowledge for teaching on student achievement." *American Educational Research Journal*, 42 (2), 371–406.
- Kilpatrick, J., Swafford, J., & Findell, B. (eds.) (2001). *Adding it up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press.
- Moses, R. & Cobb, C. (2001). *Radical Equations: Math Literacy and Civil Rights*. Boston: Beacon Press.
- Sherman, R., Safford-Ramus, K., Hector-Mason, A., Condelli, L., Olinger, A., & Jani, N. (2006). *An environmental scan of adult numeracy professional development initiatives and practices*. Washington, DC: American Institutes for Research. Available from the U. S. Department of Education, Office of Vocational and Adult Education Web site, <http://www.ed.gov/about/offices/list/ovae/pi/AdultEd/math.html>
- Ward, J.E. (2000). "Arkansas GED mathematics instruction: History repeating itself." In K. Safford-Ramus & M.J. Schmitt (Eds.), *Proceedings of the Seventh Annual Adult Learning Mathematics Conference—ALM7*. Cambridge MA: National Center for the Study of Adult Learning and Literacy.

From Standards-Led to Market-Driven: A Critical Moment for Adult Numeracy Teacher Trainers *

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Abstract

There has been a shift in the training of numeracy teachers in England away from a highly regulated 'standards-based' approach to teacher training towards one that seeks to engage employer groups and stakeholders in determining the training needs of teachers in further education. This shift has taken place within the context of rapid reform to numeracy and mathematics curricula for post-16 learners. The planned curriculum changes have again highlighted the shortage of qualified numeracy teachers needed to implement national policy initiatives, and has brought numeracy teacher training onto the policy agenda once again. This paper uses Bernstein's notions of vertical teacher knowledge and horizontal teacher knowledge to consider how trainee teachers may be supported to bridge the gap between their own mathematical knowledge and their classroom practice as numeracy teachers. It also draws on Shulman's seven types of teacher knowledge to make these connections. Recommendations made relate to the entry criteria for adult numeracy teachers, allowing 'time and space' to reflect with other trainees rather than 'immersion in practice', the benefits of practitioner-led enquiry to develop innovative pedagogies, and enhanced links between further education and school-based mathematics and between further education and higher education.

Key words: numeracy, teachers training, standards-based approach

Introduction and policy context for adult numeracy teacher training in England

The Moser report (DfEE, 1999) signalled the introduction of the 'skills for life' policy in England with a commitment to raise the literacy and numeracy skills of adults. This policy initiative was introduced in the context of a largely casualised teaching workforce where literacy and numeracy teachers often existed on the margins of further education and were sometimes perceived as lacking the subject or occupational expertise often associated with teachers of academic or vocational subjects (Lucas, 2007). The introduction of 'subject specifications for teachers of adult literacy and numeracy' (DfES/FENTO, 2002) sought to address this by ensuring "that all new teachers [of literacy and numeracy] are equipped with the appropriate knowledge, understanding and personal skills in their subject, in order to put them on a par with teachers in any other subject" (Lucas, 2007, p.127).

The drive to raise the subject knowledge of literacy and numeracy teachers in England through the introduction of the DfES/FENTO (2002) subject specifications was to some extent subsumed within the wider 'equipping our teachers for the future' initiative (DfES, 2004) that sought to raise the subject knowledge of all teachers in further education. This was partly driven by a critical Ofsted report (2003) into the initial training of further education teachers that found little systematic development of the specific skills and understanding needed for effective subject specialist teaching and that the lack of this specialist dimension to be "a major shortcoming in the present system of FE teacher training" (Ofsted, 2003, p.23).

The DfES/FENTO (2002) 'subject specifications for teachers of literacy and numeracy' were replaced in 2007 by 'new overarching professional standards for teachers, tutors and trainers in the lifelong learning sector' (LLUK, 2007a) and an application of those standards for specialist teachers of adult numeracy (LLUK, 2007b). These new professional standards were followed by a qualification framework, workforce regulations and the imposition of highly prescriptive learning outcomes that sought to regulate the competencies trainee teachers were expected to demonstrate during initial teacher training. Nasta (in Lawy and Tedder, 2009, p.56) described this policy model as driven by a "linear notion that the standards must be specified first, then regulations and qualifications must be developed that incorporate the standards, and only at the final stage are a curriculum and assessment model to be developed that will form the basis of what trainees actually experience".

Two research projects were carried out by the National Research and Development Centre (NRDC) into the DfES/FENTO (2002) 'subject specifications for teachers of numeracy and literacy'. The earlier of these studies (Lucas et al., 2004) was based on nine universities that piloted the subject specifications alongside their initial teacher training courses whilst the later study (Lucas et al., 2006) drew upon a larger sample of mostly in-service courses delivered by both universities and colleges. The key foci of these research projects included an exploration of how the subject specifications were being translated and re-contextualised into teaching practice; different approaches taken to delivering the subject specifications; and the balance to be struck between subject specific knowledge, pedagogic knowledge and practical teaching skills (Lucas, 2007). The two NRDC projects led to a number of peer-reviewed publications by the researchers involved in the projects (Lucas, Loo and McDonald, 2006; Lucas, 2007; Loo, 2007a; Loo, 2007b). These discussed issues relating to the increased subject knowledge of numeracy (and other 'skills for life') teachers and the relationship of that increased subject knowledge to classroom teaching practice using Bernstein's (2000) notions of vertical teacher knowledge and horizontal teacher knowledge.

Whilst a body of literature began to emerge specific to adult numeracy teacher training as a result of the two NRDC studies (Lucas et al., 2004; Lucas et al., 2006), this literature did not explicitly take account of the more developed debates on the nature of subject knowledge needed for teaching mathematics in schools (e.g. Ball and Bass, 2003; Davis and Simmt, 2006; Ball, Thames and Phelps, 2008; Hodgen, 2011). It is appropriate in considering subject knowledge for teaching adult numeracy to engage with the wider debate of subject

knowledge for teaching mathematics in schools, particularly given the research that has taken place into the longer-established subject knowledge enhancement courses (formerly called mathematics enhancement courses) that are by universities to prospective trainee mathematics teachers for secondary schools (e.g. Adler and Davis, 2006; Askew, 2008; Stevenson, 2008; Adler et al., 2009).

The change of government in the UK in 2010 resulted in a shift of educational policy on teacher professionalism away from centralised government-control through a standards-based and regulatory system towards one that afforded greater autonomy to employers to determine the professional qualifications their teaching workforce needed to respond to the needs of the learners and employers they seek to serve. The Lingfield review of teacher professionalism in further education (BIS, 2012, p.5) did confirm the need for specialist pre-service or early in-service teacher training for “lecturers in the foundation skills of literacy and numeracy”, albeit within the context of the revocation of the statutory regulations for teacher qualifications in further education. What Lingfield did not attempt to do was define what constitutes foundation skills in numeracy (whether it includes functional mathematics for 14 to 19 year-olds or GCSE mathematics, for example) or the specific outcomes trainee teachers should be expected to demonstrate during initial teacher training.

This article seeks to develop Bernstein's notions of vertical teacher knowledge and horizontal teacher knowledge found in the literature relating to adult numeracy teacher training in England by comparing it with Shulman's seven categories of teacher knowledge found in the literature from the more established subject knowledge enhancement courses offered by universities for intending mathematics teachers in secondary schools. Bernstein and Shulman's theoretical models will be used to analyse post-hoc three teacher training activities drawn from courses designed to meet the subject knowledge requirements of the DfES/Fento (2002) subject specifications for adult numeracy teachers.

Throughout this article the term ‘numeracy’ is used to distinguish the curriculum taught to post-16 learners in vocational contexts from ‘mathematics’ as the curriculum taught as a compulsory subject in schools. Similarly ‘numeracy teachers’ refers to those teachers qualified or training as specialist teachers of adult numeracy and ‘mathematics teachers’ to those qualified or training as specialist teachers of mathematics in secondary schools. The use of these terms to distinguish between curricula and job roles does not imply that such a simplistic division between numeracy and mathematics exists. Indeed, as will be seen in the later section *critical moment in a changing policy context*, the labels numeracy and mathematics can be used to signal the ideological perspectives of policy-makers and as such be subject to different interpretations. For a flavour of the debate on the use of the terms numeracy and mathematics see the papers presented by Kaye in earlier conference proceedings of this journal (Kaye, 2002; Kaye 2010).

Subject specifications for adult numeracy teachers - Bernstein's vertical teacher knowledge and horizontal teacher knowledge

The two NRDC studies (Lucas et al., 2004; Lucas et al., 2006) into pilot courses designed to meet the requirements of the FENTO ‘subject specifications for teachers of numeracy and literacy’ identified three different types of participant on the courses studied. These included very experienced practitioners who also held management posts and staff training roles in colleges; practicing teachers with some classroom teaching experience; and new entrants to teaching with little teaching experience. Each group had different expectations from the course with the most experienced wanting “a high level of theoretical content that would ... provide them with a synoptic perspective on their specialism” (Lucas, Loo and McDonald, 2006, p.341) whilst the newer entrants to teaching wanted an emphasis on practical teaching to prepare them for teaching practice. Lucas, Loo and McDonald (2006) applied Bernstein’s notions of horizontal teacher knowledge and vertical teacher knowledge to understand the distinction between theoretical and practical knowledge for teachers and ways in which the courses attempted to bridge these two types of knowledge through what Bernstein called ‘re-contextualisation’.

An examination of the FENTO subject specification for adult numeracy (DfES/FENTO, 2002) shows that it consisted primarily of Bernstein’s ‘vertical knowledge’ separated into the sections of number and numeric operations, geometry and spatial awareness, statistics, and working with algebra. It was primarily ‘vertical knowledge’ in the sense that the specification required an academic or theoretical understanding of the content that was independent of context or experience. A closer inspection of the elements listed in the specification revealed that most of them approximated to topics that might be found on the first year of a course in GCE Advanced Level mathematics (level 3 on the English National Qualifications Framework) whilst other topics were identifiable from the content required for higher level tier of GCSE mathematics syllabi (level 2 on the English National Qualifications Framework). The specifications immediately raised the questions of (i) how the courses can be justified as being at level 4 on the national qualifications framework (equivalent to the first year of undergraduate study) when the content was clearly a repetition of level 3 study, and (ii) how all the elements listed in the specifications can be covered in a course of one-year part-time duration.

The first of these two questions relating to academic level was the simplest to answer. In the case of the experienced practitioners seeking a theoretical and synoptic perspective of mathematics this ‘level 4-ness’ could be justified as being demonstrated through the adoption of a connectionist approach to mathematics that emphasised relational understanding over procedural understanding (Skemp, 1976; Askew, 1997). For new entrants to teaching it was the requirement for 60 hours of practical experience in teaching adult numeracy that were seen to bring the ‘level 4-ness’. In both cases there were significant challenges for numeracy teacher trainers supporting trainees in the process of re-contextualising vertical teacher knowledge of mathematical content into horizontal teacher knowledge of classroom practice in teaching adult numeracy.

The second of the two questions posed more difficulties for course designers with different approaches taken by awarding bodies and universities to the problem of achieving coverage of the specifications within the learning hours available. Lucas (2007) identified that whilst national awarding bodies adopted a ‘standards-based approach’ that emphasised ‘coverage’ and ‘mapping’ in the competency tradition, universities were more innovative in a ‘knowledge-based approach’ where they chose which elements of the specifications to emphasise and in what depth to explore them.

Three examples, one from a course that I delivered at Thames Valley University, another from a course delivered by LLU+ at London South Bank University reported in the proceedings of the 13th annual international conference of Adults Learning Mathematics (Stone and Griffiths, 2006), and a third from one of the NRDC pilot studies (Lucas et al., 2004; Lucas et al., 2006) illustrate ways in which universities developed innovative ‘knowledge-based approaches’ towards the DfES/FENTO (2002) subject specifications:

Example 1: Thames Valley University

One element of the DfES/FENTO (2002) subject specification within the statistics section required knowledge of discrete probability distributions. The direct contact-time available to the trainer to teach this topic was a single session of four hours duration, albeit with the expectation that trainees would engage in self-directed study to further their knowledge outside of the taught session. There were several problems with this. Discrete probability distributions include rectangular, binomial and Poisson distributions. Each of these constitutes a topic in its own right worthy of more than four hours of direct contact-time. Furthermore, knowledge of discrete probability distributions does not easily translate to strategies for teaching adult numeracy learners. Interestingly, coverage of the normal distribution was not required by the DfES/FENTO (2002) subject specifications since this is a continuous rather than discrete probability distribution, even though an understanding of the normal distribution is arguably more relevant to teachers than the discrete probability distributions due to its usefulness in interpreting assessment results for large populations, understanding IQ scores, and so on.

The trainer made the decision in planning the session to teach both the continuous probability distribution (normal) and the discrete probability distributions (rectangular, binomial and Poisson) within the four hour session. Being aware of the impossibility of teaching such a range of mathematical knowledge within four hours the trainer elected to see the content as a vehicle towards meeting an overarching course aim rather than specific content to be covered. The overarching aims of the trainer were (i) to provide trainees with the opportunity to carry out self-study in pairs on an area of mathematics unfamiliar to them and then teach that concept to the rest of the group, (ii) appreciate the uses of mathematical modelling (e.g. the normal distribution to interpret IQ scores and the Poisson distribution to predict volcanic activity), and (iii) to make links with own practice as teachers of adult numeracy.

Example 2: LLU+ at London South Bank University

Stone and Griffiths (2006, p.148-149), in reflecting upon their experiences as numeracy teacher trainers at LLU+, argued that:

Making teachers ‘do some hard sums’ and giving them some background information on personal and social factors affecting learning was not really equipping them to teach their subject. ... Clearly, something was missing. At LLU+ the feedback from our own teacher training programmes was that while the course sessions were fun and participants were exposed to [an] imaginative variety of teaching methods, they did not feel they were learning as much as they would have liked that would be useful to them in the numeracy classroom. To this end, we began enriching our programmes on offer with opportunities to explore mathematics and numeracy at a basic level and to discuss and evaluate ways to teach it.

This extract appears to indicate a similar orientation to the trainer in example 1 where a commitment to overarching course aims allowed the subject specifications to be interpreted creatively. In the case of the two trainers at LLU+ the overarching course aims appeared to include learning as fun, modelling variety in teaching methods, valuing the ‘student voice’, and ensuring relevance of activities to participants’ professional practice.

Example 3: Broken keys activity

Loo (2007) describes an activity used by one of the institutions in the NRDC studies called ‘broken keys’. This involved trainees creating problems for others in the group to solve using mathematical functions. These were then linked to word cards and picture cards to illustrate the links between algebraic symbolism and real life. Finally the trainees were encouraged to reflect on how the approaches could be applied to the teaching of topics from the Adult Numeracy Core Curriculum (DfES, 2001).

Whilst the starting point to the ‘broken keys’ activity was drawn from the ‘working with algebra’

section of the subject specifications a commitment on behalf of the trainers to overarching course aims such as modelling the Standards Unit approaches of learners creating problems, multiple representations and encouraging discussion (Swan, 2005) can arguably be inferred from the teaching approach described.

Subject knowledge enhancement courses for schoolteachers in secondary mathematics - Shulman's seven major categories of teacher knowledge

Subject knowledge enhancement courses (previously known as mathematics enhancement courses) are well-established in many English universities offering Post-Graduate Certificate in Education (PGCE) courses for intending mathematics teachers in secondary schools (Sheffield Hallam University, 2013). These courses are usually offered as short part-time courses to graduates who have already been offered a place on secondary mathematics PGCE courses. They are designed to meet the needs of new entrants to teaching whose

undergraduate degree is not in mathematics but in a related subject such as engineering or finance. Since such courses are more established and theorised than those developed to the DfES/FENTO (2002) subject specifications that are the subject of this article it is worth considering what lessons can be learnt from them, and whether those lessons are transferable to adult numeracy teacher training.

Shulman (1986), in developing a theoretical model for teacher knowledge that can be applied to mathematics (and adult numeracy) teacher training, defined the seven major categories of teacher knowledge shown in figure 1. The first four of these categories related to generic teaching skills and these were the mainstay of teacher education programmes at the time. These four categories were seen as relevant to all teachers irrespective of the subject-specific context of their teaching. Shulman acknowledged the crucial importance of these four categories for teaching but went on to propose three further categories that he termed content knowledge, curriculum knowledge and pedagogical content knowledge.

‘Content knowledge’ includes knowledge of the subject to be taught and how it is organised, including an understanding of which concepts are central to the discipline and which are peripheral (Ball, Thames and Phelps, 2008). This type of knowledge can be related to the expectations of the most experienced practitioners in Lucas, Loo and McDonald’s (2006) study of pilot DfES/FENTO courses who wanted a high level of theoretical content to provide them with a synoptic view of their specialism.

‘Curriculum knowledge’ relates to knowledge of the full range of courses available to teach particular subjects and topics at a particular level, including the range of instructional materials available (Ball, Thames and Phelps, 2008). It also includes ‘lateral curriculum knowledge’ (what is being taught to learners in other subject areas) and ‘vertical curriculum knowledge’ (what has been taught in the subject in previous years, and what will be taught in subsequent years).

Shulman’s final category of ‘pedagogical content knowledge’ sought to define that specific knowledge about a subject that is unique to teachers of the subject. It includes an awareness of what makes particular topics conceptually easy or difficult for learners to understand; the most useful analogies, illustrations, examples, explanations and demonstrations that can be used to support learning whilst remaining consistent to the integrity of the subject matter; and common conceptions and misconceptions of particular topics typically held by learners at different ages or ability levels (Ball, Thames and Phelps, 2008). Interestingly, Shulman’s approach was quite different to that of subject specifications and prescribed learning outcomes adopted by FENTO and its successor bodies in that he “did not seek to build a list or catalogue of what teachers need to know in any particular subject area” but instead “sought to provide a conceptual orientation and a set of analytic distinctions that would focus the attention of the research and policy communities on the nature and types of knowledge needed for teaching a subject” (Ball, Thames and Phelps, 2008, p.392).

By analysing Shulman's categorisation of different types of teacher knowledge it becomes apparent that his content knowledge related most closely to Bernstein's vertical teacher

knowledge whilst Shulman's curriculum knowledge and pedagogical content knowledge are more akin to Bernstein's horizontal teacher knowledge.

- General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter
 - Knowledge of learners and their characteristics
 - Knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures
 - Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds
 - Content knowledge
 - Curriculum knowledge, with particular grasp of the materials and programs that serve as “tools of the trade” for teachers
 - Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding
- (Shulman, 1987, p. 8)

Figure 1. Shulman’s major categories of teacher knowledge

There are currently two dominant views on the subject knowledge that mathematics teachers in secondary schools need to know to effectively teach their subject (Bell, Thames and Phelps, 2008). The first view is that they need to know whatever mathematics is in the curriculum at the level they are intending to teach plus some additional years of further study at a higher level of mathematics. The second view is that they need to know the mathematics in the curriculum at the level they are intending to teach, but that this should be a ‘deep understanding’ incorporating aspects of Shulman’s ‘pedagogical content knowledge’ (Shulman, 1986). The notion of deep understanding in mathematics is evident in the literature in a number of guises. Ma (1999), for example, refers to ‘profound understanding of fundamental mathematics’ whilst Adler and Davis (2006) use ‘understanding mathematics in depth’ to describe their conceptualisations of subject pedagogical knowledge.

Bringing together the theories of Bernstein and Shulman

Bernstein’s notion of the re-contextualisation of vertical teacher knowledge into horizontal teacher knowledge applied by Loo (2007a; 2007b) to adult numeracy teacher training and Shulman’s seven categories of teacher knowledge applied to secondary mathematics teacher training (Ball and Bass, 2003; Davis and Simmt, 2006; Ball, Thames and Phelps, 2008; Hodgen, 2011) can be brought together by considering the three examples of teacher training activities discussed earlier.

In example 1 the teaching of discrete probability distributions was discussed. Knowledge of discrete probability distributions (rectangular, binomial and Poisson) fits comfortably within Bernstein's vertical teacher knowledge in that it provides teachers with a synoptic view of their specialism. The re-contextualising of that vertical teacher knowledge into horizontal teacher knowledge is more problematic since the pedagogical techniques adopted of peer-led teaching and mathematical modelling could have been achieved more effectively through studying a numeracy concept drawn from the curriculum that trainees were being trained to teach, rather than through an unfamiliar mathematical topic that trainees themselves experienced as conceptually difficult. It could be argued, for example, that it would be more beneficial for teacher trainers to model the use of a 'washing line' strung across the classroom to order the probability of events occurring on a scale of 0 to 1 rather than being required to teach discrete probability distributions in the tradition of Bernstein's vertical teacher knowledge as a proxy for Shulman's pedagogical content knowledge.

In example 2, discussed earlier, the difficulties teacher trainers experienced in supporting trainees to re-contextualise Bernstein's vertical teacher knowledge into horizontal teacher knowledge was even starker. In this case the phrase 'do some hard sums' was contrasted negatively with what teacher trainers saw as necessary to equip trainees to teach adult numeracy effectively. Their response was to enrich the programmes (presumably by adding what they considered to be more relevant pedagogical content knowledge) to the content prescribed by the subject specification. In this case it could be argued that the trainers' pedagogical content knowledge replaced, or at least marginalised, the vertical teacher knowledge found in the subject specification in such a way as to obviate the need for the re-contextualisation by trainees of different types of teacher knowledge.

The broken keys activity described earlier in activity 3 resonates with the first example in that mathematical functions do not feature in the adult numeracy core curriculum (DfES, 2001). Nevertheless they appear to have been used with some success to introduce Shulman's pedagogical content knowledge by proxy through the use of Standards Unit (Swan, 2005) approaches to teaching mathematical functions. In spite of the apparent success of this approach it could again be argued that using the algebraic notation of functions unfamiliar to trainees adds an unhelpful layer of conceptual difficulty that clouds the more pressing concern of how to effectively teach the basic algebraic concepts found in the adult numeracy core curriculum (DfES, 2001).

Critical moment in a changing policy context

In recent times teaching has been practiced within a rapidly changing policy context (Ecclestone, 2008; Earley et al., 2012). This has led to changes in the way that the teaching role and teacher professionalism has been conceptualised, along with related changes within teacher training itself. It is within this context that a 'critical moment' for adult numeracy teacher training may emerge.

Current government policy in England raises the expectation that all school-leavers without the GCSE mathematics pass expected of sixteen year-olds should be required to retake the full GCSE in mathematics if they progress to full-time further education (DfE, 2013). Additionally, those school leavers progressing to full-time further education who have already achieved the GCSE mathematics pass expected of school-leavers should be required to continue to study mathematics to a higher level rather than being allowed to discontinue mathematics at age 16 as previously (ACME, 2012). Such an approach is seen by policy-makers as promoting the more rigorous and academic study of mathematics rather than the development of numeracy skills for vocational learners through qualifications such as adult numeracy and functional mathematics. Such curriculum reforms are seen by policy-makers as ensuring the UK can compete with leading industrialised nations (Vorderman, 2011).

Recent policy initiatives in teacher training for schools have included encouraging high-achieving graduates to enter teaching through targetted bursaries and to encourage school-centred initial teacher training (SCITT) consortia to provide teacher training as an alternative to more traditional university-led provision (DfE, 2010; DfE, 2011). Such an approach to teacher training assumes that the acquisition of subject content knowledge at a high level should be attained prior to entering teacher training, and that the practical skills of teaching itself are acquired as a 'craft' by working alongside practicing teachers. The speech by the Secretary of State for Education to the National College (Gove, 2010) expressed the view that "Teachers grow as professionals by allowing their work to be observed by other professionals, and by observing the very best in their field ..." and that "teachers ... improve their **craft** by learning from others while also deepening their academic knowledge" (my emphasis). The dichotomy between teaching as a craft and teaching as a profession was challenged by Kirk (2011) who argued that whilst teaching generates substantial personal craft knowledge, often in the form of tacit knowledge, it also required engagement with a broader type of knowledge that "... implies a professional duty to keep in touch with the literature of teaching and learning, and indeed to contribute to it as a way of raising the level of public and professional debate on teaching and learning" (Kirk, 2011).

Similar tensions have been experienced in the training of further education teachers to those found for schoolteachers. The Lingfield Report (2012) recommended the revoking of the regulatory framework for teachers in further education and called for new qualifications for teacher training to be developed by an employer-led 'guild'. However, Lingfield (2012, p.33) also called for a strong professional identity for further education teachers underpinned by increased autonomy to develop innovative pedagogies specific to the vocational focus that is unique to further education. Such practitioner-led enquiry hinted at by Lingfield (2012) is not new to further education. Previous initiatives have included the practitioner-led research initiative (NRDC) and the teacher enquiry funded projects (NCETM). Such initiatives were consistent with Hoyles' (1975) notion of extended professionalism and sit comfortably with emerging measures of professional esteem such as chartered mathematics teacher status and chartered status for further education teachers. In reflecting upon such initiatives, however, it

is necessary to sound a cautionary note concerning the culture within the further education sector that can mitigate against such initiatives. *The using research to enhance professionalism in further education project* (Economic and Social Research Project) identified that whilst practitioner research had a significant role to play in shaping the professional identities of those teachers that engaged in it, the benefits were often undermined by managerialist cultures within colleges where short-term gains, such as compliance with national policy agendas, hindered practitioners from asking more fundamental and critical questions about their practice (Goodrham, 2008).

The reforms to the post-16 mathematics curriculum described earlier in this section are a case in point where the shortage of qualified mathematics teachers to deliver the policy initiative has led to the launch of a government-subsidised six-day training programme intended to "further develop the skills of those currently teaching functional skills, preparing them to teach GCSE maths" (Education and Training Foundation, 2013). Such a quick-fix approach to numeracy training appears unlikely to provide teachers with the space or time to gain Bernstein's vertical teacher knowledge and re-contextualise it into horizontal teacher knowledge, nor to acquire those aspects of Shulman's subject pedagogical knowledge critical for effective teaching of numeracy to 'second-chance' learners in further education. Regional training programmes promoted as up-skilling teachers of numeracy by "enhance[ing] their knowledge so that they can teach GCSE effectively" (EMCETT, 2013) is likely to lower the status of numeracy teachers and undermine the gains made through the introduction of specialist teacher training for adult numeracy teaching rather than raise the quality of numeracy teaching. The ambitious targets set to engage post-16 learners in the study of mathematics up to the age of eighteen is laudable, as is the intention to enhance the subject knowledge of teachers so that they can effectively meet the challenges of the new curriculum. These targets and intentions need to be matched by a strategy for recruiting high quality graduates into teaching mathematics and then providing specialist teacher training courses to support them to re-contextualise their own knowledge of mathematics into effective numeracy pedagogies for further education. Similarly, experienced teachers of vocational subjects cannot be expected to retrain to teach GCSE mathematics without first being provided with the opportunities to increase their own mathematical knowledge to the standards that would be required for teaching in any other curriculum area.

Conclusions and recommendations

Mathematics subject knowledge should be a prerequisite for new entrants to numeracy teaching, whether for new entrants to teaching or for experienced teachers retraining to teach numeracy from other curriculum areas, in the same way that the best graduates and those with substantial vocational experience are sought as teachers for other academic and vocational subjects. Whilst it is unlikely that a consensus can be reached amongst the mathematics community on the detail of the content and level necessary, it is nevertheless important for the status of numeracy that minimum entry criteria be developed. These criteria should be credible when compared with entry requirements for teaching in other academic and vocational areas of further education.

Numeracy teachers should be given opportunities to build upon and extend their own mathematical knowledge and subject pedagogical knowledge throughout their careers, including at Masters level. They should be given support, time and space to develop innovative numeracy pedagogies related to the particular vocational contexts and specialist settings they encounter within further education. Supporting practitioner-led enquiry holds much promise as an effective form of continuous professional development for numeracy teachers.

Whilst acknowledging the benefits of observing the best teachers to learn the 'craft of teaching', it is also necessary to allow teachers the time and space to reflect on their professional learning with other trainee teachers. Such an approach is more likely to develop the critical skills to adapt to the fast-changing and policy-driven culture of further education than immersion in practice. The benefits gained from the subject specialist teacher training in adult numeracy from 2002 need to be maintained and strengthened if the challenges of post-16 curriculum reform are to be met.

Opportunities for developing links between further education and school-based mathematics and between further education and higher education should be grasped. These links can be beneficial both to share effective practice in teaching mathematics and to identify the nature of numeracy pedagogies specific to the contexts and learners in further education.

References

- Adler, J. & Davis, Z. (2006). Opening another black box: researching mathematics for teaching in mathematics teacher education. *Journal for Research in Mathematics Education*, 37(4), 270-296.
- Adler, J., Hossain, S., Stevenson, M., Grantham, B., Clarke, J. & Archer, R. (2009). Interpretations of, and orientations to, "understanding mathematics in depth": students in MEC programmes across institutions. In M. Joubert (Ed.), *Proceedings of the British Society for Research into Learning Mathematics*, 29(3).
- Askew, M., Brown, M., Rhodes, V. William, D., & Johnson, D. (1997). *Effective teachers of numeracy*. London: King's College London.
- ACME (2012) *Post-16 mathematics: a strategy for improving provision and participation - ACME post-16 mathematics project*. Last accessed 20 January 2014 at: http://www.acme-uk.org/media/10520/20121217acme_post_16_strategy.pdf.
- Askew, M. (2008). Mathematical discipline knowledge requirements for prospective primary teachers, and the structure and teaching approaches of programs designed to develop that knowledge. In P. Sullivan & T. Wood, (Eds.), *The International Handbook of Mathematics Teacher Education Volume 1: Knowledge and beliefs in mathematics teaching and teaching development*. Rotterdam: Sense Publishers.
- Ball, D. L. & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.) *A research companion to principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Ball, D. L., Thames, M. H. & Phelps, G. (2008). Content knowledge for teaching: what makes it special? *Journal of Teacher Education*, 59, 389-407.
- Bernstein, B. (2000) *Pedagogy, symbolic control and identity: theory, research and critique*. London: Taylor and Francis.
- BIS (2012). *The 'Lingfield report' professionalism in further education: final report of the independent review panel*. London: Department for Business, Innovation and Skills.

- Davis, B. & Simmt, E. (2006) Mathematics for teaching: an ongoing investigation of the mathematics that teachers (need to) know. *Education Studies in Mathematics*, 6 (3), 293-319.
- DfE (2010) *The importance of teaching: the schools white paper*. Norwich: The Stationery Office.
- DfE (2011) *Training our next generation of outstanding teachers: implementation plan*. Department for Education.
- DfE(2013) *Post-16 English and mathematics*. [Online]. Last accessed 20 January 2014 at: <http://www.education.gov.uk/childrenandyoungpeople/youngpeople/qandlearning/programmes/a00225749/post-16-eng-and-maths>.
- DfEE (1999). *The 'Moser report' improving literacy and numeracy: a fresh start*. Sudbury: Department for Education and Employment.
- DfES (2001). *Adult numeracy core curriculum*. Nottingham: DfES Publication.
- DfES (2004). *Equipping our teachers for the future: reforming initial teacher training for the further education sector*. Nottingham: DfES Publications.
- DfES/FENTO (2002). *Subject specifications for teachers of literacy and numeracy*. London: FENTO.
- Earley, P., Higham, R., Allen, R., Allen, T., Howson, J., Nelson, R., Rawar, S., Lynch, S., Morton, L., Mehta, P., Sims, D. (2012) *Review of the school leadership landscape*. Nottingham: National College for School Leadership.
- Ecclestone, K. (2008) The impact of assessment on pedagogy can have damaging consequences. In I. Nash, S. Jones, K. Ecclestone & A. Brown (Eds.), *Challenge and change in further education: a commentary by the teaching and learning research programme*. London: Teaching and Learning Research Council.
- Economic and Social Research Council. *Using research to enhance professionalism in further education*. [Online]. Last accessed 16 January 2014 at: <http://www.esrc.ac.uk/my-esrc/grants/L139271007/read>.
- Education and Training Foundation (2013) *The NCETM GCSE Mathematics Enhancement Programme*. [Online]. Last accessed 20 January 2014 at: <http://www.et-foundation.co.uk/services-and-support/maths-enhancement-programme.html>.
- EMCETT (2013) *Professional development*. [Online]. Last accessed 20 January 2014 at http://www.emcett.com/professional_development.
- Goodrham (2006) The place of research in FE professionalism must be protected. In I. Nash, S. Jones, K. Ecclestone & A. Brown (Eds.), *Challenge and change in further education: a commentary by the teaching and learning research programme*. London: Teaching and Learning Research Council.
- Gove, M. (2010). Presentation at the National College Annual Conference, Birmingham, 16 June 2010.
- Hodgen, J. (2011) Knowing and identity: a situated theory of mathematics knowledge in teaching. In Rowland, T. & Ruthven, K. (eds.) *Mathematical knowledge in teaching*. Dordrecht: Springer/Kluwer.
- Hoyle, E. (1975), Professionalism, professionalism and control in teaching. In V. Houghton et al. (eds.) *Management in education: the management of organisations and individuals*. London: Ward Lock Educational in association with Open University Press.
- Lawy, R. & Tedder, M. (2009). Meeting standards: teacher education in the further education sector. What of the agency of teacher educators? *Studies in the Education of Adults*, 41 (1), 53-67.
- Kaye, D. (2002) Defining numeracy. In J. Evans, P. Healy, D. Kaye, V. Seabright & A. Tomlin (Eds.), *Proceedings of the 9th International Conference of Adults Learning Mathematics (ALM9): policies and practices for adults learning mathematics - opportunities and risks*.
- Kaye, D. (2010) Defining numeracy - the story continues. In H. Christensen, J. Diez-Palomar, J. Kantner, & C. Klinger (Eds.), *Proceedings of the 17th International Conference of Adults Learning Mathematics (ALM): maths at work - mathematics in a changing world*.
- Kirk, G. (2011) It's not craft or profession. Teachers without both skills will be a walking disaster. [Online]. *Times Educational Supplement*, 11 February 2011. Last accessed 09 January 2014 at: <http://www.tes.co.uk/article.aspx?storycode=6069934>.
- LLUK (2007a). *New overarching professional standards for teachers, tutors and trainers in the lifelong +learning sector*. London: LLUK.
- LLUK (2007b). *New overarching professional standards for teachers, tutors and trainers in the lifelong learning sector: application of the standards for teachers of mathematics (numeracy)*. London: LLUK.
- Loo, S. (2007a). Theories of Bernstein and Shulman: their relevance to teacher training courses in England using adult numeracy courses as an example. *Journal of Further and Higher Education*, 31 (3), 203-214.

- Loo, S. (2007b). Learning to be teachers of adult numeracy. *Journal of Education for Teaching: International Research and Pedagogy*, 33 (4), 425-440.
- Lucas, N. (2007). The in-service training of adult literacy, numeracy and English for Speakers of Other Languages teachers in England; the challenges of a 'standards-led model'. *Journal of In-Service Education*, 33 (1), 125-142.
- Lucas, N., Casey, H., Loo, S., McDonald, J., Young, M. & Giannakaki, M. (2004) *New initial teacher education programmes for teachers of literacy, numeracy and ESOL 2002/3. An exploratory study*. London: NRDC.
- Lucas, N., Casey, H., Loo, S. McDonald, J., Sagon, O. & Koutsoubou, M. (2006). *Towards a professional workforce: adult literacy, ESOL and numeracy teacher education 2003-5*. London: NRDC.
- Lucas, N., Loo, S. & McDonald, J. (2006). Combining 'subject knowledge' with 'how to teach': an exploratory study of new initial teacher education for teachers of adult literacy, numeracy and ESOL for Speakers of other languages. *International Journal of Lifelong Education*, 24 (4), 337-350.
- Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, NJ: Lawrence Erlbaum.
- National Centre for Excellence in the Teaching of Mathematics. *Teacher enquiry funded projects*. [Online]. Last accessed 10 January 2014 at: https://www.ncetm.org.uk/enquiry/funded-projects/funded_projects_tefp.
- National Research and Development Centre for Adult Literacy and Numeracy. *Practitioner-led research initiative*. [Online]. Last accessed 10 January 2014 at: <http://www.nrdc.org.uk/content.asp?CategoryID=512>.
- Ofsted (2003). *The initial training of further education teachers: a survey*. London: Ofsted.
- Sheffield Hallam University (2013). *Subject knowledge enhancement (SK1) secondary mathematics education*. [Online]. Last accessed 25 July 2013 at: <http://www.shu.ac.uk/prospectus/course/1061/>.
- Shulman, L. S. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher*, 15 (2), 4-14.
- Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20-26.
- Stevenson, M. (2008) Profound understanding of fundamental mathematics: a study into aspects of the development of a curriculum for content and pedagogical subject knowledge. In M. Joubert (Ed.), *Proceedings of the British Society for Research into Learning Mathematics*, 28 (2).
- Stone, R. and Griffiths, G. (2006). What should pedagogy for adult numeracy consist of? Or it ain't what you do but the way that you do it. In V. Seabright & I. Seabright (Eds.), *Proceedings of the Adults Learning Mathematics (ALM) 13th Annual International Conference: crossing borders – research, reflection and practice in adults learning mathematics*.
- Swan, M. (2005). *Standards Unit – improving learning in mathematics: strategies and practices*. Nottingham: DfES Publications.
- Vorderman, C. (2011) *A world class mathematics education for all our young people*. Last accessed 20 January 2014 at: <http://www.nationalnumeracy.org.uk/resources/15/index.html>

Provoking Mathematical Thinking: Experiences of Doing Realistic Mathematics Tasks with Adult Numeracy Teachers *

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Abstract

This action research project looks at what happened when a small group of adult numeracy teachers with widely different experiences of learning and teaching mathematics explored their own informal numeracy practices and undertook a series of collaborative mathematical tasks. Evidence from qualitative data collected during the enquiry suggests that realistic tasks can provoke a range of mathematical thinking and learning responses which allow us to identify ways in which procedural and conceptual thinking is being used, and to track learning journeys through different stages of problem-solving. Although more experienced numeracy teachers could move between and within their 'real worlds' and 'maths worlds' with intent and ease, others had less integrated experiences, often valuing perceived mathematical powers over their own intuitive powers, with mixed success.

Key words: mathematical thinking, action research, adult numeracy teachers, realistic, realisable, mathematisation, collaborative classroom, intra- and extra-mathematical.

Introduction

Historically, within the UK, adult numeracy teaching is a field that many people move into sideways, often from teaching other disciplines. The requirement for practitioners to have a set level of personal mathematics skills was introduced only relatively recently and it is not untypical to find teachers of numeracy who lack confidence in their own mathematical ability (Cara et al., 2010). Personal mathematics development is therefore an important component within many pre- and in-service adult numeracy teacher education programmes. Teachers are encouraged to develop their mathematical thinking throughout their training, both by participating in class activities and pursuing private study. As a tutor and course leader on such a programme, I have observed that when it comes to building a personal mathematics portfolio, many teachers exhibit fairly mechanistic and unreflective ways of working. This is true not only in terms of the approaches they adopt, but also the sorts of independent tasks they choose to undertake - often a surprisingly narrow diet of content-driven and competence-based exercises. The purpose of this research project was to explore how to better support adult numeracy teachers to develop and extend their own mathematical thinking. The rationale for this extends beyond the perceived need for adult numeracy teachers to 'upskill' and is based on the underlying assumption that developing teachers'

confidence, awareness and insight into their own mathematical thinking, will better equip them to develop and extend the mathematical thinking of their learners.

Method of enquiry

The classroom, tutor and teachers

The twelve teachers in the group participating in this enquiry were aged 25-55, from socially and ethnically diverse backgrounds and included two teachers whose first language was not English. They were all undertaking professional development in adult numeracy teaching and consistently demonstrated high levels of motivation and engagement although their personal experiences of mathematics, both as teachers and learners varied tremendously.

Within the group, we had negotiated a shared sense of adult numeracy as involving more than basic mathematical skills, or the application of mathematics in everyday life but rather numeracy as a way of negotiating the world through mathematics, “*not less than maths but more*” (Johnston & Tout, cited in Coben, 2004, p3). In the course of working together during the year, we had tried to develop a co-operative and conjecturing classroom - a milieu that explicitly challenged deficit models of adult numeracy. This ethos was influenced by the idea of *funds of knowledge* describing the informal knowledge, skills and experiences that adult learners can draw on but may not be evidenced by formal qualifications (Moll et al., 1992; Baker, 2005), a concept that can be broadened to include interpersonal and metacognitive skills.

Responses to initial classroom probes into their mathematical thinking suggested that few of the teachers moved flexibly between different representational modes. The most mathematically experienced wanted to adopt a symbolic or algebraic response whenever possible, with few trying out more practical approaches. The least experienced saw this use of ‘formal’ mathematical methods as their ultimate goal, placing less value on other approaches. This apparent lack of variety on the teachers’ own mathematical journeys was often in contrast to the active learning and multi-sensory approaches they were developing to support mathematical thinking with their own learners. The initial focus of the enquiry was to explore how to provoke adult numeracy teachers to think and act less mechanistically as ‘doers’ of mathematics themselves.

Methodology

The enquiry adopted an action research approach based on the idea “*that a practitioner is involved in analysing a situation, planning an alternative action, carrying out that action, and then evaluating the effects of what they have done*” (Mason, 2002, p172). The research was broken down into three smaller cycles or phases of enquiry and reflection. These were undertaken over an eight week period in the final semester of the course.

All participants within the group were involved in research design tasks for about an hour a week in class with some out-of-class time required for auditing, self-reflection and write-ups. Although an essentially social constructivist perspective informed the research focus and the design of classroom interventions, the research methodology itself was mixed. Data collection from tutor field-notes and audio-recordings of semi-structured group discussions focussed on teachers' interpretations and evaluations of tasks undertaken in and out of the classroom, suggestive of an ethnographic approach. Other sets of data, however, were generated from audio-recordings of pair discussions, stimulated recall interviews, written work and tutor observations which aimed to capture responses to paired and individual tasks. Though more typical of positivist methodologies, these provided rich qualitative data which allowed me as a practitioner researcher to experience more fully what happened as teachers engaged in tasks.

Mason (2002, p52) suggests that in researching one's own practice, it is useful to differentiate between giving a *brief-but-vivid* "account-of what was seen, heard, experienced" and analysing, explaining or "accounting-for" incidents. *Accounts-of* will be used to illustrate salient incidents and experiences, along with excerpts from edited transcripts of audio-recordings and examples of teacher responses to tasks. Data analysis will be through a mixture of event sampling using and adapting pre-specified categories from wider theoretical and empirical research, and *accounting-for* recurring phenomena using key constructs and frameworks which are reported within each of the three action research cycles.

This paper will now outline key findings from cycle 1 of the enquiry before going on to focus in particular on significant moments arising from data generated within naturally occurring peer-peer discourse between two pairs of teachers during the second and third cycle of the enquiry.

Cycle 1—Awareness raising

Gattegno (1988, p167) highlights the importance of teachers sensitising themselves to their own behaviours, emotions, and awarenesses:

Teachers need to make themselves vulnerable to the awareness of awareness, and to mathematization, rather than to the historical content of mathematics. They need to give themselves an opportunity to experience their own creativity and when they are in contact with it, to turn to their students to give them the opportunity as well.

In considering what sorts of mathematical activities to use within this action research, I wanted tasks that would support teachers to take the initiative and become more fully engaged in their own mathematical thinking. Schoenfeld (1994) developed a broad and age-independent description of what learning to think mathematically means:

1. Developing a mathematical point of view – valuing the process of mathematisation and abstraction and having the predilection to apply them.

2. Developing competence with the tools of the trade and using these in the service of the goal of understanding structure – mathematical sense-making.

But what did mathematical thinking and mathematisation look like ‘outside formal mathematics classrooms’? Research has demonstrated that adults have access to many informal numeracy practices (Street, 1984; Nunes, Schliemann & Carraher, 1993a; Baker & Rhodes, 2007). The idea that teachers need to become aware of learners’ innate or natural powers to think mathematically (Mason and Johnston-Wilder, 2006) is echoed in a number of recent research reports (Swan, 2006; Swan and Swain, 2007). Indeed, much official discourse now actively encourages adult numeracy teachers to “*build on the knowledge learners already have*” (Swain et al., 2007, p. 7).

The belief in the importance of teachers’ recognising their own *funds of knowledge* and exploring innate *mathematical sense-making* powers themselves, provided the initial impetus for considering *everyday* contexts and numeracy in the task design. By exploring what we as adult numeracy practitioners noticed about our own numeracy practices, would any shared characteristics, prior knowledge or behaviours related to mathematical thinking emerge to inform the design of tasks for subsequent action research cycles, for both experienced and less experienced participants?

Task design 1

During the first week of the enquiry, teachers and tutors made diary notes about what they identified as their numeracy practices over the course of a ‘work-day’ and a ‘non-work day’. These were mostly handwritten on two large A3 diagrams resembling a clock face. A further record sheet was completed during the second week. This required us to identify and classify mathematical behaviours we noticed according to what Bishop (1988) identified as six universally occurring activities: counting, locating, measuring, designing, playing and explaining.

Analysis

Each week, findings were shared with peer partners. Subsequent whole group discussion were animated, as numerous and at times conflicting *accounts-of* and *accounts-for* were generated:

Accounts 1

The supermarket does all the price comparisons – I just read the labels.

We’re on a really tight budget so I’m working out stuff with money all the time.

I get the kids to help with the adding up when we’re in the supermarket.

I was quite shocked – I do more maths out of work than when I’m teaching.

It took ages to park this morning. Usually there are a few places left, but today it was practically deserted.

I didn't realise how much time I spend in the car at the moment – there's journey times, buying petrol, using maps and Google directions, speeds and signs, even working out the best lane to be in where all the road works are.

I'm totally addicted to Sudoku at the moment – my son and I try to see who can finish first.

Teacher and tutor *accounting* suggested that as well as becoming more sensitised to our own numeracy practices, we engaged in a diverse range of socially and culturally situated mathematical behaviours. Although some of us identified possible mathematical topics and themes related to particular situations or times, others discussed what they actually did. Many omitted or ignored things they did not consider mathematical but “*just common sense*”. This illustrates how difficult it is to design learning tasks tailored to each individual's particular experiences.

Data from this part of the enquiry did however suggest some common characteristics of the group's everyday numeracy practices which tended to involve purposeful activities which were often collaborative e.g. family activities involving playing, cooking, shopping or constructing. These were often linked to particular roles and could be dependent on and shaped by particular tools or realia e.g. maps, self-service checkouts, petrol pumps, Sat Navs. This is in line with findings from similar studies into everyday numeracy practices (Lave, 1988; Harris, 2000; FitzSimons, 2005). For example, in reviewing a range of empirical research some 20 years ago, Resnick (1987) noted that much activity outside classrooms is socially shared. She contrasted examples of shared knowledge and understanding, tool manipulation, contextualised reasoning and situation specific competencies from everyday numeracy practices with the sorts of individual knowledge and skills, abstraction, symbolic manipulation and generalised learning more likely to be experienced in many formal mathematics classrooms.

Implications from Cycle 1

- This initial analysis suggested that the teachers' informal numeracy practices could be drawn on more effectively by providing tasks which afforded:
- Opportunities for them to work together on problems.
- Access and use of cognitive tools.
- Direct engagement with objects and situations rather than purely symbolic thinking.
- Use of situation-specific competencies (adapted from Resnick, 1987).

However, the overall goal was to further develop and extend these teachers' mathematical thinking; to build on existing knowledge *and* ensure those with little or less successful

experience of learning maths were empowered to operate successfully within formal mathematics classrooms too. To this end, the framework above merely presented possible points of departure.

In terms of identifying an actual topic base for the mathematical tasks to be used in the next cycles of the research, I was particularly struck by the relatively infrequent use made of ‘standard’ measures or indeed measuring devices during awareness raising activities in Cycle 1. Discussions with teachers revealed resonated experiences and generated additional complex, contingent and subjective strategies for measuring and estimating everyday phenomena:

Accounts 2

In the morning I know when the bath’s getting full ... I can hear how long I’ve got to drink my coffee.

I can estimate how much it’ll cost by how full the trolley is.

I know how much squash to add by the colour – not dilution ratios!

I measure how crowded a place is by how far I have to go to get to an uncrowded place.

Buying petrol has nothing to do with gallons or litres...

Don’t need an alarm clock... my dogs tell us when it’s time to get up.

Cycle 2—Plausible estimates

Subsequent research and review of potential mathematical thinking tasks which could be adapted in accordance with the research focus and findings to date, uncovered a number of suitable open-ended tasks based on estimation and measure. A set of classroom assessment tasks (CATs) which had already been field-tested were chosen for cycle 2 of the research. These involved “*Making plausible estimates*” based on Fermi-type problems (Ridgeway and Swan, 2010).

Task design 2

Figure 1 details the task objectives presented to the whole group:

The aim of this task is to provide the opportunity for you to work with your partner to:

- Make sensible assumptions
- Develop a chain of reasoning
- Choose suitable units
- Communicate your assumptions and reasoning effectively to peers

Extension: Identify upper and lower bounds i.e. what range of values would you give in order to be pretty certain that you have included the true value being estimated?

Figure 1. Plausible Estimations

Tahta (1981) makes a useful distinction between *inner* and *outer* tasks which helps here to distinguish between the explicit *outer* task of finding a plausible estimate and the intended *inner* task which would allow both teachers and tutor to gain experience of what mathematical thinking and communicating might look, behave and feel like.

By building on a range of theoretical and empirical research, Goos et al. (2004, p100) identify five assumptions they argue are crucial to creating a culture and ‘*community of mathematical inquiry*’:

1. Mathematical thinking is an act of sense-making, and rests on the processes of specialising and generalising, conjecturing and justifying.
2. The processes on mathematical inquiry are accompanied by habits of individual reflection and self-monitoring.
3. Mathematical thinking develops through student scaffolding of the processes of enquiry.
4. Mathematical thinking can be generated and tested by students through participation in equal-status peer partnerships.
5. Interweaving of familiar and formal knowledge helps students to adopt conventions of mathematical communication.

Mindful of the desire to value and develop teachers’ informal and formal mathematical experiences, I found the first of these assumptions resonated strongly with the notion of accessing learners’ *innate powers* and the last two strongly influenced my decisions to conduct the plausible estimation sessions with particular peer partners, and to require teachers to present and justify their findings to the whole group. The focus of analysis within this cycle of the research also moved onto data generated by two pairs of teachers within the group who fulfilled certain contrasting characteristics related to previous experience of teaching and learning mathematics.

Teachers M and N were confident in using higher level mathematical skills, had studied mathematics at university level and each had at least five years' experience of teaching mathematics to adult learners mainly within further education settings. They were given the 'mummies' task in Figure 2.

An unravelled roll of paper is 33 metres or 100 feet long.

Will one roll be enough to wrap a person up?



Figure 2. Mummies

Teachers R and S were less confident in their mathematics skills and knowledge, had no formal mathematics qualifications beyond a foundation level and had quite recently become involved in teaching adults numeracy within their respective work-based training organisations. They worked on the 'briefcase of pennies' task in Figure 3.

Suppose you filled a briefcase with one penny coins.

How much money would you have?



Figure 3. The briefcase of pennies

Before considering in more detail what unfolded as these teachers engaged with their plausible estimation tasks over the next two week period, it is important to outline further theoretical frameworks which significantly impacted on both the conduct and analysis of data from this second cycle of enquiry.

Realistic maths and mathematisation

The idea of relevance and realism within mathematics teaching is complex and contested. Many *authentic* mathematics and *real* problem solving approaches advocate settings and situations which try to motivate and engage learners by using topics relevant to their immediate concerns. However Swain et al. (2005) argue that is the quality of an individual's

engagement with a problem that makes math *meaningful* rather than its utility or everydayness. Others, like Cooper and Dunne (2004) highlight the hidden rules younger learners must negotiate when tackling *contextualised* word problems and how these can adversely impact on learners from different cultural or social backgrounds.

Realistic mathematics is a term which better describes the sorts of tasks adopted within this enquiry and relates to an approach developed by Freudenthal (1991) which accentuates the actual activity of *doing* mathematics and advocates the power of learners to make things real for themselves by using their imagination. Such *realistic* tasks require learners to mathematise subject matter from *real* or *realisable* situations and reinvent mathematical insights, knowledge and procedures in the course of “*their own mathematical activity rather than from the traditional view of presenting mathematics to them as a ready-made system with general applicability*” (Gravemeijer cited in Barnes, 2004, p5). These situations can include contextual problems or mathematically authentic contexts for learners where they experience the problem presented as relevant and real. Such a process- rather than content-driven approach was consistent with the research focus of developing more relational and creative mathematical thinking and built on earlier project findings. The ‘plausible estimation’ tasks are examples of such *realistic* tasks.

Horizontal and vertical mathematisation

Additionally, in responding to the challenge that both learners and teachers often experience in trying to distinguish between concepts and procedures in mathematical thinking, Treffers (1987) developed the idea of *horizontal* and *vertical mathematisation* within this *realistic* maths framework. According to Freudenthal (1991) horizontal mathematisation ‘leads from the world of *life* to the world of *symbols*’ (p.41), which Barnes (2004) suggests happens when learners use their informal strategies to describe and solve a contextual problem. On the other hand, *vertical mathematisation* occurs when the learners' informal strategies lead them to find a suitable algorithm or to solve the problem using mathematical language. For Freudenthal (1991), this is where ‘symbols are shaped, reshaped, and manipulated, mechanically, comprehendingly, reflectingly’ (p.41). For example, in the case of the ‘plausible estimation’ tasks, the process of establishing the important information required and using an informal strategy such as trial and improvement to arrive at an estimate would be *horizontal mathematisation*. Translating the problem into mathematical language through using symbols and later progressing to selecting an algorithm such as an equation could be considered *vertical mathematisation*, as it involves working with the problem on different levels. This framework will be used within the analysis of data generated during teacher work on the ‘plausible estimation’ tasks.

Analysis

When first presented with these Fermi-type problems, teachers responded with surprise and some uncertainty:

Extract 1

R: Make it pound coins and I might have a go.

S: How are we going to do this then?

R: Haven't a clue.

This intrigue right at the start of the tasks worked effectively to harness teachers' emotions. Teachers M and N worked flexibly between mainly *iconic* or visual and *symbolic* modes of representation. They modelled a human being as a cylinder, deciding this would be the most effective way of minimising surface area rather than use a collection of smaller cylinders; agreed symbolic formulae for this surface area; assigned variables and established a direct proportional relationship between foot length and height.

R and S initially worked in an *iconic* mode with mental images of their own bags and briefcases, before looking around the room for more immediate concrete items which approximated their mental image of the briefcase on the task sheet. They stayed in this *enactive* mode, using a ruler to measure an actual briefcase and the diameter and depth of a 1p coin. Cooper & Dunne (2004) might argue that this is an example of teachers not understanding conventions for estimating and rounding. However, taking time within this mode allowed R and S to feel confident in their estimates before moving on to iconic and symbolic approaches. M and N, in contrast showed throughout the task that they knew and understood 'the rules' of the mathematics classroom:

Extracts 2

M: We've worked to the nearest whole number throughout so we should use 2m for the height.

And later:

N: The task just wants us to decide yes or no, so we've done it. We don't need to work out how much more paper we'd need.

Ironically, their numerical estimates were the weakest chain in their initial argument as they got caught up in the mathematical conventions and opted for a height of 2m rather than their original 1.8m. The extension task which M and N completed required them to establish upper and lower bounds and brought the idea of *dimensions-of-possible-variation* (Marton and Booth, 1997) into play. By considering the *range-of-permissible-change* (op cit) for the heights of adults and children, constraints on their first model were established and a new model created which would incorporate a baby's particular body shape and size. This extension task had extended the teachers' mathematical thinking by requiring them to look for *invariance in the midst of change* (op cit). Overall, M and N demonstrated sophistication in their relational understanding and worked together to integrate relevant real-life experiences as they engaged flexibly between *horizontal* and *vertical mathematisation* of the

problem (Treffers, 1987). The ease with which I was able to identify and analyse M and N's responses to this task using the frameworks provided is also significant as it may provide an *account of* what I readily recognise and implicitly value as 'mathematical thinking'.

R and S had a significantly different journey through the task. S began to *mathematise vertically* using an algorithmic procedure involving volume. She wanted to work out the volume of the case and then to divide by the volume of a single coin and remembered that $V = l \times w \times h$ "maybe". R, on the other hand, began with *horizontal mathematisation* - drawing lines of pennies as he built up a sense of volume through layering. His approach required him to find out how many coins were needed to layer out the briefcase and he struggled to make sense of the approach adopted by S:

Extract 3

R: I'm just using a practical approach that I understand but S's solution is more mathematical.

R's implicit value judgment here about some sorts of mathematical thinking being more valuable than others resonates powerfully. Barnes (2004, p59) argues that using a formula does not necessarily imply better conceptual understanding and warns of the "*danger of focusing too much on vertical mathematisation*". In fact, when R and S discovered that their initial results did not match, it was by using R's *horizontal mathematisation* that they were able to establish that an error must have been made with S's conversion rates. Indeed, R had a very solid conceptual understanding of volume whereas S had adopted *vertical mathematisation* mechanically but without Freudenthal's (op cit) other two requirements: comprehension and reflection. This prevented her from spotting the common misconception made in converting between units of volume. It was by reverting back to R's layering approach that both were able to work out that $1 \text{ cm}^3 = 10\text{mm} \times 10\text{mm} \times 10\text{mm}$ and come to an agreed plausible estimate. Having to articulate for the whole class their chains of reasoning, initial assumptions and ways of validating results helped both R and S strengthen their understanding of the general algorithms they had adopted, although more time to consider upper and lower bounds may have consolidated this further.

Although R and S were less confident in terms of their *intra-mathematical* skills, it is important to note that they brought a range of social, communication and meta-cognitive skills and experiences to the task process which allowed them to discuss, peer check and when necessary seek help from peers and tutor. They were tenacious, supportive of each other and prepared to take their time, progressing with small steps along repeated cycles of what Mason, Burton and Stacey (1985, p156) describe as "*the helix of manipulating – getting a sense of – articulating*" when thinking mathematically.

Both sets of teachers were provoked by the tasks to fall back on their own experiences and access a range of personal 'everyday' or *extra-mathematical* knowledge in diverse ways. M and N drew confidently on their experiences of child birth to establish estimates for the width

of the head and the length of a new born baby. However, when asked how they might check their findings, neither M nor N wanted to try out their solutions. The intrinsic motivation and interest was in the *intra-mathematical* process – the accuracy of their final estimation all but redundant. M and N had quickly moved from the *real* world to their own *mathematical* world and intended to stay there. In contrast, when asked how they would check their estimate, R and S went straight back to *extra-mathematical* knowledge of their *real* world:

Extract 4

S: You'd not use volume at all – you'd empty the case and weigh them.

R: Just like they do in the bank.

This willingness (or not, in the case of teachers M and N) to re-engage with the *real* world scenario in order to evaluate not only the plausibility of the estimates found but the validity of the *mathematisation* process itself may have significant implications for the teachers' own professional practice. Arguably, failing to reinterpret and validate mathematical results within *real* situations can result in leaving *unrealistic* modelling unexposed. For some learners this sort of uncritical mathematical thinking does nothing to close 'the gap' between *real* and *maths* worlds.

When asked whether the 'Pennies in a Briefcase' task had been useful, R and S responded:

Extract 5

S: Yes, it got us thinking – we had to use lots of different sort of maths.

R: We'd forgotten lots. I think I understand units of volume better now.

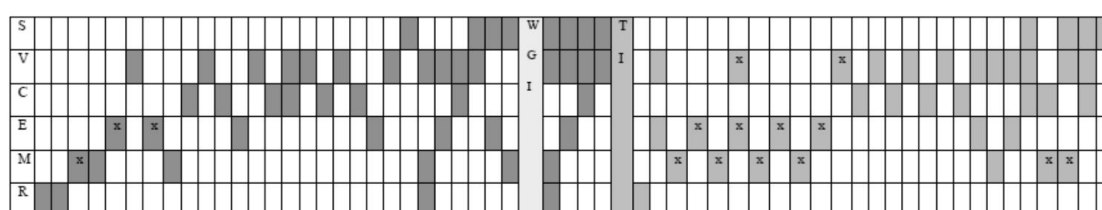
Although these 'Plausible Estimation' tasks required only a basic knowledge of geometry, numeric skills and units of measure, the teachers did engage in more relational and connected thinking. Misconceptions related to conversions of units, use of appropriate formulae and rounding errors were identified through self and peer monitoring and teachers seemed to develop a more intrinsic feeling for the plausibility of their estimates. The value of developing conceptual and procedural knowledge in tandem seemed clear to all participants, and some teachers were also able to reflect more confidently and critically on their chains of reasoning.

Coding framework for plausible estimates

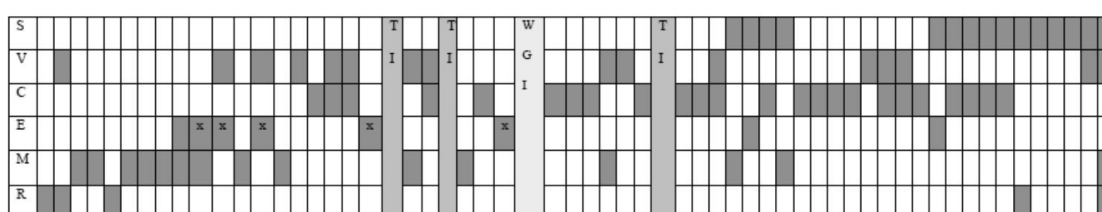
A more analytical comparison of the mathematical thinking and specific problem solving strategies the two pairs of teachers adopted in moving from *real* worlds to their *maths* worlds and (sometimes) vice-versa is difficult, not least because they were undertaking two different tasks. However, by adapting a framework devised by Arleback (2009), I was able to encode data from recordings of peer-peer discussions during the 'Pennies in a briefcase' and 'Mummies' tasks:

1. Reading: reading the task and getting an initial understanding of the task
2. Making model: simplifying and structuring the task and mathematising
3. Estimating: making estimates of a quantitative nature
4. Calculating: doing maths - performing calculations, solving equations, drawing diagrams
5. Validating: interpreting, verifying and validating results: calculations and the model itself
6. Summarising: summarising the findings and results in writing or orally

1. M and N: Plausible estimation – Wrapping a mummy



2. R and S: Plausible estimation – Pennies in a briefcase.



This pair did not have time to progress to the extension task.

R = Reading M = Making model E = Estimating C = Calculating V = Validating S = Summarising

Figure 4. Mathematical behaviours during ‘plausible estimations’

Figure 4 aims to capture a macroscopic and fairly dynamic picture of how teachers were *heard* to move between different ‘behaviours’ during the audio-recordings of the first 30 minutes of paired work on these tasks. Coded activities are identified within blocks, representing approximately 30 second time intervals. A whole group tutor intervention (WGI) took place after 15 minutes, and tutor interventions (TI) for particular pairs are also identified. X indicates where teachers have explicitly used *extra-mathematical* knowledge and experiences in diverse ways as outlined earlier.

Interestingly, although R and S had divergent calculation strategies during their tasks, the actual mathematical behaviours displayed in the diagram were similarly categorised within this framework as was the modelling stage which did not differentiate between horizontal and vertical mathematisation.

Implications from Cycle 2

Although the main value of these diagrams to me as a practitioner comes from the actual process and challenge of coding and categorising the peer-peer discussions, they do also provide some triangulation of earlier observations on how and when *extra-mathematical* knowledge is used, some new insights into the timescale of comparative progress through the tasks by both pairs, the frequency with which the teachers validated results and the time taken to summarise findings in preparation for articulation to the whole group. Arleback (2009) noted similar phenomena with his learners and observed that validation of results involved checking calculations, estimations *and* the initial model. However although both pairs of teachers here used articulation to summarise *and* peer validate their calculations, results and decision making processes throughout, M and N were more reluctant to ‘re-enter’ the messier *real* world once they had found a comfortable place of abstraction in their *maths* world.

Cycle 3—Creating measures

For the final cycle of the research, a second set of field-tested mathematical tasks were used. These aimed to prompt teachers “*to evaluate an existing measure of an intuitive concept and then create and evaluate their own measure of this concept*” (Ridgeway and Swan, 2010). A key component within this cycle would be the requirement for both pairs of teachers to test and evaluate any measures created back in the *real* world.

Task design 3

Requiring teachers to start from everyday concepts – *steep-ness, sharp-ness, awkward-ness, compact-ness, crowded-ness and square-ness* – to mathematise phenomena by creating their own measures seemed even more closely related to the experiences of awareness-raising in the first cycle of the enquiry and consistent with the sort of mathematisation and guided re-invention advocated by a *realistic* maths approach. As well as provoking mathematical thinking, I hoped these tasks would afford *meaningful* two-way connections between *real* and *maths* worlds.

Experiences during cycle two of the enquiry suggested that peer partners worked well together. This time however, I provided more scaffolding in the form of prompts in teachers’ work packs, so that tasks could be sustained and worked on independently. These included regular self-monitoring and reflection opportunities, consistent with the second and third assumption identified earlier as crucial to a *community of mathematical inquiry* (Goos et al., 2004). Teachers worked on these extended tasks in class each week for an hour over a three week period. Although they had individual work packs, pairs were expected to work collaboratively to reach a point where they would be able to go out on campus to test whether their measures actually worked. A written summary of findings ‘so far’ with commentaries, photographs and individual reflections on the creating measures process would provide evidence for teachers’ personal mathematics portfolios.

Before tasks were distributed, an introductory activity was undertaken to encourage teachers to consider themes, processes and specific features evoked by particular concepts:

- With your partner, take a few minutes to discuss what the concept of ‘**sharp-ness**’ means to you both.
- This might include thoughts, images, experiences, associations, special words or phrases, contexts or feelings.
- Use a concept map to record your initial responses.

Figure 5. Example of introductory activity for ‘creating measures’ task

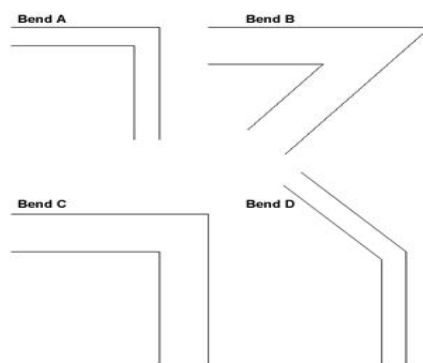
When finally presented with their actual tasks, several teachers experienced what Mason & Johnston-Wilder (2006, p96) describe as “*a contradiction of expectation*” which they argue is a useful disturbance to provoke activity:

Extract 6

M: Oh, it’s nothing to do with pain or needles ...

The actual ‘creating sharp-ness’ activity presented to teachers M and N is shown here:

Without measuring anything, put the four bends in order of "sharp-ness".



Explain your method clearly.

Figure 6. ‘Sharp-ness’ Activity 1 Warm-up

This first activity specifically invited teachers to engage with *iconic* modes of representation. By inviting them to ‘look first, and act later’, I hoped that the teachers would use their own mental imagery and innate sense-making powers to identify similarities and differences between images, to specialise and generalise, order and classify and begin to become aware of some of the properties of the bends, or in the case of teachers R and S, the staircases which they might be able to explore later:

Without measuring anything, put the staircases in order of "steep-ness".

Explain your method clearly.

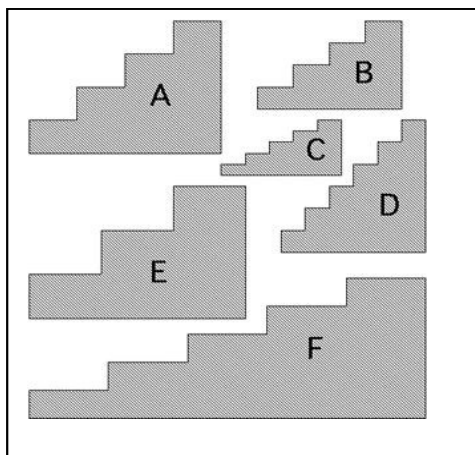


Figure 7. ‘Steep-ness’ Activity 1 Warm-up

Although these two dimensional images were less life-like than those used in the earlier ‘plausible estimation’ tasks, they were not conventionally mathematised to one-dimensional lines. Another feature of the classroom at the start of this third cycle of enquiry was the availability of mathematical equipment – tools for measuring, different sorts of paper including square, graph and dotted, calculators, counters, centicubes, etc. Indeed, all tasks required teachers to undertake some hands-on measurement, ensuring that everyone got involved at an *enactive* level, quite literally manipulating, constructing and measuring particular properties of their task concepts. Nunes et al. (1993b) identify the significance of such *measuring* tools in supporting mathematical reasoning in younger learners and increasingly adult learners are being re-introduced to the power of multi-sensory approaches to mathematical sense-making. These tasks required that my teachers did the same.

Figure 8 shows how the learning objectives for the ‘steep-ness’ task were introduced to teachers R and S:

Objectives

This problem gives you the chance to:

- criticise a given measure for the concept of "steep-ness"
- invent your own ways of measuring this concept
- examine the advantages and disadvantages of different methods.

Figure 8. ‘Steep-ness’ Task objectives

Analysis

In their initial discussions on the staircases, R and S identified a range of factors influencing their perceptions of steepness: personal preferences about heights and depths of steps, fitness and stamina, carrying shopping bags, going up or down, taking single or multiple steps, individual heights and builds, disabilities, indoor or outdoor steps, surfaces, ‘length’ of staircases. Their considerations were very much rooted in the social context of the staircase journeys – who, why, when, where, how often. Rather than a straightforward exercise in finding gradients, R and S were tackling a much more complex modeling task within the real-world scenario they had created.

M and N on the other hand again moved ALMost immediately to abstract mathematisation, exploring how they could use trigonometry to create a measure of ‘sharpness’, focusing solely on angles and width with no consideration of other contextual factors. When prompted, they were able to generate other variables: roads, lanes, vehicles, weather, surface, speed, visibility, gradient, etc. but the relevance of these only really became apparent to them when they went outside to test their new measure in the messier real world. Figure 9 provides a brief *account-*of their measure for ‘sharpness’:

Measures of Sharpness

M:
$$r = \frac{x}{1 - \sin \theta}$$

N:
$$\frac{r}{x} = \left(1 + \frac{\tan \theta}{\tan\left(\frac{90 - \theta}{2}\right)} \right)$$

They prove:
$$\frac{1}{1 - \sin \theta} \equiv \left[1 + \frac{\tan \theta}{\tan\left(\frac{90 - \theta}{2}\right)} \right]$$

when $\theta \neq 90^\circ$

And test their measures outside with real world bends.

Figure 9. ‘Measures of sharpness’ invented by teachers M and N

This may also convey some the unconscious assumptions and value judgements that I, as someone more comfortable within the abstract maths world of algebra myself, make about what mathematical thinking looks like. It certainly *accounts-for* some of my confidence that such realisable tasks can provide effective points of departure for diverse groups of teachers to

engage in doing, thinking and communicating mathematically and to recognise what this engagement entails.

Although data from this third cycle of the enquiry provided many other textured examples of ways in which the ‘creating measures’ tasks provoked teachers to engage in mathematical thinking, I will finish by focusing on one further incident that was particularly significant and indeed disturbing:

Extract

S: Before today I thought I could look at a slope and know how steep it was. But when you do the measurements, you realise it’s different. I’ll never decide about steepness by eye again.

What had happened for this teacher to conclude that her intuitive understanding and experience of steepness in the *real* world was wrong? Data from the audio-recording of S and R’s work and stimulated recall interviews suggest that S drew this puzzling conclusion as a result of some very ‘logical’ deduction:

Account 8

R and S measure the height and slope of staircases on campus.

Back in the classroom, they use Pythagoras to calculate length.

They produce scale drawings - ‘staircase triangles’.

They measure the angles.

The steepest staircase isn’t the one they thought it would be.

You can’t trust your eyes to measure steepness.

Do it by measuring in future!

Ironically, R and S had no need to use Pythagoras at all but had been so excited in “*finally understanding how to do it*” that they built it into what was otherwise a reasonable algorithmic approach to measuring steepness, believing their calculations would be more accurate if they only had to use two real-life measurements. However, rather than consider that they might have made a calculation error, S instantly gave up her own internal sense of what a reasonable result should look like, trusting to the “*power of mathematics*” and in particular, the power of formulae, over-riding the evidence of her own eyes. R who was much less critically engaged in the process, was happy to concur with S and seemed unconcerned that evidence from calculations totally contradicted his initial observations ‘by eye’.

This episode suggests that for S, the world of formal maths although exciting was still very external to her own internal world. It also suggests something about how she valued different sorts of knowledge – with formal mathematical powers at the top of the hierarchy and her own at the bottom. It took time, considerable peer checking and more experiences of measuring and

testing staircases around the campus before S's mathematical and personal worlds began to reintegrate. R and S may have recovered from this incident but it continues to resonate strongly with previous personal experiences.

If learners override *extra-mathematical* understanding, how can they develop their ability to judge whether their answers are sensible and how often do they leave classes not knowing any more how to do something that made sense to them at the start of the lesson? In the case of R and S, the incident actually provided a sort of dissonance that generated another very fruitful point of departure. However, in a short time-restrained session where curriculum and assessment demands may prevent teachers and learners taking the time to move within this *horizontal* phase of mathematisation to deal constructively with misconceptions and bridge gaps between real and maths worlds, how damaging might this sort of mathematical experience be to learners' self-confidence and self-concept?

While researching this phenomenon further, I found an article in which Meissner (2006) suggests that we have a number of internalised representations or micro-worlds which inform our subjective domains of experience. He identifies a reflective and subjective domain of experience (SDE) and argues that although both are important for flexible thinking one can often be more dominant over another, particularly when a new problem or conflict arises:

The individual prefers to ignore the conflict rather than modify the SDE or adopt another SDE. In mathematics education it is quite natural that an 'analytical-logical' behaviour remains dominant and that conflicting, common-sense experiences or spontaneous ideas get ignored. (Meissner, 2006, p3)

This is an interesting theoretical construct with which to try to understand why R seemed relatively unperturbed by cognitive dissonance, while S was so easily enticed to relinquish her own *common sense experiences*.

At this stage of the enquiry then, my initial disappointment that carefully selected and adapted mathematical tasks had resulted in some teachers dismissing rather than valuing their own intuitive mathematical powers, was tempered by the fact that engagement with these same tasks had generated phenomena that provided insight into another interesting and valuable point of departure related to how we move between and within our formal and informal, real and mathematical worlds.

Summary discussion

The initial focus of this action research project was to improve practice in supporting adult numeracy teachers develop and extend their own mathematical thinking. At each stage of this inductive process, as a participant observer I have collected, reflected on and evaluated data related to teachers' responses to a series of research design tasks. In particular, using audio-recordings to reflect on classroom discourse during collaborative work on mathematical tasks and in oral presentations to peers generated evidence of rich, cyclical and non-linear problem

solving and mathematical thinking processes. It was a real privilege to listen to teachers interacting together with energy, trust, humour, perseverance, intelligence and humanity.

During this enquiry, I hoped to gain insights into a group of adult numeracy teachers' mathematical thinking but learned a great deal more about my own assumptions, beliefs, and expectations. In focusing on the quality of my own interventions and interactions with teachers, I need to recognise that I can be just as *mechanistic* and *instrumental* in supporting work on mathematical tasks as they can be in solving them. I also recognise, value and am more likely to favour mathematical thinking and behaviours which mirror my own formal mathematical experiences and interests and need to be fully conscious of this if I am to further develop my own inclusive practices in supporting teachers to develop mathematical thinking.

Teachers and tutors come to formal mathematics classrooms with *funds of knowledge*, which include diverse and contingent informal numeracy practices which are culturally and socially situated. These often go unrecognised, are not valued or are held subconsciously. Raising awareness of these through systematic reflection can provide valuable insights into hidden personal and interpersonal resources and propensities which can be harnessed or challenged to support teachers' own mathematical thinking and, hopefully, their professional practice.

More *enactive* and *iconic* approaches can open up or close down possible lines of inquiry in unexpected ways. Similarly, tasks which specifically require teachers to take more time in *manipulating* and *getting-a-sense of* the mathematical structures of a problem, though often more time-consuming, are less likely to result in teachers adopting mechanistic or instrumental approaches.

Conclusion

What unfolded during this small-scale practitioner enquiry suggests that doing *realistic* mathematics tasks within a *community of inquiry* can provoke a range of mathematical thinking and learner responses. These allow us to identify ways in which procedural and conceptual thinking can be used within *horizontal* and *vertical mathematisation*, and how learner journeys can be tracked through different stages of problem solving. Such tasks can also provide *meaningful* starting points to teachers with varying levels of prior mathematical experience. However, teacher and tutor beliefs and assumptions about what constitutes mathematical behaviour can support or constrain the intent and ease of movement within and between their *real* and *mathematical* worlds, and vice versa. While teachers with more experience of mathematics could do this flexibly, despite preferences and predispositions to reside in more formal mathematical mental environments, others with less confidence or less well developed *intra-mathematical* knowledge and skills dismissed their own innate *sense-making* and *extra-mathematical* knowledge too readily, with mixed success.

Recommendations for future practice

Adult numeracy and mathematics teacher education courses need to support students to engage regularly in a variety of sustained, open-ended and *realistic* mathematical tasks, with further extended tasks signposted for independent study.

If teachers are to develop greater awareness of what mathematical thinking looks, feels and sounds like, more self and group reflection and evaluation tasks need to take place with explicit reflections on *inner*, *outer* and *meta*- tasks encouraged within personal maths portfolios and group discussions.

New mobile technologies are being used increasingly and naturalistically within sessions: listening to, watching and analysing targeted audio- and video-recordings of engagement in their own mathematical thinking tasks will support teachers to develop *awareness of awareness* further.

The key literature, frameworks and constructs which informed the context and conduct of this enquiry along with the specific mathematical tasks used could be shared and contribute to reading lists used on other adult numeracy teacher education courses.

Throughout this paper, there has been an underlying assumption that developing teachers' confidence, awareness and insight into their own mathematical thinking, will better equip them to develop and extend the mathematical thinking of their learners. Adult numeracy teacher educators need to identify and value further opportunities for students to explicitly evidence and reflect on how they are using their own experiences of thinking and acting mathematically to inform their practice with learners.

References

- Allen, B. and Johnston-Wilder, S. (Eds) (2004). *Mathematics education: exploring the culture of learning*. London: Routledge FALMer.
- Arleback, J. B. (2009). On the use of *realistic* fermi problems for introducing mathematical modelling in school. *The Montana Mathematics Enthusiast*, 6(3), 331- 364. Retrieved from http://www.math.umt.edu/TMME/vol6no3/Arleback_article3_vol6no3_pp331_364.pdf.
- Baker, D. (2005). Numeracy and "funds of knowledge". *Reflect* 3, 16-19. Retrieved from <http://www.nrdc.org.uk/content.asp?CategoryID=755>
- Baker, D. & Rhodes, V. (2007). *Research report. Making use of learners' funds of knowledge for mathematics and numeracy: Improving Teaching and Learning of mathematics and numeracy in adult education*. [online] https://www.ncetm.org.uk/public/files/254456/research_funds_of_knowledge.pdf.
- Barnes, H. (2004). Realistic mathematics education: Eliciting alternative mathematical conceptions of learners. *African Journal of Research in SMT Education*, 8(1), 53-64.
- Bishop, A. J. (1988). *Mathematical enculturation: a cultural perspective on mathematics education*. Dordrecht, D: Reidel Publishing Company.
- Cara, O., Lister, C., Swain, J. and Vorhaus, J. (2010). *The teacher study: The impact of the skills for life strategy on teachers*. London: NRDC.
- Coben, D. (2004). What is specific about research in adult numeracy and mathematics? *Adult and Lifelong Education 10th International Congress on Mathematical Education – www.icme-10.dk – Programme – TSG6*. Retrieved from <http://www.icme-organisers.dk/tsg06/Papers/CobenD.pdf>.

- Cooper, B. & Dunne, M. (2004). Constructing the 'legitimate' goal of a 'realistic' maths item: a comparison of 10-11 and 13-14 year olds, in Allen, B. and Johnston-Wilder, S. (Eds) *Mathematics Education: exploring the culture of learning*. Oxon: Routledge Farmer.
- FitzSimons, G. (2005). Numeracy and Australian workplaces: Findings and implications. *Australian Senior Mathematics Journal*, 19(2), 27–40.
- Freudenthal, H. (1991). *Revisiting mathematics education*. Dordrecht: Kluwer Academic Publishers.
- Gattegno, C. (1988). *The science of education, part 2b: The awareness of mathematization*. New York: Educational Solutions.
- Goos, M., Galbraith, G. & Renshaw, P. (2004). Establishing a community of practice in a secondary mathematics classroom. In Allen, B. & Johnston-Wilder, S. (Eds), *Mathematics Education: exploring the culture of learning* (pp. 91-116). Oxon: Routledge Farmer.
- Harris, M. (2000). Women, mathematics and work., in D. Coben, J. O'Donoghue & G.E. FitzSimons (Eds) *Perspectives on Adults Learning Mathematics: Research and Practice* (pp. 171–90). Dordrecht: Kluwer Academic Publishers.
- Lave, J. (1988). *Cognition in practice*. Cambridge: Cambridge University Press.
- Martin, F. & Booth, S. (1997). *Learning and awareness*. New Jersey: Erlbaum.
- Mason, J. (2002). *Researching your own practice: The discipline of noticing*. London: Routledge FALMer.
- Mason, J., Burton, L. & Stacey, K. (1985). *Thinking mathematically*. Harlow: Pearson Education Limited.
- Mason, J. & Johnston-Wilder, S. (Eds) (2005). *Fundamental constructs in mathematics education*. London: Routledge FALMer.
- Mason, J. & Johnston-Wilder, S. (2006). *Designing and using mathematical tasks*. St Albans: Tarquin Publications and OU.
- Meissner, H. (2005). Creativity and mathematics education – challenges to provoke creativity. *Proceedings of the 3rd East Asia Regional Conference on Mathematics Education (EARCOME 3)*, Shanghai. Retrieved from <http://math.ecnu.edu.cn/earcome3/sym1/sym104.pdf>.
- Moll, L., Amanti, C., Neff, D. & González, N. (1992). Funds of knowledge for teaching: using a qualitative approach to connect homes and classrooms. *Theory into Practice*, 31, 132-141.
- Nunes, T., Schliemann, A. & Carraher, D. (1993a). *Street mathematics and school mathematics*. Cambridge: Cambridge University Press.
- Nunes, T., Light, P., & Mason, J. (1993b). Tools for thought: The measurement of length and area. *Learning and Instruction*, 3(1), 39–54.
- Resnick, L.B. (1987). *Learning in school and out*. AERA Presidential Address. Washington, DC, April.
- Ridgeway, J. & Swan, M. (2012). *Classroom assessment techniques mathematical thinking*. Retrieved from <http://www.flaguide.org/cat/math/estimation/estimation7.php>.
- Schoenfeld, A.H. (1994). *Mathematical thinking and problem solving*. New Jersey: Lawrence Erlbaum Associates.
- Street, B. (1984). *Literacy in theory and practice*. Cambridge: Cambridge University Press.
- Street, B., Baker, D. & Tomlin, A. (2005). *Navigating numeracies: Home/school numeracy practices*. Dordrecht: Kluwer Academic Publishers.
- Swain, J., Baker, E., Holder, D., Newmarch, B. & Coben, D. (2005). 'Beyond the daily application': making numeracy teaching meaningful to adult learners. London: NRDC.
- Swain, J., Newmarch, B. & Gormley, O. (2007). *Numeracy. Developing adult teaching and learning: Practitioner Guides*. Leicester: NIACE.
- Swan, M. (2006). *Collaborative learning in mathematics: a challenge to our beliefs and practices*. London: NRDC.
- Swan, M. & Swain, J. (2007). *Thinking through mathematics research report*. London: NRDC.
- Tahta, D. (1980). About geometry. *For the learning of mathematics*. 1(1), 2-9.
- Treffers, A. (1987). *Three dimensions: A model of goal and theory description in mathematics instruction- The Wiskobas Project*. Dordrecht: Reidel.

Critical Issues in Adult Numeracy Practice—Contradictions and Strategies

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Abstract

This paper discusses the critical situations I have been asked to ‘improve’ by providing professional development for teams of adult numeracy and functional mathematics teachers in the post-16 sector in London. These situations have not been identified through any research process, but arise from internal management reviews of course outcomes and staff development provision. The assessment by the institution’s management of these situations is often very different from that of the teaching staff. And my view as a teacher trainer is probably different again. The main focus of my intervention is to suggest changes to planning and teaching strategies. However, organisational structures have also to be considered. The author argues that three significant theories, ‘multiple intelligences’, ‘a profound understanding of fundamental mathematics’ and ‘how the mind creates mathematics’ provided guidance for the reflection of practice. The approach taken is supported by the Open University’s guide to action research.

Key words: numeracy, mathematics skills, adult mathematics learning, critical issues, strategies.

Introduction

This paper provides a review of a series of interventions into adult numeracy teaching in London, United Kingdom (UK) over a two-year period from 2011 to 2013. The interventions were made at the request of Further Education Colleges to improve the standard of teaching. With reflection, the concerns of the local management have been identified as critical issues in the teaching of numeracy to adults. Similar issues were identified in a number of Colleges and contradictions between the teaching aims and methods were also identified. To help improve the outcomes some activities were suggested. These have since been reviewed and can now be examined as a set of strategies to improve teaching and learning. This paper recounts this journey from support for professional staff to a set of key theories that underpin innovative interventions in practice.

Though I aim to analyse a range of research sources that are relevant to this journey, I will follow a narrative that is founded on the experiences of giving support both in structured sessions and during teaching practice observations. This set of experiences was not designed at the time as a research project, but do now form the basis for a, retrospective, critical review of strategies for improving adults learning mathematics.

Critical Situations

The critical situations that form the teaching practice, core to this analysis, arose out of the formal provision of professional development to improve teaching and learning standards. Over a number of years the UK government funded support to educational and training institutions on a national basis. Such support reflected various formats, and included partner institutions supporting each other, banks of on-line resources or the provision of specialist trainers. There was, for a short period, a particular focus on adult numeracy, and it was this situation that provided the opportunity for staff development visits to be made. See, for example, the pages on “Whole Organisation Approach to literacy, language and numeracy (LLN) Framework” on the Excellence Gateway site for Supporting Skills and Improving Practice.

In retrospect, I identified the following as the main situations that concerned the institutional management about their numeracy teaching, and for which they requested some specialist help:

- Working with students on vocational courses
- Working with ESOL students
- Raising students’ level from Level 1 to Level 2
- Preparing for functional mathematics assessments
- Making the numeracy class more interesting

Let us look at these in a little more detail.

Working with students on vocational courses comes out of a long history of adult numeracy and mathematics being seen as one of the basic skills that underpin success in ALMost all vocational education and training. Those familiar with the policy issues in this field in the UK since 2000 might be aware of the debates that have developed over the issues of integration, embedding and context. (See for example the NRDC report on embedding literacy, language and numeracy [Casey, H. et al. 2006]). The iColleges were concerned about attendance and outcomes on numeracy and mathematics support classes.

Teaching ESOL students is a particularly large part of the work of adult numeracy practitioners working in London. The expression, ESOL, a contraction for “English for speakers of other languages” is used as shorthand for students who do not have English as their first language, whether or not they are attending language classes. Many numeracy teachers work with classes that are largely or entirely comprised of ‘ESOL’ students and so institutions are concerned with how best to serve this cohort.

Raising students' level from Level 1 to Level 2 is with reference to the levels of the Adult Numeracy Core Curriculum (ANCC) and to the more recently introduced Functional Mathematics. The content of these levels can be explored on the Excellence Gateway site for Skills for Life Core Curriculum, particularly in the “numeracy progression overview” document⁶⁹. For some teachers and curriculum managers the change of content from Level 1 to Level 2 is seen as a much larger challenge than movement between other levels, when planning teaching and learning.

Preparing for functional mathematics assessments is particular to the English situation, as this was a new form of assessment for the sector introduced in pilot form in 2007. It has, only quite recently, become the main form of assessment for adult students. It is a very different form of assessment compared to the national tests that were used previously. The national tests were multiple-choice questions, whereas functional mathematics aims to measure process skills and requires more writing and explanation. The mathematical content however, is very similar.

Making the numeracy class more interesting is a very broad category that in practice covers issues in which the curriculum managers considered that the mathematics teaching was too traditional, and the teaching staff were not open to new approaches. See for example, the approaches associated with collaborative learning, such as described by Malcolm Swan (Swan, 2007]).

The practice for this ‘reflection-on-action’ [Schön quoted in OU (2005) p24] comprised staff development sessions devised by the author. These took place in colleges of further education and local adult education services in the London (UK) area. The courses at these institutions were for students aged 16 or above. However for organisational and funding purposes the courses are usually organised separately for young adults, aged 16 to 19 and adult classes for those aged 20 and above. The courses can generally be classified under three headings: vocational, functional mathematics and English language (ESOL). The teachers attending the staff development sessions included mathematics and numeracy specialists, support teachers (for literacy and numeracy) and specialist vocational teachers.

Contradiction and Strategy 1—Order of Numeracy Topics

The impact of reports about poor numeracy, particularly the Moser report (DfEE 1999), led to the publication of the Adult Numeracy Core Curriculum (ANCC) in 2001 (Basic Skills Agency). There remains considerable dispute whether this document is properly described as a curriculum, despite its name. However, what is certain is that it set out a list of topics divided into sections, sub-sections and curriculum elements. These curriculum elements were presented across three levels: Entry Level, Level 1 and Level 2. The Entry Level was itself sub-divided into three, Entry 1, 2 and 3.

The three sections of the ANCC are number, measure, shape & space and handling data. These were based on a model established by the National Curriculum for primary school

mathematics. An example of an element from the number section at Entry 3 is “add and subtract using three-digit whole numbers”. An example of an element from the measure shape & space section at Level 1 is “work out the perimeter of simple shapes”. The Core Curriculum is now available as an on-line document, however, for about 5 years the printed document was the only version available and the order of the elements tended to be followed as a syllabus by many numeracy teachers. This close adherence to the printed document was further compounded by the common practice in many institutions to encourage (at the very least) numeracy teachers to identify the numeracy elements covered in their lesson plan by their distinct element reference number. The ANCC itself encourages this.

The curriculum elements must be clear and used with learners. The aim must be that learners develop the concepts and the language that will help them make sense of their learning and go on doing it. Evidence shows that the inclusion of explicit curriculum targets in learning programmes has resulted in a clearer identification of outcomes by learners, and in better attendance and progression by learners (BSA, 2001, p. 8).

As the core curriculum has been used as a syllabus, schemes of work begin with the four arithmetic operations, proceed through whole numbers and then decimals and fractions and percentages. Here is the contradiction. All of these teachers are aware that it is considered good practice to take the students experience into account and place calculating techniques into context. Yet the part of the curriculum most removed from any context is introduced first and can easily take up half of the course time. The ANCC itself emphasises the need to take into account the students’ past experiences.

The skills and knowledge elements in the adult numeracy core curriculum are generic. They are the basic building blocks that everyone needs in order to use numeracy skills effectively in everyday life. What is different is how adults use these skills and the widely differing past experience that they bring to their learning. This is the context that the learner provides . . . (BSA, 2001, p.8)

The strategy I propose is simple and straightforward, yet experience has shown it is frequently condemned and rejected. The ANCC is divided into three sections, Number, Measure, shape & space and Handling data. That strategy is simply to start the course in a section other than Number - to begin the course with some aspect of measuring or collecting data. There are three main advantages to be gained from this strategy.

1. It avoids presenting adult students with the mathematical techniques, such as mental calculations, that they probably find the most difficult, if not impossible, right at the start of the course. This is often countered with the argument that they ‘have to know how to multiply – to know their tables’. Perhaps they do. However, I question whether this traditional approach can work with most adult students. The students are in the adult numeracy class because they have not achieved previously. If they have completed secondary school they will have been shown the techniques for multiplication at least 10 times; if they have already had additional help in school and

attended other post-16 classes this is probably closer to 15 times. Why should this occasion be any different?

2. Measure shape & space and handling data provide ‘in-built’ context for working with numbers. Something must always be measured or a shape must be a shape of something, and have dimensions. If data is being collected it must be about something. Starting a programme of study with topics drawn from these sections provides the opportunity for the examples used to be relevant to the students’ lives or the other courses they are studying. All of the calculating techniques from the ‘Number’ section occur when manipulating problems within these topics. Over time, appropriate support can be developed where it is necessary.
3. By starting with measuring or data not only are we avoiding starting with topics that are likely to be the most challenging for students – there is the opportunity to start with topics that the students are more familiar with. Very often the students themselves do not identify what they can do as mathematics at all (Colwell, 1997). For example, a student may have poor multiplication skills, and therefore have considerable difficulty in converting measurements. However they may have excellent estimating skills, demonstrating a thorough understanding of measurement, but the students may well consider this ‘just common sense’.

Contradiction and Strategy 2—Numeracy for Speakers of Other Languages

The problem as it is posed is ‘what we have to do to teach mathematics to the students who do not have English as their first language?’ In discussing this further with the teachers concerned, there appears to be a contradiction between what the teachers think they should be teaching and what the students need to learn. For teachers in stand-alone adult numeracy classes, this problem has been compounded by the recent introduction of Functional Mathematics. As was described above the new Functional Mathematics assessments require more writing to explain why a particular solution has been chosen. Part of the strategy here is to know about and understand the background of these students. The term ‘ESOL’ is used to refer to a very wide range of students. Many of the students will have lived in Britain for a comparatively short period of time, and therefore their schooling or education would have taken place elsewhere. In many institutions ESOL students are placed in classes according to the level they have been assessed at in English, and these are often at Entry Level. In the mathematics class the teaching is likely to begin with calculating methods, as discussed above. This may well be totally unnecessary and even cause confusion.

The students may well (currently) have a low level of English, but that does not mean that they cannot calculate; they may well have a good knowledge of mathematics. If a student has completed their secondary education in another country they are likely to be fully competent in their calculating skills. They may well be proficient using other methods, and this is where confusion can occur. If a different way to calculate is demonstrated, they may well think that they are doing something wrong using the one they have been taught previously in school. Given they may have a low level of English it will be difficult to discuss this, and so care needs to be taken to ensure previously acquired skills are recognised and supported. It is

important to recognise that the skill some students need to learn is the language of mathematics. There is quite a complex relationship between the language in which mathematics was learnt and the current learning medium of English. Dhamma Colwell (1997) gives examples of the processes people experience as they move from one language to another in their mathematical practices. For example M changed from Cantonese speaking school to an English-speaking one at the age of eleven. She found that maths was the only subject that she could understand easily, because the symbols used were the same in both languages (Colwell, 1997 p67)

In this example skills in mathematics are compensating for the lack of skills in English, by depending on familiar symbols.

The other part of the strategy is to ensure that connections are repeatedly made between the mathematical items, saying and writing the words that describe it, and the symbols used to represent it. An example of this is 'ratio'. This is the mathematical item. It is written as '3:2' and said as "three to two". The concept of manipulating quantities in ratio may well be understood, but to discuss it and ask questions it is necessary to have the written and spoken language of '3:2' and "three to two".

This can be represented by an image using the concept of a number.

Symbol

Word

Concept

Figure 1. Representations for number three

Explanations given in a numeracy or mathematics lesson usually use all three - representing the concept in some form, saying or writing in words a definition or explanation and presenting the concept in symbolic form. These are very often not presented at the same time and moving from one to the other, with the intention to explain more clearly, can cause confusion.

Contradiction and Strategy 3—Numbers with and without Context

The need to consider the context is particularly relevant to working with students on vocational courses. The pressure on institutions to ensure students have the mathematical skills to achieve their primary learning goal on a vocational course has long been an issue. For example Gail FitzSimmons discusses this in the Australian context in the late 1990s (FitzSimmons 1997). It is still a very live issue. At the time of writing, August 2013, the UK Government has just announced new measures for 17 year olds to continue to learn English and mathematics. Professor Alison Wolf, who headed a government review of vocational qualifications, described continuing in the two subjects as the most important

recommendation of her inquiry. “Good English and maths grades are fundamental to young people’s employment and education prospects,” she added. “Individuals with very low literacy and numeracy are severely disadvantaged in the labour market.” (Wolf, 2013)

The contradiction is that mathematics can be presented differently in a vocational class to how it is introduced in the mathematics support class. An example of this was observed during an LSIS support session (see LSIS Support Programme – Barking and Dagenham College). In a painting and decorating class the students had to make a six-colour wheel on the doors they were decorating. Under the instruction of the painting and decorating teacher the students drew a circle. They then marked out the length of the radius of the circle six times around the circumference of the circle and drew lines from these marks to the centre. They completed the activity by painting in the three primary colours of red, yellow and blue, and by overlapping creating the three secondary colours of green, orange and purple (or violet). The mathematical solution to this problem would involve considering that a circle can be divided into 360 degrees and that to divide the circle into six equal parts then requires the calculation of 360 divided by six. To complete the practical task angles of 60° would then need to be measured or constructed.

This situation raises many more questions about the purpose of certain problems, and the reasons given for doing certain calculations. However, for the purposes here the strategy to be noted is that practical solutions are used in vocational classes that are different from those a mathematics teacher is likely to use. If this is not taken into account the numeracy / mathematics support classes are likely to be seen as irrelevant. The recognition of different sorts of mathematics in vocational and cultural contexts has been developed far more deeply, practically, pedagogically and theoretically under the heading of ‘ethnomathematics’, particularly in South America. (See for example Knijnik’s (2007) study of the mathematical practices in the Brazilian Landless Movement).

Theoretical Inspiration

The contradictions and strategies I have been discussing arose out of my own practice in teaching adult numeracy, in teacher education and in professional development. This practice was informed by reflection on the feedback received from teachers and discussions with colleagues and also on a whole body of theories and research on teaching adults mathematics. In reflecting on my own practice, , I realised that I was concerned that such reflection and evaluation should lead to change This was associated with certain approaches to action research, such as that described in the Open University guide for action research:

The second approach has other attractions. As noted, it draws upon Schön’s (1983; 1987) ideas of ‘the reflective practitioner’ and ‘reflection-on-action’: the active and critical consideration and reflection by us, as practitioners, on such aspects as the motives behind and the consequences of our professional practice. This is achieved through a process of action-

reflection-action and is what permits us as teachers to analyse our practice, both for ourselves and for others, and thus to change and develop. (OU, 2005, p.24)

The next section provides a summary of my thoughts about the contradictions and strategies in teaching adults under three headings:

- Different ways of thinking about a problem . . . and solutions
- A deeper understanding of how people calculate
- Considering how the brain manipulates numbers

These ideas have been inspired by the work of three very different researchers, whose work has helped to explain the contradictions and inspire the new strategies. The first is the theory of ‘Multiple intelligences’. Howard Gardner first published this in 1983 in *Frames of Mind*. Since then he has updated the theory by taking into account how others have used this theory and adding one more intelligence to the original seven (Gardner, 2006). Gardner’s theory, in its current form, identifies eight different sorts of intelligences. Two of these are linguistic and logical-mathematical, and in his debate with the psychometricians (those who work with intelligence tests) he argued that the traditional tests primarily measured these two only.

The other intelligences that Gardner describes are musical, spatial, bodily kinaesthetic, interpersonal, intrapersonal and naturalist. What I found particularly helpful from this theory is that it provides a theoretical basis for recognising that people can be poor at some tasks but very good at others. Particularly they may have poor mathematical skills, or mathematics approached in a particular way, but have many other talents. If that is the case, then these talents can be used to build their numeracy experience, rather than continuing to focus most on the parts of mathematics they cannot do.

The second source is that of the researcher Liping Ma, in her study *Knowing and Teaching Elementary Mathematics* (Ma, 1999). The main focus of this study is to compare the mathematical knowledge and teaching practices of teachers trained in the USA and China. The examples she focuses on are very instructive, such as looking at how teachers understand the rules for dividing a fraction by a fraction. However, what I found particularly instructive was the section entitled: ‘Profound Understanding Of Fundamental Mathematics’ (Ma, 1999, pp118).

What this provides is an argument for having a deep understanding of the concepts that underpin the processes involved in basic calculating. This, once again, provides support for the development of alternative strategies. With this ‘profound understanding of fundamental mathematics,’ a teacher would be easily able to adapt a calculating process to suit a particular student, and would have the personal skills to evaluate a different method used by a student. Without such understanding, the teacher is left with only the method they have learnt, which they may be able to perform by rote, but cannot be explain or deviate from.

The third source is the work of Stanislas Dehaene (1999). His ideas are summarised in the book, *The Number Sense* which is sub-titled ‘how the mind creates mathematics’. There are

three things that give me inspiration from this book. The first is the introduction the author presents to the neuroscientific approach to understanding mathematics. He introduces the reader to studies of the brain, which show where, and possibly how, numbers and quantities are manipulated. Much of the work of the neuroscientists Dahanne showcased has been to work with patients who have lost specific number skills, after an illness or an accident, and identify which parts of the brain have been damaged. The second is the introduction to the concept of ‘subitizing’. This is described as a particular ability that enables one, two or three (and possibly four) objects to be recognised and distinguished without one to one counting. It is used to show there is a number sense in very young babies and animals and to support arguments for some aspects of understanding numbers as being innate.

The third is introducing the term ‘numerosity’. This is the attribute of a group of things that gives it countable quantity. It is recognising amounts. This I found useful in discussing at a fundamental level what we mean when we talk about ‘a number’ or ‘numbers’. The word ‘numbers’ has so many meanings that having a specific word which refers to the concept of ‘amounts’ rather than how a number is written or said can help clarify thinking and from that, how number concepts are explained and demonstrated.

Finally there is one more source that needs to be noted. I have spoken briefly about the importance of collaborative work with colleagues. Over recent years my reflections and self-evaluation of staff development initiatives have been supported by discussions and joint work with my colleagues and by the initiatives in teacher training for adult numeracy specialists. This body of knowledge and experience can be found summarised in ‘Teaching in Adult Numeracy’ (Griffiths & Stone, 2013).

Conclusion

In this paper the experience of working with a wide range of adult numeracy professionals is reflected upon in order to identify the key changes to teaching strategies that were being promoted. The key changes to teaching strategies are recognised as being underpinned by three diverse theories: Howard Gardner’s ‘multiple intelligences’, (2006); Liping Ma’s ‘a profound understanding of fundamental mathematics’ (1999) and Stanislas Dehaene’s ‘how the mind creates mathematics’ (1999). The process has been seen to have similarities with the reflective practices associated with action research.

References

- Adult Numeracy Core Curriculum on the Excellence Gateway. Retrieved from:
<http://www.excellencegateway.org.uk/node/1514>.
 Basic Skills Agency ([BSA], 2001). *Adult numeracy core curriculum*. London: Cambridge Training and Development on behalf of the Basic Skills Agency.
 BIS (2011). *Skills for life survey: Headline findings*. BIS Research paper number 57. London: Department for Business, Innovation and Skills.
 Casey, H., Cara, O., Eldred, J., Grief, S., Hodge, R., Ivani,c R., Jupp, T., Lopez, D. & McNeil, B. (2006). *You wouldn't expect a maths teacher to teach plastering.... Embedding literacy, language and numeracy in*

- post-16 vocational programmes – the impact on learning and achievement*. Summary Report London National Research and Development Centre for adult literacy and numeracy.
- Coates, M. (2005). *Action research: a guide for associate lecturers*. Milton Keynes Open University Press.
- Colwell, D. (1997). Adults' experiences of learning and using maths in a second language in Coben, D. (comp) *Adults Learning Mathematics – 3*. Proceedings of ALM3 the Third International Conference of Adults Learning Maths - A Research Forum. London: Goldsmiths College, University of London in association with ALM.
- Dehaene, S. (1999). *The Number sense – how the mind creates mathematics*. London: Penguin Books.
- DfEE (1999). *A fresh start: Improving literacy and numeracy*. The report of the Working Group chaired by Sir Claus Moser. London Department for Education and Employment.
- FitzSimmons, G. E. (1997). Mathematics in the vocational education and training sector: the professional development implications for teachers. In Coben, D. (comp) *Adults Learning Mathematics – 3*. Proceedings of ALM3 the Third International Conference of Adults Learning Maths - A Research Forum. London: Goldsmiths College, University of London in association with ALM.
- Gardner, H. (2006). *Multiple Intelligences – New Horizons*. New York: Basic Books.
- Griffiths, G. & Stone, R. (Eds.) (2013). *Teaching Adult Numeracy – principles and practice*. Maidenhead: Open University Press.
- Jorgensen, M. (2014). Using mathematics autobiographies to help students identify critical moments important to their success. Proceedings of the 20th International Conference of Adults Learning Mathematics (ALM), 200.
- Knjnik, G. (2007). Cultural differences, adult education and mathematics in Seabright, V. & Seabright, I. (Eds) *Crossing Borders – Research, Reflection, & Practice in Adults Learning Mathematics*. Proceedings of the 13th International Conference of Adults Learning Mathematics – A Research Forum. Dublin. ALM with CAMET (Centre for Advancement of Mathematics Education in Technology) & Institute of Technology Tallaght Dublin.
- Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, New Jersey: Lawrence Erlbaum Associates.
- Swain, J. Newmarch, B. & Gormley, O. (2007). *Numeracy – Developing Adult Teaching and learning: practitioners guides*. NRDC / NIACE Leicester (Crown copyright)
- Swan, M. (2006). *Collaborative learning in mathematics: a challenge to our beliefs and practices* London: NRDC / NIACE.
- Wolf, A (2103). The Independent (online). Retrieved from <http://www.independent.co.uk/news/education/education-news/pupils-who-fail-gcse-english-or-maths-will-be-forced-to-stay-on-at-school-and-resit-their-exams-8793614.html>.

Resources from the Excellence Gateway

- Functional mathematics: Standards: <http://www.excellencegateway.org.uk/node/20517>
- Teaching and learning support material <http://www.excellencegateway.org.uk/node/22188>
- LSIS Support Programme – Barking and Dagenham College Collaboration between functional skills specialists and vocational specialists
- <http://repository.excellencegateway.org.uk/fedora/objects/eg:5390/datastreams/DOC/content> accessed September 2013
- Whole Organisation Approach to literacy, language and numeracy (LLN) Framework<http://www.excellencegateway.org.uk/node/1151> accessed 28th August 2013